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[Continued from May and June issue.]

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A paper presented at the General Meeting of the
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DERIVATION AND DISCUSSION OF THE GENERAL SOLUTION FOR THE CURRENT FLOWING IN A CIRCUIT CONTAINING RESISTANCE, SELF-INDUCTION AND CAPACITY, WITH ANY IMPRESSED ELECTROMOTIVE FORCE.

BY FREDERICK BEDELL, PH. D., AND ALBERT C. CREHORE, PH. D.

The problems arising in the case of circuits containing resistance and self-induction, or resistance and capacity have been treated by various writers and a discussion of such circuits by the authors has recently appeared as a series of articles in the *Electrical World* which form an introduction to the present paper. The treatment of circuits containing all three, resistance, self-induction, and capacity, is not so familiar, and the solutions which have been hitherto given apply only to certain particular cases.

The object of this paper is to treat the problem of a single circuit containing resistance, self-induction, and capacity, in the most general way and to derive a general solution for the current and charge at any time, when there is any impressed E. M. F. acting in the circuit.

All possible cases which may arise can be divided into four divisions according to the nature of the impressed E. M. F. The general solution will, therefore, be discussed in turn for these four cases which arise according to whether, (1) the E. M. F. is suddenly removed; (2) a constant E. M. F. is suddenly inserted; (3) there is an harmonic E. M. F.; or (4) there is any periodic E. M. F. not harmonic. Although many of the particular solutions are familiar, these will be obtained independently from the differential equations, and the results identified with those obtained from the general solution.

Throughout the discussion the following symbols will be used :

R = resistance.
L = coefficient of self-induction (a constant).
C = capacity.
E = (*a*) constant electromotive force ; or
(*b*) maximum value of harmonic electromotive force.
e = instantaneous value of electromotive force.
I = (*a*) constant current ; or
(*b*) maximum value of harmonic current.
i = instantaneous value of current.
Q = constant charge of condenser.
q = instantaneous value of charge of condenser.
$\omega = 2\pi n = 2\pi \times$ frequency = angular velocity.
$+\theta$ = angle of advance.
$-\theta$ = angle of lag.
T = (*a*) period ; or (*b*) time-constant.
$j = \sqrt{-1}$.
ε = base of Naperian logarithms = 2.71828.
c = arbitrary constant of integration.

The meaning of other letters when used will be evident.

The starting point will be the differential equation of energy for a circuit with resistance, self-induction, and capacity. This equation is the mathematical expression of the fact, readily derived from the law of the conservation of energy, that the total energy supplied to a circuit is equal to the sum of the several expenditures of energy.

The equation of energy is :

$$(1) \quad e\,i\,d\,t = R\,i^2\,d\,t + L\,i\,\frac{d\,i}{d\,t}d\,t + \frac{i\,d\,t\int i\,d\,t}{C}.$$

The first member of this differential equation $e\,i\,d\,t$ represents the total energy supplied to the circuit in the time $d\,t$. A part of this energy represented by $R\,i^2\,d\,t$ is used in heating the conductor. A second part $L\,i\,\frac{d\,i}{d\,t}\,dt$ is expended in creating a magnetic field in the space surrounding the conductor. A third part, represented by $\frac{i\,dt\int i\,dt}{C}$, is expended in charging the condenser. Equation (1) is the general differential equation of energy, in terms of the current which flows in the circuit, the E. M. F. which drives the current, and the time, for a circuit containing resistance, self-induction, and capacity in series.

The equation of energy may be expressed as a differential equation in terms of the quantity of electricity in the condenser, or charge of the condenser, the E. M. F., and time, by means of the relation $d\,q = i\,d\,t$, or $q = \int i\,d\,t$. On substituting in (1) $i = \frac{d\,q}{d\,t}$, we have

$$(2) \qquad e\frac{dq}{dt}\,dt = R\left\{\frac{dq}{dt}\right\}^2 d\,t + L\frac{d^2q}{d\,t^2}\,\frac{dq}{d\,t}\,d\,t + \frac{q}{C}\frac{dq}{dt}\,d\,t..$$

Each term in this equation is equal to the corresponding term in equation (1), since it is obtained by direct substitution. The first member, $e\frac{d\,q}{d\,t}\,d\,t$, is the total energy supplied to the circuit, and the three terms of the second member represent the three ways in which this energy is expended, viz., in heat, in creating the field, and in charging the condenser.

If equation (1) is divided through by $i\,d\,t$, it becomes an equation of E. M. F.'s, thus

$$(3) \qquad e = R\,i + L\frac{d\,i}{d\,t} + \frac{\int i\,d\,t}{C}.$$

If equation (2) is divided through by $\frac{dq}{dt}\,d\,t$, it likewise becomes an equation of E. M. F.'s, thus

$$(4) \qquad e = R\frac{d\,q}{d\,t} + L\frac{d^2\,q}{d\,t^2} + \frac{q}{C}.$$

These are equations of E. M. F.'s, equation (3) in terms of current, E. M. F., and time; equation (4) in terms of the charge of the condenser, E. M. F., and time. Each term in (4) is equal to the corresponding term in (3). The first member e is the E. M. F. impressed upon the circuit. That part of e necessary to overcome the resistance is $R\,i$, or $R\frac{d\,q}{d\,t}$. That part of e necessary to overcome the counter E. M. F. of self-induction is $L\dfrac{d\,i}{d\,t}$ or $L\,\dfrac{d^2q}{d\,t^2}$. The third part of e necessary to overcome the counter E. M. F. of the condenser is $\dfrac{\int i\,d\,t}{C}$, or $\dfrac{q}{C}$.

These differential equations may be written in forms more convenient for solving. Differentiating equation (3) with regard to t, to free it from the integral sign, we obtain

$$\frac{d^2 i}{d\,t^2} + \frac{R}{L}\,\frac{d\,i}{d\,t} + \frac{i}{L\,C} = \frac{1}{L}\,\frac{d\,e}{d\,t}. \tag{5}$$

By transposition (4) becomes

$$\frac{d^2 q}{d\,t^2} + \frac{R}{L}\,\frac{d\,q}{d\,t} + \frac{q}{L\,C} = \frac{e}{L}. \tag{6}$$

We know that the impressed E. M. F. has one value at one particular time and is therefore a single-valued function of the time, that is, $e = f(t)$. When we introduce this relation into (5) and (6), the general solution of each of these equations may be readily obtained. The solution of equation (5) will give the value of the current at any time, and the solution of equation (6) will give the value of the charge of the condenser at any time.

If $e = f(t)$, and $\dfrac{d\,e}{d\,t} = f'(t)$, upon substitution in equations (5) and (6), we have

$$\frac{d^2 i}{d\,t^2} + \frac{R}{L}\,\frac{d\,i}{d\,t} + \frac{i}{L\,C} = \frac{1}{L}\,f'(t). \tag{7}$$

$$\frac{d^2 q}{d\,t^2} + \frac{R}{L}\,\frac{d\,q}{d\,t} + \frac{q}{L\,C} = \frac{1}{L}\,f(t). \tag{8}$$

General Solution for Current at any Time.

In solving equation (7) to obtain the value of the current at any time, it is convenient to make use of the symbolic method for linear equations. (See page 101, Johnson's Differential Equations.)

$$\text{Let } D = \frac{d\,[\;]}{d\,t}. \qquad D^2 = \frac{d^2\,[\;]}{d\,t^2}.$$

Writing (7) in symbolic form, we have

$$\left\{ D^2 + \frac{R}{L} D + \frac{1}{L\,C} \right\} i = \frac{1}{L} f'(t), \quad \text{or}$$

$$(9) \qquad i = \frac{1}{L \left\{ D^2 + \frac{R}{L} D + \frac{1}{L\,C} \right\}} f'(t) .$$

Resolving the inverse operator $\dfrac{1}{D^2 + \frac{R}{L} D + \frac{1}{L\,C}}$ into partial fractions, we have the identical equation,

$$(10) \qquad \frac{1}{D^2 + \frac{R}{L} D + \frac{1}{L\,C}} = \frac{L\,C}{\sqrt{R^2\,C^2 - 4\,L\,C}}$$

$$\left\{ \frac{1}{D + \frac{R\,C - \sqrt{R^2C^2 - 4\,L\,C}}{2\,L\,C}} - \frac{1}{D + \frac{R\,C + \sqrt{R^2\,C^2 - 4\,L\,C}}{2\,L\,C}} \right\}.$$

$$(11) \qquad \text{Let } T_1 = \frac{2\,L\,C}{R\,C - \sqrt{R^2\,C^2 - L\,C}} .$$

$$\text{and } T_2 = \frac{2\,L\,C}{R\,C + \sqrt{R^2\,C^2 - 4\,L\,C}} .$$

Placing these values in (10), and substituting (10) in (9), we obtain

$$(12) \; i = \frac{C}{\sqrt{R^2C^2 - 4\,L\,C}} \left\{ \frac{1}{D + \frac{1}{T_1}} f'(t) - \frac{1}{D + \frac{1}{T_2}} f'(t) \right\}.$$

The linear equation of the first order, $\dfrac{d\,y}{d\,x} + a\,y = f(x)$, when written in the symbolic form is $(D + a)\,y = f(x)$ or

$$(13) \qquad y = \frac{1}{D + a} f(x) .$$

The solution of this linear equation of the first order is known to be (see Johnson's Diff. Equations, page 31)

$$(14) \qquad y = \varepsilon^{-a\,x} \int \varepsilon^{a\,x} f(x)\,d\,x + c\,\varepsilon^{-a\,x} .$$

Here c is the arbitrary constant of integration, and none other must be added when the integration is performed.

By (13) and (14), we have

$$\frac{1}{D+a} f(x) = \varepsilon^{-ax} \int \varepsilon^{ax} f(x)\, dx + c\, \varepsilon^{-ax}. \tag{15}$$

If we replace a, in this general formula, by the constant $\frac{1}{T_1}$, and $f(x)$ by $f'(t)$, we have

$$\frac{1}{D+\frac{1}{T_1}} f'(t) = \varepsilon^{-\frac{t}{T_1}} \int \varepsilon^{\frac{t}{T_1}} f'(t)\, dt + c\, \varepsilon^{-\frac{t}{T_1}}. \tag{16}$$

But this is the value of the first term in the parenthesis of equation (12). The value of the second term in that parenthesis may be found in the same manner, and (12) may be finally written

$$i = \frac{C}{\sqrt{R^2 C^2 - 4 L C}} \left\{ \varepsilon^{-\frac{t}{T_1}} \int \varepsilon^{\frac{t}{T_1}} f'(t)\, dt - \varepsilon^{-\frac{t}{T_2}} \int \varepsilon^{\frac{t}{T_2}} f'(t)\, dt \right\} + c_1\, \varepsilon^{-\frac{t}{T_1}} + c_2\, \varepsilon^{-\frac{t}{T_2}}. \tag{17}$$

This is the general solution of equation (7) and gives the current which flows at any time in a circuit having resistance, self-induction and capacity.

Since the differential equation (8) for the charge becomes identical with the differential equation (7) for the current when we write $f'(t)$ instead of $f(t)$, and since f denotes any arbitrary single-valued function whatever, we may in the general solution (17) suppress the accents on the arbitrary functions and write the solution for q instead of i. Thus

$$q = \frac{C}{\sqrt{R^2 C^2 - 4 L C}} \left\{ \varepsilon^{-\frac{t}{T_1}} \int \varepsilon^{\frac{t}{T_1}} f(t)\, dt - \varepsilon^{-\frac{t}{T_2}} \int \varepsilon^{\frac{t}{T_2}} f(t)\, dt \right\} + c_1\, \varepsilon^{-\frac{t}{T_1}} + c_2\, \varepsilon^{-\frac{t}{T_2}}. \tag{18}$$

PARTICULAR ELECTROMOTIVE FORCES.

These equations, (17) and (18), express the values of the current and charge at any time, when the impressed E. M. F. is anything whatever, since f is any arbitrary single-valued function whatever.

There are four cases, covering all possible cases, which arise according to the nature of the impressed E. M. F. These are

Case I. $e = f(t) = 0$.
Case II. $e = f(t) = E =$ constant.
Case III. $e = f(t) = E \sin \omega t$.
Case IV. $e = f(t) = \sum_{E,b,\theta} E \sin (b \omega t + \theta)$.

The meaning of the first assumption is that the impressed E. M. F. is to be zero at every point of time. This condition is fulfilled if we charge a condenser with some quantity Q and then suddenly remove the impressed E. M. F. The impressed E. M. F. remains zero at every point of time after the removal of the source of E. M. F. and consequently satisfies the condition $e = f(t) = 0$. The solutions of the differential equations under this assumption give the current at any time flowing in the circuit and the charge at any time remaining in the condenser, when an impressed E. M. F. is suddenly removed from the circuit. It may be any circuit whatever containing any combination of resistance, self-induction and capacity, that is, a circuit containing R and L alone, R and C alone, or R, L and C together. In case the circuit has R and C, or R, L and C, the solutions will give the current i and quantity q at any time during the *discharge* of the condenser. If the circuit contains R and L alone, the solution will give the current at any time as it dies away after the removal of the E. M. F.

When we assume $e = f(t) = E =$ a constant, we mean that the E. M. F. is to be equal to E at every point of time. This condition will be fulfilled if the source of E. M. F. in any circuit is suddenly changed from one constant value to another constant value, either of which may be zero. If the circuit contains R and C, or R, L and C, the solutions give the current flowing in the conductor and the charge of the condenser at any time after the change in the E. M. F. If the circuit contains R and L only, the solution gives the value of the current at any time as it changes to its final steady value.

The third assumption, $e = E \sin \omega t$, means that the circuit contains an impressed E. M. F. varying harmonically with the time. The solutions of the general equations for q and i show that when the impressed E. M. F. is harmonic, both the current and the charge are likewise simple sine-functions of the time, having the same period as the E. M. F.

The fourth assumption, $e = \sum\limits_{E,b,\theta} E \sin (b \omega t + \theta)$ — where b takes in succession any integer values—means that the circuit contains an impressed E. M. F. which is any periodic function of the time whatsoever.

The solution and discussion of these four cases will be considered in the above order.

CASE I.—VALUE OF CURRENT AND CHARGE AT ANY TIME DURING DISCHARGE.

Here the E. M. F. is assumed to be zero at every point of time, and the equation expressing this is, as above,

$$e = f(t) = 0.$$

If the value $f'(t) = 0$ is substituted in the general equation (17) for current, and if the value $f(t) = 0$ is substituted in equation (18) for charge, we have

$$i = c_1 \varepsilon^{-\frac{t}{T_1}} + c_2 \varepsilon^{-\frac{t}{T_2}}. \tag{19}$$

$$q = c_3 \varepsilon^{-\frac{t}{T_1}} + c_4 \varepsilon^{-\frac{t}{T_2}}. \tag{20}$$

Had the value $e = 0 = f(t)$ been substituted in the differential equation (8), and $f'(t) = 0$ in equation (7), we should have had

$$\frac{d^2 i}{d t^2} + \frac{R}{L} \frac{d i}{d t} + \frac{i}{L C} = 0. \tag{21}$$

$$\frac{d^2 q}{d t^2} + \frac{R}{L} \frac{d q}{d t} + \frac{q}{L C} = 0. \tag{22}$$

It is to be noticed that the form of the differential equation for i is identical with that for q. Hence their integrals (19) and (20) have the same form, although with different arbitrary constants of integration. The solutions of the differential equations

(21) and (22)—which are identical with (7) and (8) when their second members are zero—give what is called the "complementary function." The complementary function contains all the arbitrary constants of integration. The sum of the particular integral—found to satisfy equations (7) or (8) when the second member is not zero—and the complementary function gives the complete integral of the general differential equations (7) or (8).

The particular case of the *discharge* of a condenser through a circuit possessing resistance and self-induction, has been fully discussed by Sir Wm. Thomson and was published as early as 1853 in the *Philosophical Magazine.* His results were obtained from the solution of equation (22), which was solved directly in the usual manner by assuming $q = \varepsilon^{mt}$. This gave equation (20) as his result, which he showed could be expressed in two different forms, according as T_1 and T_2 are real or imaginary.

Writing equation (19) in full, by replacing the values of T_1 and T_2 given in (11), we have

$$(23)\quad i = c_1\,\varepsilon^{-\frac{RC-\sqrt{R^2C^2-4LC}}{2LC}t} + c_2\,\varepsilon^{-\frac{RC+\sqrt{R^2C^2-4LC}}{2LC}t}.$$

If the value of $R^2\,C$ is greater than $4\,L$, the value of i is real; but if $R^2\,C$ is less than $4\,L$, i apparently assumes an imaginary form. It will be shown, however, that i can by a trigonometric transformation be expressed in a real form when $R^2\,C$ is less than $4\,L$.

When $R^2\,C$ is equal to $4\,L$ and we have the critical case, it is evident that the two terms of equation (23) may be written as one, and thus the two arbitrary constants combine into one. The complete solution, which must contain two arbitrary constants, inasmuch as it is derived from a differential equation of the second order, cannot be readily obtained therefore in this case from equation (23); but it will be directly obtained from the differential equations (21) and (22).

If $R^2\,C$ is less than $4\,L$, after factoring out the common factor $\varepsilon^{-\frac{RC}{2LC}t}$, or $\varepsilon^{-\frac{Rt}{2L}}$, we may write (23) in another form, thus:

$$(24)\quad i = \varepsilon^{-\frac{Rt}{2L}}\left\{c_1\varepsilon^{\frac{j\sqrt{4LC-R^2C^2}}{2LC}t} + c_2\varepsilon^{-\frac{j\sqrt{4LC-R^2C^2}}{2LC}t}\right\}.$$

Here j is used to represent $\sqrt{-1}$. If we write

$$(25)\qquad \theta = \frac{\sqrt{4LC - R^2C^2}}{2LC}\,t,$$

then (24) becomes

$$(26)\qquad i = \varepsilon^{-\frac{Rt}{2L}} \left\{ c_1 \varepsilon^{j\theta} + c_2 \varepsilon^{-j\theta} \right\}.$$

The sine and cosine may be written in exponential form thus:

$$(27)\qquad \sin\theta = \frac{\varepsilon^{j\theta} - \varepsilon^{-j\theta}}{2j}\text{; and } \cos\theta = \frac{\varepsilon^{j\theta} + \varepsilon^{-j\theta}}{2}.$$

Therefore

$$(28)\qquad \cos\theta + j\sin\theta = \varepsilon^{j\theta},$$

$$(29)\qquad \text{and}\quad \cos\theta - j\sin\theta = \varepsilon^{-j\theta}.$$

Multiplying through by c_1 and c_2 respectively, and adding, we have

$$(30)\qquad c_1\varepsilon^{j\theta} + c_2\varepsilon^{-j\theta} = (c_1 + c_2)\cos\theta + (c_1 - c_2)\,j\sin\theta.$$

If c_1 and c_2 are conjugate imaginary quantities, they may be written

$$c_1 = \frac{A + Bj}{2},$$

$$c_2 = \frac{A - Bj}{2},$$

where A and B are both real quantities. Taking the sum and difference

$$c_1 + c_2 = A,$$

$$c_1 - c_2 = Bj,$$

and substituting these values in (30), we have

$$(31)\qquad c_1\varepsilon^{j\theta} + c_2\varepsilon^{-j\theta} = A\cos\theta + B\sin\theta,$$

where c_1 and c_2 are imaginary, while A and B are real quantities. Substituting (31) in (26)

$$(32) \qquad i = \varepsilon^{-\frac{Rt}{2L}} (A \cos \theta + B \sin \theta).$$

By the trigonometric formula

$$(33) \quad A \cos \theta + B \sin \theta = \sqrt{A^2 + B^2} \sin \left(\theta + \tan^{-1} \frac{A}{B} \right) .$$

we may finally write equation (32), after restoring the value of θ from (25), in the form

$$(34) \qquad i = A \, \varepsilon^{-\frac{R\,t}{2\,L}} \sin \left\{ \frac{\sqrt{4\,L\,C - R^2\,C^2}}{2\,L\,C} \; t + \Phi \right\},$$

where A and Φ are the arbitrary constants of integration. Here A is not the same as in equation (32), but stands for $\sqrt{A^2 + B^2}$, and Φ stands for $\tan^{-1} \frac{A}{B}$.

If $R^2\,C$ is equal to $4\,L$, then the differential equations (21) and (22) become

$$(35) \qquad \frac{d^2\,i}{d\,t^2} + \frac{R}{L} \frac{d\,i}{d\,t} + \frac{R^2}{4\,L^2} \; i = 0, \quad \text{and}$$

$$(36) \qquad \frac{d^2\,q}{d\,t^2} + \frac{R}{L} \frac{d\,q}{d\,t} + \frac{R^2}{4\,L^2} \; q = 0 .$$

Upon substituting $i = \varepsilon^{m\,t}$, we have

$$(37) \qquad m^2 + \frac{R}{L} \, m + \frac{R^2}{4\,L^2} = 0,$$

which is seen to be a perfect square as it stands, and consequently the two values of m become equal, and $m = - \frac{R}{2\,L}$. When there are equal roots, the solution is of the form

$$i = c_1 \, \varepsilon^{m\,t} + c_2 \, t \, \varepsilon^{m\,t}$$

(see Johnson's Diff. Equations, page 95) or, replacing m by its value $- \frac{R}{2\,L}$, we have as the complete solution

$$i = c_1 \, \varepsilon^{-\frac{R\,t}{2\,L}} + c_2 \, t \, \varepsilon^{-\frac{R\,t}{2\,L}},$$

$$q = c' \, \varepsilon^{-\frac{R\,t}{2\,L}} + c'' \, t \, \varepsilon^{-\frac{R\,t}{2\,L}} .$$

Returning to equation (21), we may write its solution (19), the complementary function, in three different real forms, according as the value of $R^2 C$ is greater than, less than, or equal to $4 L$. These forms are

When $R^2 C > 4 L$,

$$(38) \qquad i = c_1 \varepsilon^{-\frac{RC-\sqrt{R^2C^2-4LC}}{2LC}t} + c_2 \varepsilon^{-\frac{RC+\sqrt{R^2C^2-4LC}}{2LC}t}.$$

When $R^2 C < 4 L$,

$$(39) \qquad i = A\, \varepsilon^{-\frac{Rt}{2L}} \sin \left\{ \frac{\sqrt{4\, L\, C - R^2 C^2}}{2\, L\, C}\, t + \Phi \right\}.$$

When $R^2 C = 4 L$,

$$(40) \qquad i = c_1\, \varepsilon^{-\frac{Rt}{2L}} + c_2\, t\, \varepsilon^{-\frac{Rt}{2L}}.$$

The value of the charge q given by equation (20), being of the same form as (19), may take three different forms according as $R^2 C$ is greater than, less than, or equal to $4 L$; and these forms only differ from (38), (39) and (40) in the arbitrary constants, thus

When $R^2 C > 4 L$,

$$(41) \qquad q = c' \varepsilon^{-\frac{RC-\sqrt{R^2C^2-4LC}}{2LC}t} + c'' \varepsilon^{-\frac{RC+\sqrt{R^2C^2-4LC}}{2LC}t}.$$

When $R^2 C < 4 L$,

$$(42) \qquad q = A'\, \varepsilon^{-\frac{Rt}{2L}} \sin \left\{ \frac{\sqrt{4\, L\, C - R^2 C^2}}{2\, L\, C}\, t + \Phi' \right\}.$$

When $R^2 C = 4 L$,

$$(43) \qquad q = c'\, \varepsilon^{-\frac{Rt}{2L}} + c''\, t\, \varepsilon^{-\frac{Rt}{2L}}.$$

The constants of integration in the equations (38), (39), (40) and (41), (42), (43) are determined by the initial conditions imposed by the problem. For instance if a condenser charged with a quantity Q, is suddenly discharged through a circuit with resistance and self-induction, we may count the time from the moment of discharge, and thus have $q = Q$ and $i = 0$ when $t = 0$, and $q = 0$ and $i = 0$ when $t = \infty$.

NON-OSCILLATORY DISCHARGE.

The equations (38) and (41) may be written as in (19) and (20), in terms of the time-constants T_1 and T_2, thus

$$(44) \qquad i = c_1 \varepsilon^{-\frac{t}{T_1}} + c_2 \varepsilon^{-\frac{t}{T_2}}.$$

$$(45) \qquad q = c' \varepsilon^{-\frac{t}{T_1}} + c'' \varepsilon^{-\frac{t}{T_2}}.$$

The arbitrary constants c_1, c_2, c', c'' of these equations will be determined according to the conditions mentioned above, viz., when $t = 0$, $i = 0$ and $q = Q$; when $t = \infty$, $i = 0$ and $q = 0$.

Substituting in (44) $i = 0$ when $t = 0$, and in (45) $q = Q$ when $t = 0$, we have

$$(46) \qquad 0 = c_1 + c_2 \quad \text{or} \quad c_1 = -c_2,$$

$$(47) \qquad Q = c' + c'',$$

Since we have the relation $d\,q = i\,d\,t$, we may differentiate (45) and write

$$(48) \qquad i = -\frac{c'}{T_1} \varepsilon^{-\frac{t}{T_1}} - \frac{c''}{T_2} \varepsilon^{-\frac{t}{T_2}}.$$

Equating (48) and (44), we find

$$(49) \qquad c_1 = -\frac{c'}{T_1} \quad \text{or} \quad c' = -c_1 T_1.$$

$$c_2 = -\frac{c''}{T_2} \quad \text{or} \quad c'' = -c_2 T_2.$$

Remembering the relation in (46), we may write

$$(50) \qquad c'' = c_1 T_2. \qquad \text{Adding (49) and (50)}$$

$$c' + c'' = c_1 (T_2 - T_1) = Q. \quad \text{[See (47).]}$$

Hence

$$c_1 = \frac{Q}{T_2 - T_1}$$

$$c_2 = \frac{Q}{T_1 - T_2}$$

$$c' = \frac{Q\,T_1}{T_1 - T_2}$$

$$c'' = \frac{Q\,T_2}{T_2 - T}$$

Substituting in (44) and (45) the constants c_1, c_2, c', c'', as finally determined, we have

$$(51) \qquad i = \frac{Q}{T_2 - T_1} \left\{ \varepsilon^{-\frac{t}{T_1}} - \varepsilon^{-\frac{t}{T_2}} \right\}.$$

$$(52) \qquad q = \frac{Q}{T_1 - T_2} \left\{ T_1 \varepsilon^{-\frac{t}{T_1}} - T_2 \varepsilon^{-\frac{t}{T_2}} \right\}.$$

These equations give the complete solution and express the current or the charge at any time after discharge (see Fleming's Alternate Current Transformer, page 376). They show that if we have the relation $R^2 C > 4 L$, the discharge is a gradual dying away without oscillation. Since T_1 and T_2 are each of them positive when $R^2 C > 4 L$, [see (11)], i or q may be represented geometrically as the difference of two decreasing logarithmic curves. To see this more clearly, the values of the time-constants T_1 and T_2 may be substituted in the coefficients of equations (51) and (52). The result is

$$(53) \qquad i = \frac{Q}{\sqrt{R^2 C^2 - 4 L C}} \left\{ \varepsilon^{-\frac{t}{T_2}} - \varepsilon^{-\frac{t}{T_1}} \right\}.$$

$$(54) \qquad q = \frac{Q}{2} \left\{ \frac{R C}{\sqrt{R^2 C^2 - 4 L C}} + 1 \right\} \varepsilon^{-\frac{t}{T_1}}$$

$$- \frac{Q}{2} \left\{ \frac{R C}{\sqrt{R^2 C^2 - 4 L C}} - 1 \right\} \varepsilon^{-\frac{t}{T_2}}$$

These equations may be more easily explained by referring to figures 1 and 2, which represent the plot of these equations for particular assumed values of R, C and L. The values assumed for the constants of the circuit are

$R = 100$ ohms, $C = 1$ microfarad, $L = .0016$ henrys.

By calculating the values of T_1 and T_2 [equation (11)], $T_1 = 8 \times 10^{-5}$, and $T_2 = 2 \times 10^{-5}$, and the equations (53) and (54), with these particular values, become

$$(55) \qquad i = \frac{Q}{.6 \times 10^{-5}} \left\{ \varepsilon^{-\frac{t}{2 \times 10^{-5}}} - \varepsilon^{-\frac{t}{8 \times 10^{-5}}} \right\}.$$

$$(56)\quad q=\frac{Q}{2}\left[1\frac{2}{3}+1\right]\varepsilon^{-\frac{t}{8\times10^{-5}}}-\frac{Q}{2}\left[1\frac{2}{3}-1\right]\varepsilon^{-\frac{t}{2\times10^{-5}}}.$$

If the condenser was charged to a potential of 2,000 volts, the capacity being .000001 farads, the charge is .002 coulombs. Substituting this value for Q, we have

$$i = 33.33\left[\varepsilon^{-\frac{t}{2\times10^{-5}}}-\varepsilon^{-\frac{t}{8\times10^{-5}}}\right]$$

$$q=\frac{1}{1000}\left[1\frac{2}{3}+1\right]\varepsilon^{-\frac{t}{8\times10^{-5}}}-\frac{1}{1000}\left[1\frac{2}{3}-1\right]\varepsilon^{-\frac{t}{2\times10^{-5}}}$$

where i is in amperes and q in coulombs.

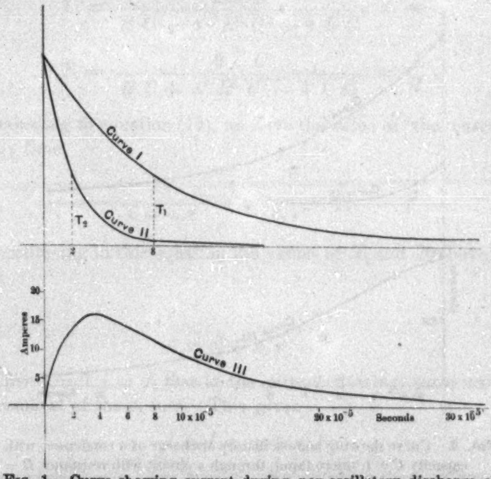

FIG. 1. Curve showing current during non-oscillatory discharge of condenser with capacity $C=1$ micro-farad, through a circuit with resistance $R = 100$ ohms, and self-induction $L = .0016$ henrys when originally charged to a potential of 2000 volts.

In Fig. 1, curves I. and II. represent the two component logarithmic curves, corresponding to the first and second terms respectively of equation (53), whose difference gives the resultant current curve III. Curve I., corresponding to the first term of equation (53), has the larger time-constant, and is therefore the more important curve. The area included between curve III. and the axis of abscissae is equal to $\int i\,d\,t = Q$, and is therefore independent of the constants of the circuit through which the condenser is discharged.

The current reaches a maximum at a point which may be determined by differentiating equation (53) and equating the first derivative to zero in the usual manner for a maximum. The time t_m at which the current is a maximum is thus found to be

$$t_m = \frac{L\,C}{\sqrt{R^2\,C^2 - 4\,L\,C}} \log \frac{T_1}{T_2}.$$

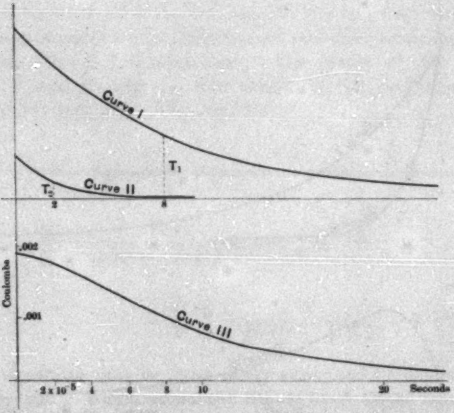

FIG. 2. Curve showing non-oscillatory discharge of a condenser, with capacity $C = 1$. micro-farad, through a circuit with resistance $R =$ 100 ohms, and self-induction $L = .00016$. henrys. when originally charged to a potential of 2000 volts.

Substituting the particular values used in plotting Fig. 1,

$$t_m = 3.78 \times 10^{-5}.$$

In Fig. 2 curves I. and II. are the two component logarithmic curves, corresponding to the first and second terms respectively of equation (54) for charge. Curve III. is plotted by subtracting II. from I., and represents the charge of the condenser at any time, according to equation (54). It is noticeable that the upper curve, I., has the larger initial value, and as T_1 is larger than T_2, decreases the slower. It is therefore this curve which is the more important in determining the discharge of the condenser.

DYING AWAY OF CURRENT UPON REMOVAL OF E. M. F. FROM A CIRCUIT CONTAINING RESISTANCE AND SELF-INDUCTION.

In this case there is no condenser in the circuit, that is, the capacity is infinite. Substituting $C = \infty$ in the equation (11) for the time-constants, we have

$$T_1 = \frac{2\,L\,C}{R\,C - \sqrt{R^2\,C^2 - 4\,L\,C}} = \infty\,.$$

$$T_2 = \frac{2\,L\,C}{R\,C + \sqrt{R^2\,C^2 - 4\,L\,C}} = \frac{L}{R}.$$

According to equation (19), we have the value of the current at any time

$$i = c_1\,\varepsilon^{-\frac{t}{T_1}} + c_2\,\varepsilon^{-\frac{t}{T_2}}.$$

Substituting in this equation the values of T_1 and T_2 above, we have

$$i = c_1 + c_2\,\varepsilon^{-\frac{R\,t}{L}}.$$

When $t = 0$, $i = I$, that is, the current flowing previous to the removal of the E. M. F. This gives

$$i = I = c_1 + c_2\,I.$$

But when $t = \infty$, $i = c_1 = 0$. Substituting these values for the constants, we have

$$(57) \qquad i = I\,\varepsilon^{-\frac{R\,t}{L}},$$

a result which is well known.[1]

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[1] Fleming's Alternate Current Transformer, page 102.

DISCHARGE OF A CONDENSER THROUGH A CIRCUIT CONTAINING RESISTANCE, BUT NO SELF-INDUCTION.

Upon substituting $L = 0$ in the values of the time constants T_1 and T_2, (11), the expressions become indeterminate, but can readily be evaluated by differentiating numerator and denominator, and then substituting $L = 0$ as in ordinary vanishing fractions.

$$T_1 = \frac{2\,L\,C}{R\,C - \sqrt{R^2\,C^2 - 4\,L\,C}}.$$

Differentiating numerator and denominator with respect to L, we have

$$\frac{\dfrac{d}{d\,L}(2\,L\,C)}{\dfrac{d}{d\,L}\left(R\,C - \sqrt{R^2\,C^2 - 4\,L\,C}\right)} = \frac{2\,C}{\dfrac{+\,4\,C}{2\,\sqrt{R^2\,C^2 - 4LC}}}$$

$$= \sqrt{R^2\,C^2 - 4\,L\,C}.$$

Now letting $L = 0$, we have

$T_1 = R\,C.$ Similarly $T_2 = -\,R\,C.$

Substituting these values in equations (19) and (20), we have

$$(58) \qquad i = c_1\,\varepsilon^{-\frac{t}{RC}} + c_2\,\varepsilon^{+\frac{t}{RC}}.$$

$$(59) \qquad q = c_3\,\varepsilon^{-\frac{t}{RC}} + c_4\,\varepsilon^{+\frac{t}{RC}}.$$

c_2 and c_4 must each be zero or else when $t = \infty$ we would have $i = \infty$ and $q = \infty$. When $t = 0, q = Q = c_3$. By differentiating (59) and equating to (58), we have

$$i = \frac{d\,q}{d\,t} = -\frac{c_3}{R\,C}\,\varepsilon^{-\frac{t}{RC}} = c_1\,\varepsilon^{-\frac{t}{RC}},$$

and, therefore, $c_1 = -\dfrac{c_3}{R\,C} = -\dfrac{Q}{R\,C}.$

Substituting in (58) and (59) the values found for the constants c_1, c_2, c_3, c_4, we have

$$(60) \qquad i = -\frac{Q}{R\,C}\,\varepsilon^{-\frac{t}{RC}} = I\,\varepsilon^{-\frac{t}{RC}}.$$

(61) $$q = Q\,\varepsilon^{-\frac{t}{RC}}.$$

These are the well-known results for the case of discharge through a circuit with no self-induction.

OSCILLATORY DISCHARGE.

In the case of oscillatory discharge, the equations for current and charge at any time are

(62) $$i = A\,\varepsilon^{-\frac{Rt}{2L}} \sin\left\{\frac{\sqrt{4LC-R^2C^2}}{2LC}\,t + \Phi\right\}.$$

(63) $$q = A'\,\varepsilon^{-\frac{Rt}{2L}} \cos\left\{\frac{\sqrt{4LC-R^2C^2}}{2LC}\,t + \Phi'\right\}.$$

The arbitrary constants A, A', Φ and Φ' will be determined according to the same conditions as those mentioned above, viz., when $t = 0, i = 0$ and $q = Q$; also when $t = \infty, i = 0$ and $q = 0$. Substituting in (62) $i = 0$ when $t = 0$, and in (63) $q = Q$ when $t = 0$, we have

$$0 = A \sin \Phi,$$

(64) $$\text{and} \qquad Q = A' \sin \Phi'.$$

Since A and $\sin \Phi$ are constants, and their product is zero, one of them must be zero. But if A were zero, i would be zero for every point of time, which is impossible. Therefore

(65) $$\Phi = 0.$$

Differentiating (63) and remembering that $i = \frac{dq}{dt}$, we have

(66) $$i = \frac{dq}{dt} = -\frac{A'R\,\varepsilon^{-\frac{Rt}{2L}}}{2L} \sin\left\{\frac{\sqrt{4LC-R^2C^2}}{2LC}\,t+\Phi'\right\}$$
$$+\frac{A'\varepsilon^{-\frac{Rt}{2L}}\sqrt{4LC-R^2C^2}}{2LC}\cos\left\{\frac{\sqrt{4LC-R^2C^2}}{2LC}\,t+\Phi'\right\}.$$

Substituting $i = 0$, when $t = 0$;

$$0 = -R \sin \Phi' + \frac{\sqrt{4LC-R^2C^2}}{C} \cos \Phi'.$$

(67) $$\text{Hence,} \quad \Phi' = \tan^{-1}\frac{\sqrt{4LC-R^2C^2}}{RC}.$$

By (64) $A' = \frac{Q}{\sin \Phi'}$. And, by (67),

$$(68) \quad A' = \frac{Q}{\sin \tan^{-1} \frac{\sqrt{4LC - R^2C^2}}{RC}} = \frac{Q}{\sqrt{1 - \frac{R^2C^2}{4LC}}}.$$

To determine the constant A, transform (66) by the formula (33) so as to write it in terms of a sine only. The coefficient of the sine in the equation as transformed will be

$$A' \sqrt{\frac{1}{LC}}.$$

Since equations (66) and (62) are each equations for i, we may equate the coefficients of the sine, and have

$$A = \frac{A'}{\sqrt{LC}}. \quad \text{And, by (68),}$$

$$(69) \quad A = \frac{2Q}{\sqrt{4LC - R^2C^2}}.$$

Substituting A and Φ, as determined in (69) and (65), in equation (62), and substituting A' and Φ', as determined in (68) and (67), in equation (63,) we have

$$(70) \quad i = \frac{2Q}{\sqrt{4LC - R^2C^2}} \varepsilon^{-\frac{Rt}{2L}} \sin \left\{ \frac{\sqrt{4LC - R^2C^2}}{2LC} t \right\}.$$

$$(71) \quad q = \frac{2Q\sqrt{LC}}{\sqrt{4LC - R^2C^2}} \varepsilon^{-\frac{Rt}{2L}}$$

$$\sin \left\{ \frac{\sqrt{4LC - R^2C^2}}{2LC} t + \tan^{-1} \frac{\sqrt{4LC - R^2C^2}}{RC} \right\}.$$

These equations may be more readily understood by referring to Fig. 3, in which curves showing the current and charge, according to these equations, are drawn for the discharge of a condenser for particular values of L, C and R, assumed. The particular constants assumed are

$R = 100.$ ohms. $C = 1.$ micro-farad. $L = .0125$ henrys.

If the condenser be originally charged to 2,000 volts, $Q = .002$ coulombs. On substituting these values in (70) and (71), the equations for current and charge become

$$i = 20\,\varepsilon^{-4,000\,t} \sin 8,000\,t, \text{ and}$$

$$q = .00224\,\varepsilon^{-4,000\,t} \sin (8,000\,t + \tan^{-1} 2),$$

where i is in amperes and q in coulombs. Curve I, representing the current, is a sine-curve with a constant period, but with an amplitude decreasing according to the logarithmic curve $20\,\varepsilon^{-4,000\,t}$. The period is $\frac{2\pi}{8,000} = .000785$ seconds; that is, there are 1,275 complete oscillations per second. A very few oscillations are sufficient for a complete discharge.

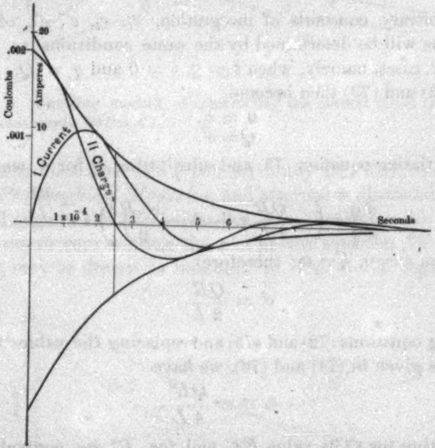


FIG. 3. Oscillatory discharge of a condenser with capacity $C = 1$ microfarad, through a circuit with resistance $R = 100$ ohms, and self-induction $L = .0125$ henrys, when originally charged to a potential of 2000 volts.

The charge at any time is shown in curve II, which is likewise a sine-curve with an amplitude decreasing according to the logarithmic curve, in this case $.00224\,\varepsilon^{-4,000\,t}$. The scale in Fig. 3 is such that the same logarithmic curve is an envelope for the current and the charge curve. The periods of the two are the same, but the curves differ in phase by an angle $= \tan^{-1} 2$, that is, the charge is ahead of the current by an angle of advance of 63° 27′.

When $R^2\,C = 4\,L$.

This is the critical case, when the discharge is just non-oscillatory. The equations for the current and charge, as previously determined, (40) and (43), are

$$i = c_1\,\varepsilon^{-\frac{R\,t}{2\,L}} + c_2\,t\,\varepsilon^{-\frac{R\,t}{2\,L}}. \tag{72}$$

$$q = c'\,\varepsilon^{-\frac{R\,t}{2\,L}} + c''\,t\,\varepsilon^{-\frac{R\,t}{2\,L}}. \tag{73}$$

The arbitrary constants of integration, c_1, c_2, c', c'', of these equations will be determined by the same conditions as in the previous cases, namely, when $t = 0$, $i = 0$ and $q = Q$. Equations (72) and (73) then become

$$\begin{aligned} 0 &= c_1, \\ Q &= c'. \end{aligned} \tag{74}$$

Differentiating equation (73) and substituting Q for c', we obtain

$$i = \frac{d\,q}{d\,t} = \left(-\frac{Q\,R}{2\,L}\,\varepsilon + c'' - \frac{c''\,R\,t}{2\,L}\right)\varepsilon^{-\frac{R\,t}{L}}. \tag{75}$$

But when $t = 0$, $i = 0$; therefore

$$c'' = \frac{Q\,R}{2\,L}. \tag{76}$$

Equating equations (72) and (75) and replacing the values for the constants given in (74) and (76), we have

$$c_2 = -\frac{Q\,R^2}{4\,L}. \tag{77}$$

Substituting for Q its value $E\,C$, and for R^2 its equivalent, in this particular case $\frac{4\,L}{C}$, (77) becomes

$$c_2 = -\frac{E}{L}.$$

Having thus determined the values of the arbitrary constants, the equations (72) and (73) for current and charge may be written

$$i = -\frac{E}{L}\,t\,\varepsilon^{-\frac{R\,t}{L}}. \tag{78}$$

$$q=\left(1+\frac{Rt}{2L}\right)Q\,\epsilon^{-\frac{Rt}{2L}} \tag{79}$$

These equations give the value of the current and charge at any time during the discharge of the condenser in the case where the discharge is just non-oscillatory. This case is often called the case of quickest discharge, as was pointed out by Dr. W. E. Sumpner, in the *Philosophical Magazine*, and afterwards discussed by Dr. Oliver Lodge in the *Electrician* for May 18, 1888.

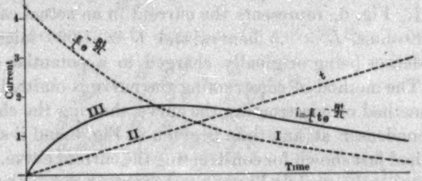


FIG. 4. Showing method of constructing the current curve in the case where $R^2 C = 4L$.

The curve representing the plot of the equation (78) for the current, may be drawn as indicated in Fig. 4. A logarithmic curve I., having its initial value $\frac{E}{L}$ and time constant $\frac{2L}{R}$, is drawn to represent the equation as it would be with t omitted from the coefficient; and each ordinate is then multiplied by the

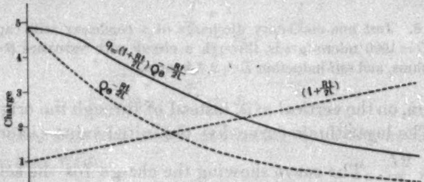


FIG. 5. Showing the method of constructing the curve representing the charge left in the condenser at any time after discharge.

ordinate of a straight line II. which passes through the origin and represents the uniform increase of the time t. The product of the ordinates of curves I. and II. at each point gives the ordinate of the current curve III. at that point. When actual values of R, L and C are assumed, it is found to be difficult to represent these curves to scale, so that Fig. 4 is shown simply as an illustration of the method of constructing the current curve. Curve I., Fig. 6, represents the current in an actual case where $R = 100$ ohms, $L = 2.5$ henrys and $C = 1{,}000$ micro-farads, the condenser being originally charged to a potential of 2,000 volts. The method of constructing the curve is omitted.

The method of constructing the curve, showing the charge left in the condenser at any time is given in Fig. 5 and is similar to the method just shown for constructing the current curve. The difference is that the straight line passes through a point one unit above the origin, on the vertical axis, instead of through the origin as before. The logarithmic curve has the initial value Q and a time constant $\frac{2L}{R}$. The curve showing the charge for the actual case where $R = 100$ ohms, $L = 2.5$ henrys and $C = 1{,}000$ micro-farads, the condenser being originally charged to a potential of 2,000 volts, is represented by curve II., Fig. 6.

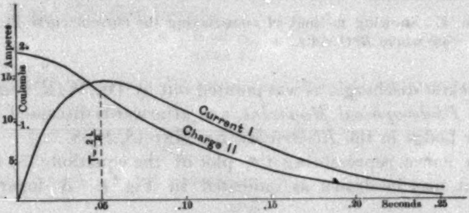


FIG. 6. Just non-oscillatory discharge of a condenser with capacity $C = 1000$ micro-farads, through a circuit with resistance $R = 100$ ohms, and self-induction $L = 2.5$ henrys.

CASE II. VALUE OF CURRENT AND CHARGE AT ANY TIME DURING CHARGING.

The E. M. F., instead of being zero as in the previous case, is assumed to be a constant E, and

$$e = f(t) = E.$$

This is the case when an E. M. F. is suddenly changed from one constant value to another constant value in a circuit, and it includes Case I. as a particular case, since E may be zero. By differentiation,

$$\frac{d\,e}{d\,t} = f'(t) = 0.$$

Substituting these values in equations (5) and (6), they become

$$(80) \qquad \frac{d^2 i}{d t^2} + \frac{R}{L}\frac{d\,i}{d\,t} + \frac{i}{L\,C} = 0,$$

$$(81) \qquad \text{and} \qquad \frac{d^2 q}{d\,t^2} + \frac{R}{L}\frac{d\,q}{d\,t} + \frac{q}{L\,C} = \frac{E}{L}.$$

It is seen that the equation for current (80) is identical with that of the previous case (21), while the equation for charge (81) has its second member equal to $\frac{E}{L}$, a constant, instead of zero. By substituting a new variable, $q' = q - E\,C$, this equation may be transformed into one having its second member zero; thus,

$$(82) \qquad \frac{d^2 q'}{d\,t^2} + \frac{R}{L}\frac{d\,q'}{d\,t} + \frac{q'}{L\,C} = 0.$$

The solutions of (80) and (82) are, as in the previous case,

$$(83) \qquad i = c_1\,\varepsilon^{-\frac{t}{T_1}} + c_2\,\varepsilon^{-\frac{t}{T_2}}.$$

$$q' = c_3\,\varepsilon^{-\frac{t}{T_1}} + c_4\,\varepsilon^{-\frac{t}{T_2}}.$$

Replacing the value of q' and remembering $Q = E\,C =$ the final charge, we may write

$$(84) \qquad q = Q + c_3\,\varepsilon^{-\frac{t}{T_1}} + c_4\,\varepsilon^{-\frac{t}{T_2}}.$$

These equations, (83) and (84), for current and charge might have been obtained directly from the general solutions (17) and (18) by substituting $e = f(t) = E$, and $\frac{d\,e}{d\,t} = f'(t) = 0$. Upon substituting $f'(t) = 0$ in (17), we obtain (83) directly, and upon substituting $f(t) = E$ in (18), we have

$$q = \frac{E\,C}{\sqrt{R^2\,C^2 - 4\,L\,C}}\,(T_1 - T_2) + c_3\,\varepsilon^{-\frac{t}{T_1}} + c_4\,\varepsilon^{-\frac{t}{T_2}},$$

But, by the values of T_1 and T_2 in (11), we find that $T_1 - T_2 = \sqrt{R^2\,C^2 - 4\,L\,C}$; and hence this equation is identical with (84), since $Q = E\,C$.

As in Case I, where $f(t) = 0$, the equations just obtained for current (83) and charge (84) when $f(t) = E$, assume three forms.

When $R^2\,C > 4\,L$,

$$(85) \qquad i = c_1\,\varepsilon^{-\frac{t}{T_1}} + c_2\,\varepsilon^{-\frac{t}{T_2}}.$$

$$(86) \qquad q = Q + c'_1\,\varepsilon^{-\frac{t}{T_1}} + c'_2\,\varepsilon^{-\frac{t}{T_2}}.$$

When $R^2\,C < 4\,L$,

$$(87) \qquad i = A\,\varepsilon^{-\frac{R\,t}{2\,L}} \sin\left\{\frac{\sqrt{4\,L\,C - R^2\,C^2}}{2\,L\,C}\,t + \Phi\right\}.$$

$$(88) \qquad q = Q + A'\,\varepsilon^{-\frac{R\,t}{2\,L}} \sin\left\{\frac{\sqrt{4\,L\,C - R^2\,C^2}}{2\,L\,C}\,t + \Phi'\right\}.$$

When $R^2\,C = 4\,L$,

$$(89) \qquad i = c_1\,\varepsilon^{-\frac{R\,t}{2\,L}} + c_2\,t\,\varepsilon^{-\frac{R\,t}{2\,L}}.$$

$$(90) \qquad q = Q + c'_1\,\varepsilon^{-\frac{R\,t}{2\,L}} + c'_2\,t\,\varepsilon^{-\frac{R\,t}{2\,L}}.$$

The constants of integration must be determined by the conditions of the problem as to the previous state of the circuit, the changes made, and the final state.

NON-OSCILLATORY CHARGING.

When $R^2\,C > 4\,L$.

The constants c_1, c_2, c'_1 and c'_2 of equations (85) and (86) will be determined by the following conditions:

When $t = 0$, $i = 0$ and $q = Q_0$.

When $t = \infty$, $i = 0$ and $q = Q$.

This means that the condenser is suddenly charged or discharged from the initial charge Q_0 to the final charge Q. Determining the constants by the same method as in Case I., we find that

$$c_1 = \frac{Q_0 - Q}{T_2 - T_1},$$

$$c_2 = \frac{Q_0 - Q}{T_1 - T_2},$$

$$c'_1 = \frac{(Q_0 - Q)\, T_1}{T_1 - T_2},$$

$$c'_2 = \frac{(Q_0 - Q)\, T_2}{T_2 - T_1}.$$

Substituting in (85) and (86) the values of the constants just determined, we have

$$(91) \qquad i = \frac{Q_0 - Q}{T_2 - T_1} \left\{ \varepsilon^{-\frac{t}{T_1}} - \varepsilon^{-\frac{t}{T_2}} \right\}.$$

$$(92) \qquad q = Q + \frac{Q_0 - Q}{T_1 - T_2} \left\{ T_1\, \varepsilon^{-\frac{t}{T_1}} - T_2\, \varepsilon^{-\frac{t}{T_2}} \right\}.$$

For Q_0, the original charge, we may write $C\,E_0$, and for Q, the final charge, we may write $C\,E$.

These equations, (91) and (92), give the value of the current and charge at any time after the change of E. M. F. from E_0 to E in a circuit with $R^2\,C > 4\,L$. As the equations now stand in their general form, they hold true for either total or partial charge or discharge according to the values of E_0 and E, and consequently Q_0 and Q, assumed. If the final charge is $Q = 0$, we have the case of complete discharge and the equations take the form of (51) and (52). If the original charge $Q_0 = 0$, we have the case of charge from zero to Q.

These equations will perhaps be better understood by referring to Fig. 7, which represents the equations with particular values assumed. These values are the same as in the preceding case, namely, $R = 100.$ ohms, $C = 1.$ micro-farad, $L = .0016$ henrys. The condenser originally had no charge, and when charged to a potential of 2,000 volts, has a charge of .002 coulombs. The current curve I., Fig. 7, is identical with curve III., Fig. 1, which represents the current during discharge. The curve of charge, II., is the same as curve III., Fig. 2, inverted and plotted downwards from the horizontal line $Q = .002$. It is noticeable that the ordinates of curve I., expressing the current, are proportional to the tangents of the angle of inclination of curve II. at every point, since the current $i = \frac{dq}{dt}$, and $\frac{dq}{dt}$ is the tangent of the angle of inclination of the curve of charge II. It is seen that the point of inflection in curve II. comes at the maximum value of the current curve I., as the tangent is the greatest at this point; indeed, curve I. might be constructed geometrically simply from the foregoing consideration.

ESTABLISHMENT OF CURRENT UPON INSERTING AN E. M. F. IN A CIRCUIT WITH RESISTANCE AND SELF-INDUCTION.

In this case there is no condenser in the circuit, that is the capacity is infinite. Substituting $C = \infty$ in the values of the time constants (11), we have

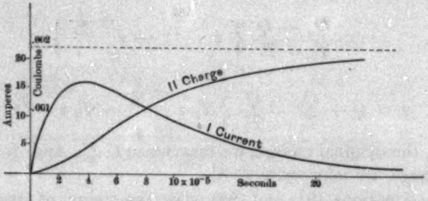


FIG. 7. Non-oscillatory charge of a condenser with capacity C 1 = micro farad, through a circuit with resistance $R = 100$ ohms and self-induction $L = .0125$ henrys when subjected to a potential of 2000 volts.

$$(93) \qquad T_1 = \infty, \qquad T_2 = \frac{L}{R},$$

as in Case I, where the current dies away after the removal of the E. M. F. Substituting these values in (19), we have

$$i = c_1 + c_2 \varepsilon^{-\frac{Rt}{L}}.$$

When $t = 0$, $i = 0 = c_1 + c_2$.

When $t = \infty$, $i = I = c_1$. $\therefore c_2 = -I$.

Substituting these values for the constants c_1 and c_2, we have

$$i = I\left(1 - \varepsilon^{-\frac{R\,t}{L}}\right).$$

I is the final steady value of the current, and is equal to $\frac{E}{R}$; hence

$$(94) \qquad i = \frac{E}{R}\left(1 - \varepsilon^{-\frac{R\,t}{L}}\right),$$

which is the well-known expression for the establishment of a current in a circuit with self-induction.

Charging a Condenser Through a Circuit Containing Resistance, but no Self-Induction.

Upon substituting $L = 0$ in the values of the time constants T_1 and T_2, the expressions become indeterminate, but can be evaluated as before by differention of numerator and denominator before substituting $L = 0$. We thus find the values, when $L = 0$,

$$(95) \qquad T_1 = R\,C. \qquad T_2 = -\,R\,C.$$

Substituting these values in the equations of current (83) and current (84), we have

$$(96) \qquad i = c_1\,\varepsilon^{-\frac{t}{RC}} + c_2\,\varepsilon^{+\frac{t}{RC}}.$$

$$(97) \qquad q = Q + c_3\,\varepsilon^{-\frac{t}{RC}} + c_4\,\varepsilon^{+\frac{t}{RC}}.$$

Q_0 is the previous charge of the condenser, and Q the final charge. The constants c_2, c_4 must be zero, or else when $t = \infty$ we would have $i = \infty$, $q = \infty$. When $t = 0$, (97) becomes

$$Q_0 = Q + c_3. \qquad \therefore c_3 = Q_0 - Q.$$

By differentiating (97) and equating to (96), we have

$$i = \frac{d\,q}{d\,t} = -\,\frac{c_3}{R\,C}\,\varepsilon^{-\frac{t}{RC}} = c_1\,\varepsilon^{-\frac{t}{RC}}.$$

Therefore

$$c_1 = -\,\frac{c_3}{R\,C} = -\,\frac{Q_0 - Q}{R\,C}.$$

Substituting in (96) and (97) the values for the constants c_1, c_2, c_3, c_4, as determined,

$$(98) \qquad i = -\frac{Q_0 - Q}{R\,C}\,\varepsilon^{-\frac{t}{RC}}.$$

$$(99) \qquad q = Q + (Q_0 - Q)\,\varepsilon^{-\frac{t}{RC}}.$$

These equations are true for the charge or discharge from Q_0 to Q, through a resistance with no self-induction. When the final charge Q is zero, we have the case of complete discharge and the equations become the same as (60) and (61). When the original charge Q_0 is zero, we have the case of charging from zero to Q.

$$(100) \qquad i = \frac{Q}{R\,C}\,\varepsilon^{-\frac{t}{RC}}.$$

$$(101) \qquad q = Q\left(1 - \varepsilon^{-\frac{t}{RC}}\right).$$

It is noticeable that the current equation is the same as that for discharge equation (60) and that the charge equation is analogous to that in the case of the establishment of the current in a circuit with resistance and self-induction, equation (94).

OSCILLATORY CHARGING.

When $R^2 C < 4 L$.

The constants A, A', Φ and Φ' in equations (87) and (88) will be determined by the same conditions as before, namely,

When $t = 0$, $i = 0$ and $q = Q_0$.
When $t = \infty$, $i = 0$ and $q = Q$.

The meaning of this supposition is the same as in the preceding case, namely, that the condenser is suddenly charged or discharged from the initial charge Q_0 to the final charge Q. The constants, determined by the same method as in Case I, are

$$A = \frac{2\,(Q_0 - Q)}{\sqrt{4\,L\,C - R^2\,C^2}}.$$

$$A' = \frac{2\,(Q_0 - Q)\,\sqrt{L\,C}}{\sqrt{4\,L\,C - R^2\,C^2}}.$$

$$\Phi = 0.$$

$$\Phi' = \tan^{-1}\frac{\sqrt{4\,L\,C - R^2\,C^2}}{R\,C}.$$

With the constants thus determined, equations (87) and (88) become

$$(102) \quad i = \frac{2\,(Q_0 - Q)}{\sqrt{4\,L\,C - R^2\,C^2}}\,\varepsilon^{-\frac{R\,t}{2\,L}} \sin \frac{\sqrt{4\,L\,C - R^2\,C^2}}{2\,L\,C}\,t.$$

$$(103) \quad q = Q + \frac{2\,(Q_0 - Q)\,\sqrt{L\,C}}{\sqrt{4\,L\,C - R^2\,C^2}}$$

$$\varepsilon^{-\frac{R\,t}{2\,L}} \sin \left\{ \frac{\sqrt{4\,L\,C - R^2\,C^2}}{2\,L\,C}\,t + \tan^{-1}\frac{\sqrt{4\,L\,C - R^2\,C^2}}{R\,C} \right\}.$$

We may write $C\,E_0$ for the original charge Q_0, and $C\,E$ for the final charge Q.

These equations, (102) and (103), give the value of the current and charge at any time after the change of the electromotive force from E_0 to E in a circuit with $R^2\,C < 4\,L$. As the equations now stand in their general form, they are true for either total or partial charge or discharge, according to the values assigned to Q_0 and Q. If the final charge Q is zero, we have the case of complete discharge, and the equations take the form of (70) and (71). When Q is less than Q_0 we have partial discharge; if Q is greater than Q_0 we have partial charging. If the original charge $Q_0 = 0$, we have the case of charge from zero to Q.

Fig. 8 illustrates the case of oscillatory charge through a circuit having the same constants as those of Fig. 3. The current curve I. is the same as that in Fig. 3, and the charge is represented by curve II., which is the same as in that figure, but inverted and plotted from the horizontal line $Q = .002$. It is seen that in charging the condenser, the charge rises at first higher than its final value, and then oscillates about that final value until it has become steady.

When $R^2\,C = 4\,L$.

This is the critical case, where the charging is just non-oscillatory. The equations for current and charge are,

$$(104) \qquad i = c_1\,\varepsilon^{-\frac{R\,t}{2\,L}} + c_2\,t\,\varepsilon^{-\frac{R\,t}{2\,L}}.$$

$$(105) \qquad q = Q + c' \varepsilon^{-\frac{R\,t}{2\,L}} + c''\, t\, \varepsilon^{-\frac{R\,t}{2\,L}}$$

The initial charge is Q_o, and the final charge Q. To determine the arbitrary constants of integration, let $t = 0$. Then $i=0$, and $q = Q_o$. Equations (104) and (105) then become

$$c_1 = 0.$$

$$c' = Q_o - Q.$$

Differentiating equation (105) and substituting the value of c', we have

$$(106) \quad i = \frac{d\,q}{d\,t} = \left(-\frac{(Q_o - Q)\,R}{2\,L} + c'' - \frac{c''\,R\,t}{2\,L}\right) \varepsilon^{-\frac{R\,t}{2\,L}}$$

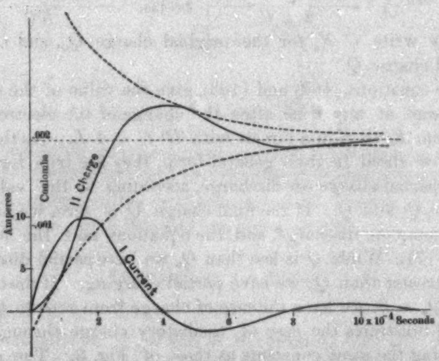


FIG. 8. Oscillatory Charge of a condenser with capacity $C = 1$ micro-farad, through a circuit with resistance $R = 100$ ohms and self-induction $L = .0125$ henrys when subjected to a potential of 2000 volts.

When $t = 0$, $i = 0$, therefore

$$c'' = \frac{(Q_o - Q)\,R}{2\,L}.$$

Equating equations (104) and (106), and replacing the values for c_1, c' and c'', we have

$$c_2 = -(Q_o - Q)\frac{R^2}{4\,L^2}.$$

If E_0 and E are the initial and final potentials respectively, we may write $E_0\,C$ for Q_0, and $E\,C$ for Q. Making this substitution and remembering that in this particular case $R^2 = \dfrac{4L}{C}$, we have

$$c_2 = \frac{E - E_0}{L}\,.$$

Replacing the values of the arbitrary constants, the equations (104) and (105) for current and charge may be written

$$(107) \qquad i = \frac{E - E_0}{L}\, t\, \varepsilon^{-\frac{Rt}{2L}}.$$

$$(108) \qquad q = Q + (Q_0 - Q)\left(1 + \frac{Rt}{2L}\right) \varepsilon^{-\frac{Rt}{2L}}$$

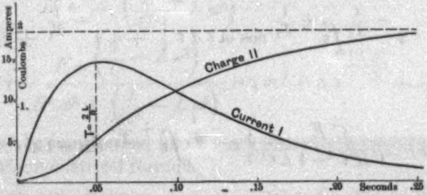


FIG. 9. Just non-oscillatory charge of a condenser with capacity $C = 1000$ micro-farads through a circuit with resistance $R = 100$ ohms and self-induction $L = 2.5$ henrys.

The current curve in the case of charging a condenser, represented by equation (107), is the same as in the case of discharge, equation (78). It is represented in Fig. 9, curve I., and may be constructed by the method shown in Fig. 4. The curve showing the charge represented in Fig. 9, curve II., is constructed in a similar manner to curve II, Fig. 6, and, indeed, curve II. of Fig. 9 is identical with curve II. of Fig. 6, it being inverted and plotted downwards from the horizontal line.

CASE III. HARMONICALLY VARYING E. M. F.

In the preceding cases considered, those of discharge and charge, the solutions for the value of the current and charge at any time were obtained in two ways, first from the general solution, and then directly from the differential equations, by substituting $e = f(t) = 0$, and $e = f(t) = E$, respectively.

The case of a circuit containing resistance, self-induction and capacity, in which there is an impressed E. M. F. varying harmonically, will now be considered, and the solution derived first from the general equations (17) and (18), and then directly from the differential equations (5) and (6). In this case,

$$e = f\ (t) = E \sin \omega\, t,$$

$$\text{and } \frac{d\,e}{d\,t} = f'\ (t) = E\, \omega \cos \omega\, t.$$

Substituting these values in (17) and (18), we have

$$(109)\quad i = \frac{C\,E\,\omega}{\sqrt{R^2C^2-4\,LC}} \left\{ \varepsilon^{-\frac{t}{T_1}} \int \varepsilon^{+\frac{t}{T_1}} \cos \omega\, t\, d\, t - \varepsilon^{-\frac{t}{T_2}} \int \varepsilon^{+\frac{t}{T_2}} \cos \omega\, t\, d\, t \right\} + c_1\, \varepsilon^{-\frac{t}{T_1}} + c_2\, \varepsilon^{-\frac{t}{T_2}},$$

and

$$(110)\quad q = \frac{C\,E}{\sqrt{R^2C^2-4\,L\,C}} \left\{ \varepsilon^{-\frac{t}{T_1}} \int \varepsilon^{+\frac{t}{T_1}} \sin \omega\, t\, d\, t - \varepsilon^{-\frac{t}{T_2}} \int \varepsilon^{+\frac{t}{T_2}} \sin \omega\, t\, d\, t \right\} + c_3\, \varepsilon^{-\frac{t}{T_1}} + c_4\, \varepsilon^{-\frac{t}{T_2}}.$$

The solution for q being similar to that for i, we will give the integration and reduction of (109) alone, and simply give the resulting expression for q. The integrals may be found by the formulæ of reduction obtained by integrating by parts. The integration of each term in (109) is

$$(111)\quad \varepsilon^{-\frac{t}{T}} \int \varepsilon^{+\frac{t}{T}} \cos \omega t d t = \frac{1}{\frac{1}{T^2}+\omega^2} \left\{ \frac{1}{T} \cos \omega t + \omega \sin \omega t \right\}.$$

For convenience in transformation and reduction, put $\tau_1 = \frac{1}{T_1}$ and $\tau_2 = \frac{1}{T_2}$. Integrating the two terms in (109) and making this substitution, we have

$$(112)\quad i = \frac{C E \omega}{\sqrt{R^2 C^2 - 4 L C}} \left\{ \left(\frac{\tau_1}{\tau_1^2 + \omega^2} - \frac{\tau_2}{\tau_2^2 + \omega^2} \right) \cos \omega t \right.$$

$$\left. + \left(\frac{\omega}{\tau_1^2 + \omega^2} - \frac{\omega}{\tau_2^2 + \omega^2} \right) \sin \omega t \right\} + c_1 \varepsilon^{-\frac{t}{T_1}} + c_2 \varepsilon^{-\frac{t}{T_2}}$$

We may simplify (112) by substituting the values of τ_1 and τ_2 [see (11)].

$$\tau_1 = \frac{1}{T_1} = \frac{R C - \sqrt{R^2 C^2 - 4 L C}}{2 L C}.$$

$$\tau_2 = \frac{1}{T_2} = \frac{R C + \sqrt{R^2 C^2 - 4 L C}}{2 L C}.$$

Then, after a few simple algebraic transformations in the coefficients of the sine and cosine, (112) becomes

$$(113)\quad i = \frac{E R \omega^2}{R^2 \omega^2 + \left(\frac{1}{C} - L \omega^2 \right)^2} \sin \omega t$$

$$+ \frac{E \omega \left(\frac{1}{C} - L \omega^2 \right)}{R^2 \omega^2 + \left(\frac{1}{C} - L \omega^2 \right)^2} \cos \omega t + c_1 \varepsilon^{-\frac{t}{T_1}} + c_2 \varepsilon^{-\frac{t}{T_2}}$$

This may be transformed into a more convenient form by means of the trigonometrical formula,

$$A \sin x + B \cos x = \sqrt{A^2 + B^2} \sin \left(x + \tan^{-1} \frac{B}{A} \right),$$

and when transformed is written,

$$(114)\quad i = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C \omega} - L \omega \right)^2}} \sin \left\{ \omega t \right.$$

$$\left. + \tan^{-1} \left(\frac{1}{C R \omega} - \frac{L \omega}{R} \right) \right\} + c_1 \varepsilon^{-\frac{t}{T_1}} + c_2 \varepsilon^{-\frac{t}{T_2}}$$

The corresponding equation for charge, being the integral of this, according to the relation $q = \int i \, d t$, may be written

$$(115)\quad q = \frac{-E}{\omega \sqrt{R^2 + \left(\frac{1}{C \omega} - L \omega \right)^2}} \cos \left\{ \omega t \right.$$

$$\left. + \tan^{-1} \left(\frac{1}{C R \omega} - \frac{L \omega}{R} \right) \right\} + c_3 \varepsilon^{-\frac{t}{T_1}} + c_4 \varepsilon^{-\frac{t}{T_2}}$$

These equations, (114) and (115), are the complete solutions for current and charge in a circuit with resistance, self-induction and capacity, when there is an harmonic impressed E. M. F.

Let us now proceed to obtain this same solution by solving the original differential equation, with the assumption that the E. M. F. varies harmonically, that is, $e = E \sin \omega t$. Substituting $\frac{d e}{d t} = E \omega \cos \omega t$ in the differential equation (5), we have

$$\frac{d^2 i}{d t^2} + \frac{R}{L} \frac{d i}{d t} + \frac{i}{L C} = \frac{E \omega}{L} \cos \omega t. \tag{116}$$

This is a linear equation of the second order with constant coefficients. The complete integral of such an equation consists of the sum of two parts, namely, the particular integral and the complementary function. The complementary function is the integral obtained by equating the first member to zero, and contains two arbitrary constants. The particular integral contains no arbitrary constants. The complementary function, obtained by equating the first member of (116) to zero and solving, is

$$i = c_1 \varepsilon^{-\frac{t}{T_1}} + c_2 \varepsilon^{-\frac{t}{T_2}}. \tag{117}$$

To find the particular integral, it is convenient to use the symbolic notation,

$$D = \frac{d\,(\,)}{d t}. \qquad D^2 = \frac{d^2\,(\,)}{d t^2}.$$

With this notation (115) is written

$$\left(D^2 + \frac{R}{L} D + \frac{1}{L C}\right) i = \frac{E \omega}{L} \cos \omega t, \qquad \text{or}$$

$$i = \frac{\frac{E \omega}{L} \cos \omega t}{\left(D^2 + \frac{R}{L} D + \frac{1}{L C}\right)}. \tag{118}$$

Next, to find the value of D^2, we have

$$\frac{d \cos \omega t}{d t} = D \cos \omega t = - \omega \sin \omega t,$$

$$\frac{d^2 \cos \omega t}{d t^2} = D^2 \cos \omega t = - \omega^2 \cos \omega t.$$

$$\text{Therefore} \qquad D^2 = - \omega^2$$

Substituting in (117) $D^2 = - \omega^2$, we have

$$(119)\quad i=\frac{E\,\omega}{L\left(\frac{R}{L}\,D+\frac{1}{L\,C}-\omega^2\right)}\cos\omega\,t=\frac{E\,\omega}{R\,D+\frac{1}{C}-L\,\omega^2}\cos\omega\,t.$$

Multiplying numerator and denominator of the coefficient of $\cos\omega\,t$ in (119) by $R\,D-\left(\frac{1}{C}-L\,\omega^2\right)$, we obtain

$$i=\frac{E\,\omega\left\{R\,D-\left(\frac{1}{C}-L\,\omega^2\right)\right\}}{R^2\,D^2-\left(\frac{1}{C}-L\,\omega^2\right)^2.}\cos\omega\,t.$$

Substituting $-\omega^2$ for D^2, and separating into two terms,

$$(120)\quad i=\frac{-E\,\omega\,R\,D\cos\omega\,t+E\,\omega\left(\frac{1}{C}-L\,\omega^2\right)\cos\omega\,t}{R^2\,\omega^2+\left(\frac{1}{C}-L\,\omega^2\right)^2}.$$

But $D\cos\omega\,t=-\omega\sin\omega\,t$. Hence,

$$(121)\quad i=\frac{E\,\omega^2\,R\sin\omega\,t}{R^2\,\omega^2+\left(\frac{1}{C}-L\,\omega^2\right)^2}+\frac{E\,\omega\left(\frac{1}{C}-L\,\omega^2\right)\cos\omega\,t}{R^2\,\omega^2+\left(\frac{1}{C}-L\,\omega^2\right)^2}.$$

This is the particular integral, to which must be added the complementary function (117) in order to obtain the complete integral. The complete integral is thus found to be

$$(122)\quad i=\frac{E\,\omega^2\,R\sin\omega\,t}{R^2\,\omega^2+\left(\frac{1}{C}-L\,\omega^2\right)^2}+\frac{E\,\omega\left(\frac{1}{C}-L\,\omega^2\right)\cos\omega\,t}{R^2\,\omega^2+\left(\frac{1}{C}-L\,\omega^2\right)^2}$$
$$+\,c_1\,\varepsilon^{-\frac{t}{T_1}}+c_2\,\varepsilon^{-\frac{t}{T_2}}.$$

This solution for the current obtained from the differential equation (5) is seen to be identical with (113), the result obtained from the general solution (17). The solution for charge could be obtained in a similar manner from the differential equation (6).

Discussion of Case III.—Harmonic E. M. F.

These solutions, (114) and (115), show that, after a very short time has elapsed, so that the exponential terms containing the arbitrary constants of integration become inappreciably small and can be neglected, both the current and the charge are simple harmonic functions and may either lag behind or advance ahead of the impressed E. M. F. The current lags behind the impressed E. M. F. when $L\omega > \frac{1}{C\omega}$, and advances ahead of it when $L\omega < \frac{1}{C\omega}$. When $L\omega = \frac{1}{C\omega}$, that is, when $\omega = \frac{1}{\sqrt{LC}}$, there is no lag or advance and the current is exactly in phase with the impressed E. M. F. In this case the current equation becomes

$$(123) \qquad i = \frac{E}{R}\sin \omega t,$$

which is identical with the current equation obtained from Ohm's law, without considering either self-induction or capacity. When the sine is unity in equation (114), the maximum value of the current, represented by I, is

$$(124) \qquad I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}}.$$

From the analogy of this equation to Ohm's law, we see that the expression $\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}$ is of the nature of a resistance, and is the apparent resistance of a circuit containing resistance, self-induction and capacity. This expression would quite properly be called "impedance," but the term impedance has for several years been used as a name for the expression $\sqrt{R^2 + L^2\omega^2}$, which is the apparent resistance of a circuit containing resistance and self-induction only. We would suggest, therefore, that the word "impediment" be adopted as a name for the expression $\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}$, which is the apparent resistance of a circuit containing resistance, self-induction and capacity, and that the term impedance be retained in the more limited meaning it has come to have, that is, $\sqrt{R^2 + L^2\omega^2}$, the apparent resistance of a circuit containing resistance and self-induction only. Equation (124) may be written

$$(125) \qquad \text{Maximum current} = \frac{\text{Maximum E. M. F.}}{\text{Impediment.}}$$

Since the effective current (the square root of the mean square of the instantaneous values of the current) is equal to $\frac{1}{\sqrt{2}}$ times the maximum value of the current. and since the effective E. M. F. $= \frac{1}{\sqrt{2}}$ times the maximum E. M. F.,

$$\text{Effective current} = \frac{\text{Effective E. M. F.}}{\text{Impediment.}} \tag{126}$$

It is convenient to consider the impediment as a resistance, and we are justified in so doing inasmuch as it has the same dimensions as a resistance, that is, a velocity in the electromagnetic system of units.

$$\omega = \frac{2\pi}{\text{Time}}.$$

$$L = \text{Length}.$$

Therefore,

$$L\omega = \frac{\text{Length}}{\text{Time}} = \text{velocity}.$$

$$C = \frac{(\text{Time})^2}{\text{Length}}.$$

$$\frac{1}{C\omega} = \frac{\text{Length}}{\text{Time}} = \text{velocity}.$$

This gives the dimensions of a velocity to the whole expression for the impediment, which may therefore be considered as a resistance.

The several particular cases of circuits containing various combinations of resistance, self-induction and capacity may readily be found by means of the general solution, equation (114).

CASE A. CIRCUITS CONTAINING RESISTANCE AND SELF-INDUCTION ONLY.

In this case the circuit has resistance R and self-induction L, and an harmonic E. M. F., $E \sin \omega t$. There being no condenser in the circuit, the capacity C is infinite. After the lapse of a very small time the terms containing the constants of integration in the general solution may be neglected as explained above. Substituting in (21) $C = \infty$, we have

$$i = \frac{E}{\sqrt{R^2 + L^2\omega^2}} \sin \left\{ \omega t - \tan^{-1} \frac{L\omega}{R} \right\}. \tag{127}$$

The current must always lag behind the impressed E. M. F. by an angle whose tangent is $\frac{L\omega}{R}$. In this case the impediment takes the particular value $\sqrt{R^2 + L^2\omega^2}$, which is known as the impedance of the circuit.

CASE B. CIRCUITS CONTAINING RESISTANCE AND CAPACITY ONLY.

In this case the circuit has resistance R and capacity C, with an harmonic E. M. F., $e = E \sin \omega t$. Substituting $L = 0$ in the general equation (114), we have

$$(128) \quad i = \frac{E}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} \sin \left\{ \omega t + \tan^{-1} \frac{1}{CR\omega} \right\}.$$

The current must always advance ahead of the impressed E. M. F. when there is resistance and capacity only in the circuit, by an angle whose tangent is $\frac{1}{CR\omega}$.

CASE C. CIRCUITS CONTAINING RESISTANCE ONLY.

In this case the self-induction $L = 0$ and the capacity $C = \infty$. Substituting these values in the general solution (114), we have

$$i = \frac{E}{R} \sin \omega t.$$

This result is immediately derivable from Ohm's law. Thus

$$\text{Since } e = E \sin \omega t,$$

$$\frac{e}{R} = \frac{E}{R} \sin \omega t,$$

$$\text{or } i = \frac{E}{R} \sin \omega t.$$

CASE D. CIRCUITS CONTAINING CAPACITY ONLY.

In this case $R = 0$ and $L = 0$. Substituting in the general equation (21), we have

$$(129) \quad i = CE\omega \sin \left\{ \omega t + \frac{\pi}{2} \right\}.$$

EFFECT OF VARYING THE CONSTANTS OF A CIRCUIT.

The general equation (114) enables us to ascertain the current which will flow in a circuit, when we know its resistance, self-induction and capacity, the value of the impressed E. M. F. and its frequency. It is important to know two things about the current: first, its maximum value I; and second, the angle θ, by which it lags behind or advances ahead of the impressed E. M. F. The mean square value is readily obtained from the maximum value. We are given R, C, L, E and ω. The angle of lag or advance is

$$(130)\qquad \theta = \tan^{-1}\left(\frac{1}{C\,R\,\omega} - \frac{L\,\omega}{R}\right),$$

$$\text{or}\quad \tan\theta = \frac{1}{C\,R\,\omega} - \frac{L\,\omega}{R}.$$

This is an angle of advance or lag, according as $\frac{1}{C\,R\,\omega}$ is greater or less than $\frac{L\,\omega}{R}$. The maximum value for the current

$$(131)\qquad I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\,\omega} - L\,\omega\right)^2}} = \frac{\frac{E}{R}}{\sqrt{1 + \tan^2\theta}}.$$

It is interesting to note how any change in R, L, C, ω or E affects the value of θ and the current.

First. If the impressed E. M. F. E, is varied and R, L, C and ω are maintained constant, tan θ is not affected, and the angle of lag or advance remains unchanged. The value of the current is varied in direct proportion to E.

$$I \propto E.$$

Second. If the resistance R, of the circuit is varied, and L, C, ω and E are maintained constant, as R is increased, the angle of lag or advance is diminished.

$$\tan\theta \propto \frac{1}{R}.$$

The sign of tan θ is positive or negative, and the angle therefore one of advance or lag, according to the values of L, C and ω, and is independent of any variations in the resistance. The current is in all cases diminished by an increase of resistance, but the amount of this decrease depends not only upon R, but upon the relation between $\frac{1}{C\,\omega}$ and $L\,\omega$.

In Fig. 10 are shown two particular cases of the variation in the current produced by change in the resistance. Curve I. is for a circuit in which

Self-induction $L = 2.$ henrys $= 2 \times 10^9$ c. g. s. units.
Capacity $C = .55$ micro-farads $= .55 \times 10^{-15}$ c. g. s. units.
Impressed E.M.F. $E = 200.$ volts $= 200 \times 10^8$ c. g. s. units.
$2 \pi n = \omega = 955.$

The abscissæ represent resistance in ohms (1 ohm $= 10^9$ c. g. s. units of resistance). The ordinates represent current in amperes (1 ampere $= 10^{-1}$ c. g. s. units). The relation between L, C and ω here taken is such that $\frac{1}{C\omega} = L\omega$, or $\omega = \frac{1}{\sqrt{LC}}$, which is the relation that gives no angle of lag or advance. The relation between current and resistance is the same as in Ohm's law, and when plotted gives the hyperbola curve I. In the same figure, curve II. represents the value of the current with different resistances in a circuit in which

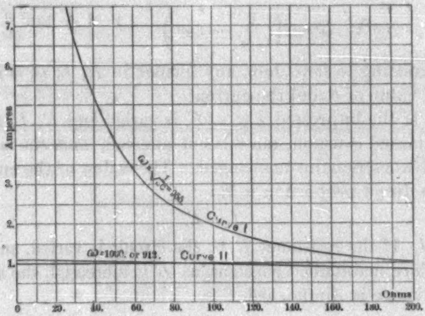


FIG. 10. Variation of Current with Change in Resistance in a Circuit in which $E = 200$. $C = .55$ $L = 2$.

$L = 2.$ henrys, $E = 200$ volts,
$C = .55$ micro-farads, $\omega =$ either 1,000 or 912.

The constants of this circuit are the same as in the previous case, with the exception of ω, which has been changed from 955, that is $\frac{1}{\sqrt{LC}}$, to either 1,000 or 912. Any change in ω from the value $\frac{1}{\sqrt{LC}}$, whether it be an increase or decrease, causes the curve to depart from the hyperbola, curve I. It is to be noticed that a change in frequency of only seven alternations per second will change the curve from I. to II.

Third. If the coefficient of self-induction L is varied while R, C, ω and E are maintained constant,

When $L < \frac{1}{C\,\omega^2}$, $\tan\theta$ is positive and θ is an angle of advance.

$$\left.\begin{array}{l}\theta \text{ becomes less}\\ I \text{ becomes greater}\end{array}\right\}\text{ as } L \text{ increases.}$$

When $L > \frac{1}{C\,\omega^2}$, $\tan\theta$ is negative and θ is an angle of lag.

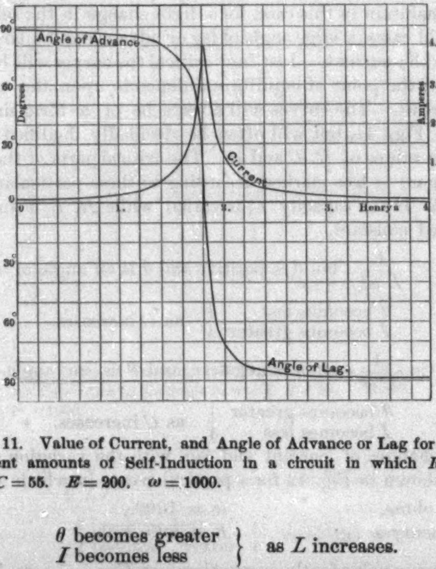


FIG. 11. Value of Current, and Angle of Advance or Lag for different amounts of Self-Induction in a circuit in which $R = 50$. $C = 55$. $E = 200$. $\omega = 1000$.

$$\left.\begin{array}{l}\theta \text{ becomes greater}\\ I \text{ becomes less}\end{array}\right\}\text{ as } L \text{ increases.}$$

These changes in the angle of lag or advance and the current, due to change in the self-induction, are better seen from the consideration of a particular case. In Fig. 11 the values of θ and I are plotted for various values of L in a circuit in which

$R = 50.$ ohms, $\omega = 1000.$
$C = .55$ micro-farads, $E = 200.$ volts.

Where $L = \dfrac{1}{C\,\omega^2} = 1.82$ the current has its maximum value equal to $\dfrac{E}{R}$, and $\theta = 0$. This is a critical point, and a slight change of L in either direction will cause θ to reach a considerable value and the current to fall to a small part of the maximum value. If L be increased from 1.82 to 1.92, θ changes from zero to — 63°, an angle of lag, and the current falls from 4. to 1.8 amperes. If L be made 1.72, θ becomes an angle of advance of 63° and the current will be 1.8 amperes. It is thus seen that an exact balance of self-induction and capacity would be exceedingly hard to maintain in this case, for a slight change in the self-induction would cause a large angle of lag or advance and a large diminution in the current. Just how critical the curves will be in the vicinity of the point of equilibrium depends upon the constants of the circuit. The curves will always be of a form similar to those in Fig. 11, but will often be decidedly modified by the particular values of R, C and ω. The critical parts of the curves may be more or less marked according to these particular values.

Fourth. If the capacity C is varied while R, L, ω and E are maintained constant,

When $C < \dfrac{1}{L\,\omega^2}$, tan θ is positive and θ is an angle of advance.

$$\left.\begin{array}{l}\theta \text{ becomes less}\\ I \text{ becomes greater}\end{array}\right\}\text{ as } C \text{ increases.}$$

When $C > \dfrac{1}{L\,\omega^2}$, tan θ is negative and θ is an angle of lag.

$$\left.\begin{array}{l}\theta \text{ becomes greater}\\ I \text{ becomes less}\end{array}\right\}\text{ as } C \text{ increases.}$$

These changes of current and lag, with the variation in capacity, are shown in Fig. 12 for a particular case in which

$R = 50.$ ohms, $\omega = 1000.$,
$L = 2.$ henrys, $E = 200.$ volts.

The maximum value for the current is where $C = \dfrac{1}{L\,\omega^2} = .5$ micro-farads. This is a critical point in the curve similar to that in the curves where the self-induction was varied. Here θ is zero, and so the current, being in phase with the impressed E. M. F., has a value of 4 amperes in accordance with Ohm's law. The critical nature of the curves here is seen by the fact that when $C = .55$ there is an angle of lag of 75° and $I = 1.07$; when $C = .458$ there is an angle of advance of 75°. When C is changed from .5 to .488, the current falls from 4 to 2.83 amperes and is put 45° out of phase in advance of the E. M. F.

Fifth. If the frequency is varied while R, C, L and E are maintained constant, still more marked changes occur in the values of I and θ.

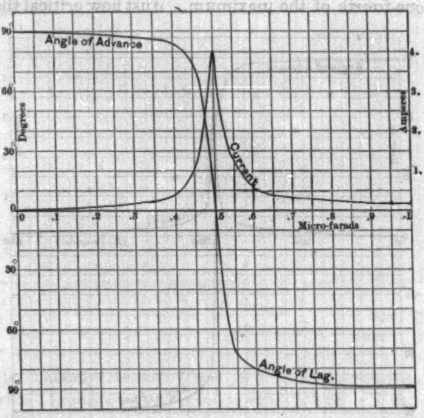


FIG. 12. Value of Current, and Angle of Advance or Lag for different Capacities in a circuit in which $R = 50$. $L = 2$. $E = 200$. $\omega = 1000$.

When $\omega < \dfrac{1}{\sqrt{L\,C}}$, $\tan\theta$ is positive and θ is an angle of advance.

$$\left.\begin{array}{l}\theta \text{ becomes less}\\ I \text{ becomes greater}\end{array}\right\} \text{ as } \omega \text{ increases.}$$

When $\omega > \dfrac{1}{\sqrt{L\,C}}$, $\tan\theta$ is negative and θ is an angle of lag.

$$\left.\begin{array}{l}\theta \text{ becomes greater}\\ I \text{ becomes less}\end{array}\right\} \text{ as } \omega \text{ increases.}$$

In Fig. 13 the values of the current and angle of lag are shown for different values of ω in a circuit in which

$R = 50.$ ohms, $C = .55$ microfarads,
$L = 2.$ henrys, $E = 200$ volts.

When $\omega = \dfrac{1}{\sqrt{LC}} = 955$, the current has its maximum value of 4 amperes, in accordance with Ohm's law. Here $\theta = 0$. A change of five per cent. one way or the other in this critical value for ω causes an angle of lag or advance of 75°, and the current falls to one-fourth of the maximum. Just how critical the curves are, in the vicinity of this point of equilibrium, depends upon the particular values of R, C and L.

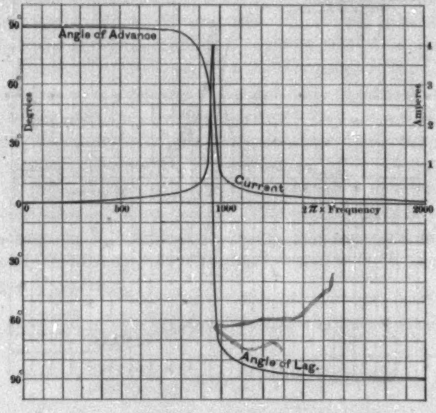


FIG. 13. Value of Current and Angle of Advance or Lag for different Frequencies in a circuit in which $R = 50$. $L = 2$. $C = .55$. $E = 200$.

In Fig. 14 is shown the E. M. F. necessary to cause a constant current to flow in a circuit in which R, C and ω are constant. In the particular case plotted,

$R = 50.$ ohms, $I = 1$ ampere,
$C = .55$ micro-farads, $\omega = 1000.$

As the self-induction is increased up to the value $L = \dfrac{1}{C\omega^2} = 1.82$, the E. M. F. needed to drive the current becomes less and less, and when $L = 1.82$ the E. M. F. needed is only 50 volts. As L increases past this critical value, the value of the E. M. F. needed increases. Except very near the critical point, the change in the necessary E. M. F. is almost directly proportional to the change in the self-induction, that is, the curve is formed of two straight lines with a rounded point. This curve is the reciprocal of the corresponding curve for current, with E constant and L variable, as shown in Fig. 11.

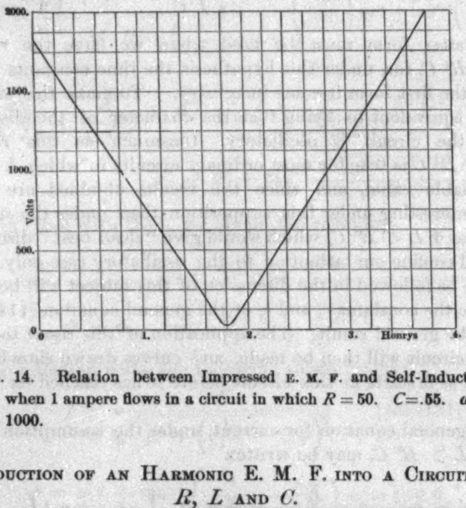


FIG. 14. Relation between Impressed E. M. F. and Self-Induction when 1 ampere flows in a circuit in which $R = 50$. $C = .55$. $\omega = 1000$.

INTRODUCTION OF AN HARMONIC E. M. F. INTO A CIRCUIT WITH R, L AND C.

Returning to the complete solutions for the current [equation (114)] and the charge [equation (115)] at any time after a simple periodic E. M. F. has been introduced into a circuit containing R, L and C, although it has been stated that after a very short time everything comes to a steady state and the complementary function may be neglected, yet it may be interesting to inquire what happens just at the moment of the introduction of the periodic E. M. F. The E. M. F. may be introduced at any point of its phase, that is, it may be zero or may have its maximum or any intermediate value; but, in any case, the complete equations (114) and (115) show just what happens, provided we determine the constants c_1 and c_2 of the complementary function, so that they correspond to the particular hypothesis made.

It has been noticed (39) that the complementary function $c_1 \varepsilon^{-\frac{t}{T_1}} + c_2 \varepsilon^{-\frac{t}{T_2}}$ may be written in another form, viz.:

$$A\, \varepsilon^{-\frac{R t}{2 L}} \sin \left\{ \frac{\sqrt{4 L C - R^2 C^2}}{2 L C}\, t + \Phi \right\}.$$

This latter form must be used when we have the relation $4 L > R^2 C$, for, under this hypothesis, the time constants T_1 and T_2 of the first form become imaginary. To make this supposition is equivalent to saying that the character of the discharge from the circuit is oscillatory. Inasmuch as this relation $4 L > R^2 C$ is true for most ordinary circuits in which L has an appreciable value, and since the results obtained are rather more interesting under this supposition than under the supposition that $4 L < R^2 C$, which would give "dead beat" discharge, we will confine our attention to the oscillatory case only. The plan to be followed in the discussion of this subject will be to determine the constants c_1 and c_2 of the general equation (114), and write the general result. The application of this result to a particular circuit will then be made, and curves drawn showing the current as it starts in this circuit before it has reached its steady state.

The general equation for current, under the assumption made that $4 L > R^2 C$, may be written

$$(132) \quad i = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C \omega} - L \omega\right)^2}} \sin \left\{ \omega t + \tan^{-1} \left(\frac{1}{C R \omega} - \frac{L \omega}{R} \right) \right\} + A\, \varepsilon^{-\frac{R t}{2 L}} \sin \left\{ \frac{\sqrt{4 L C - R^2 C^2}}{2 L C}\, t + \Phi \right\},$$

where A and Φ are the constants of integration to be determined and are each of them real. Likewise, the equation expressing the quantity of charge on the condenser at any moment may be written [see (115) and (42)]

$$(133)\quad q = \frac{-E}{\omega\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}} \cos\left\{\omega t + \tan^{-1}\left(\frac{1}{CR\omega}\right.\right.$$

$$\left.\left. - \frac{L\omega}{R}\right)\right\} + A' \varepsilon^{-\frac{Rt}{2L}} \sin\left\{\frac{\sqrt{4LC - R^2C^2}}{2LC}\, t + \Phi'\right\}.$$

Remembering the relation $d q = i d t$, we may differentiate (133) and write

$$(134)\quad i = \frac{dq}{dt} = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}} \sin\left\{\omega t + \tan^{-1}\left(\frac{1}{CR\omega}\right.\right.$$

$$\left.\left. - \frac{L\omega}{R}\right)\right\} + \frac{A'}{\sqrt{LC}} \varepsilon^{-\frac{Rt}{2L}} \sin\left\{\frac{\sqrt{4LC - R^2C^2}}{2LC}\, t\right.$$

$$\left. + \Phi' + \tan^{-1}\frac{\sqrt{4LC - R^2C^2}}{RC}\right\}.$$

Equating (134) with (132), we obtain the relations

$$(135)\qquad A = \frac{A'}{\sqrt{LC}},$$

$$(136)\qquad \Phi = \Phi' - \tan^{-1}\frac{\sqrt{4LC - R^2C^2}}{RC}.$$

For simplification make the following substitutions:

$$(137)\qquad I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}} \qquad \text{[see (131)].}$$

$$(138)\qquad \psi = \omega t + \tan^{-1}\left(\frac{1}{CR\omega} - \frac{L\omega}{R}\right) = \omega t + \theta.$$

$$(139)\qquad a = \frac{\sqrt{4LC - R^2C^2}}{2LC}.$$

The frequency of oscillation is $\frac{a}{2\pi}$, and the period $\frac{2\pi}{a}$. Then we may write, after substituting in (133) the values of A' and Φ', as determined,

$$(140)\qquad i = I \sin\psi + A\, \varepsilon^{-\frac{Rt}{2L}} \sin\left\{a t + \Phi\right\}.$$

$$(141)\quad q = -\frac{I}{\omega}\cos\psi + A\sqrt{LC}\,\varepsilon^{-\frac{Rt}{2L}}\sin\left\{at + \Phi + \tan^{-1}\frac{\sqrt{4LC - R^2C^2}}{RC}\right\}.$$

In these equations time is counted from the point when the impressed E. M. F. is zero. Let t_1 be the time when the E. M. F. is introduced. We know then that the current and the charge of the condenser are each zero at the time t_1, the condenser having no initial charge. These conditions alone, namely, that $i = 0$ and $q = 0$ when $t = t_1$, are sufficient to determine the constants. In equations (140) and (141) make $i = 0$, $q = 0$ when $t = t_1$, and call ψ_1 the value of ψ when $t = t_1$, and we have

$$(142)\qquad 0 = I\sin\psi_1 + A\,\varepsilon^{-\frac{Rt_1}{2L}}\sin\left\{at_1 + \Phi\right\}.$$

$$(143)\quad 0 = -\frac{I}{\omega}\cos\psi_1 + A\sqrt{LC}\,\varepsilon^{-\frac{Rt_1}{2L}}\sin\left\{at_1 + \Phi + \tan^{-1}\frac{\sqrt{4LC - R^2C^2}}{RC}\right\}.$$

Eliminating A between these equations, we obtain

$$(144)\qquad \Phi = \cot^{-1} - \left\{\frac{2\cot\psi_1 + RC\omega}{\omega\sqrt{4LC - R^2C^2}}\right\} - at_1.$$

Substituting this value of Φ in (142),

$$(145)\qquad A = -I\varepsilon^{+\frac{Rt_1}{2L}}\frac{\sin\psi_1}{\sin\cot^{-1} - \left\{\frac{2\cot\psi + RC\omega}{\omega\sqrt{4LC - R^2C^2}}\right\}}.$$

This expression for A may be reduced by simple trigonometrical operations to the form

$$(146)\quad A = -\frac{2I\varepsilon^{+\frac{Rt_1}{2L}}}{\omega\sqrt{4LC - R^2C^2}}\sqrt{(LC\omega^2 - 1)\sin^2\psi_1 + \frac{1}{2}RC\omega\sin 2\psi_1 + 1}.$$

Substituting these values of A and Φ in equation (132), we may write the complete solution with constants determined,

$$(147)\ i = I \sin \phi - \frac{2I\sqrt{(L\,C\,\omega^2 - 1)\sin^2\phi_1 + \frac{1}{2}\,R\,C\,\omega \sin 2\,\phi_1 + 1}}{\omega\,\sqrt{4\,L\,C - R^2\,C^2}}$$

$$\varepsilon^{-\frac{R}{2L}(t - t_1)} \sin \left\{ a\,(t - t_1) + \cot^{-1} - \left[\frac{2\cot\phi_1 + CR\omega}{\omega\,\sqrt{4LC - R^2\,C^2}}\right] \right\}.$$

There are several general conclusions which can be made in interpreting the meaning of this equation. It is evident that there will be an oscillation of the current when the E. M. F. is first introduced, which gradually dies away, the rate of dying away depending upon the exponent of ε in the equation or, in other words, upon the time-constant of the circuit, namely, $\frac{2L}{R}$. The initial value of this logarithmic decrement curve, which is the value when $t = t_1$, is expressed by the coefficient of ε in the equation. It is evident that this initial value depends upon the value of ϕ_1 for its value, or is a function of ϕ_1. The initial value of the logarithmic curve has, then, a different value for every value of ϕ_1, *i. e.*, for every point of the phase of what the current would have been if it had reached its steady state. Again, at the time $t = t_1$ the value of the last term of the equation becomes $- I \sin \phi_1$. This will be evident upon replacing the coefficient of ε by its value given in (145). The first term becomes $I \sin \phi_1$, when $t = t_1$, and the two terms together show that the equation makes the value of the current zero at the time t_1, that is, at the time the E. M. F. is introduced.

In order to show the meaning of this equation more clearly, a particular example will be assumed. Suppose we have a circuit with a resistance of 50 ohms, a self-induction of 2 henrys and a capacity of .55 micro-farads, all in series. Such a circuit would correspond nearly to the fine wire coil of a small 10-light Westinghouse transformer connected in series with a condenser of .55 micro-farads capacity. Let an E. M. F. of 100. volts (maximum value), having a periodicity of 159, be impressed upon the circuit, (that is the angular velocity $\omega = 2\pi \times 159 = 1000$ approx.) We have, then, with these values assumed,

$E =$ 100 volts (max.) $= 100 \times 10^8$ C. G. S. units.
$R =$ 50 ohms $= 50 \times 10^9$ C. G. S. units.
$L =$ 2 henrys $= 2 \times 10^9$ C. G. S. units.
$C =$ .55 micro-farads $= .55 \times 10^{-15}$ C. G. S. units.

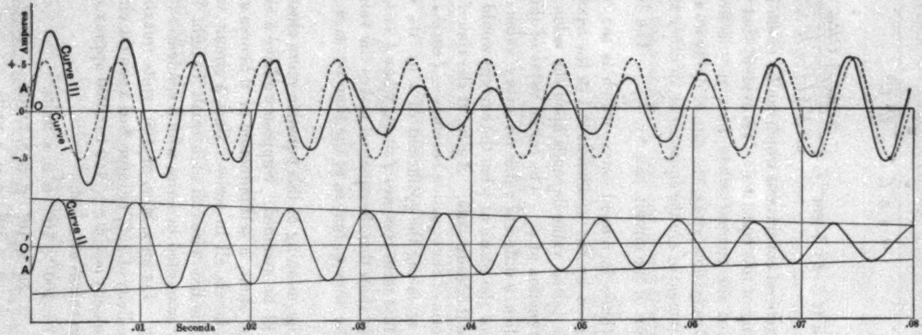


FIG. 15. Curve III. shows the current which flows after the introduction of an harmonic E. M. F. into a circuit with R, L, and C. It is the sum of the two component curves, I. a sine-curve, and II. a sine-curve with an amplitude decreasing according to a logarithmic decrement.

$\omega = 1000.$

$T = \frac{2L}{R} = \frac{4 \times 10^9}{50 \times 10^9} = .08$ seconds.

$I = .53$ amperes (max.) [See equation (137).]

$\theta = -74^\circ\ 30'.$

$a = 955 = 2\,\pi \times$ frequency of oscillation $= 2\,\pi \times 151.$ [See (139).]

The equation for the current in this particular case is

$$(148) \quad i = .53 \sin \psi - .477 \sqrt{.1 \sin^2 \psi_1 + .0137 \sin 2\,\psi_1 + 1}\ \varepsilon^{-\frac{t-t_1}{.08}} \sin \left\{ 955\,(t - t_1) + \chi \right\}.$$

Curve III., Fig. 15, represents the plot of this equation when the particular value of ψ_1 is 30°. This means that the E. M. F. is introduced into the circuit at the particular time at which the normal current curve is 30° from its zero value. The value of the coefficient of ε when ψ_1 is 30° is .495, and the equation reads

$$(149) \quad i = .53 \sin \psi - .495\, \varepsilon^{-\frac{t-t_1}{.08}} \sin \left\{ 955\,(t - t_1) + \chi \right\}.$$

Here χ stands for angle of lag expressed in equation (147), and is not expressed in figures inasmuch as it is not necessary to know it in order to draw the curves, because the phase is determined by the fact that we know the distance $O'\ A'$, Fig. 15, it being equal but of opposite sign to the distance $O\ A$. It will be noticed that the initial value of the logarithmic decrement is nearly the same for any value of ψ_1 in this particular case. Moreover, as it happens, the initial value of the logarithmic decrement is nearly the same as the maximum value of the current I. Curve I. is a sine-curve representing the first term in equation (149), and curve II. a sine-curve with logarithmic decrement representing the second term in the equation. The current curve, III., is the sum of curves I. and II. After about one-tenth of a second, curve II. becomes inappreciable and the current follows a simple sine-curve.

As a second example, let us consider the same circuit as before. But now suppose the frequency is just half what it was in the first example, namely, 79.5, or that $\omega = 500$. Furthermore, suppose the E. M. F. is such that it will send a maximum current of .5 of an ampere through the circuit. It will be found, upon calculation, that the E. M. F. must be 1320 volts, maximum. With these values, then,

$E = 1320.,$
$R = 50.,$
$L = 2.,$
$C = .55,$
$\omega = 500,$
$T = .08$ seconds,
$I = .5$ amperes,
$\theta = 88^\circ\ 55',\qquad \tan\theta = 52.8,$
$a = 955,$

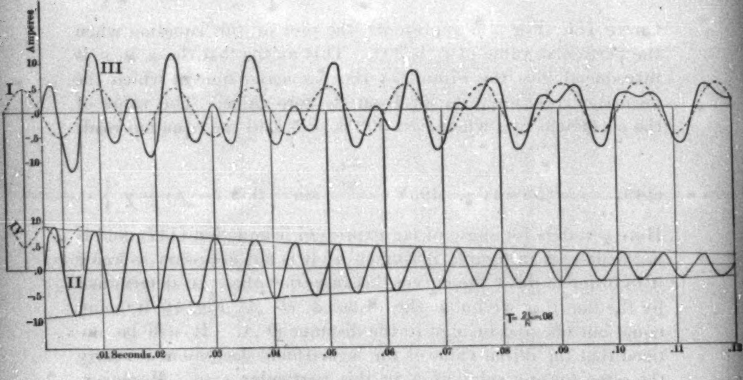


FIG. 16. Curve III. shows the current which flows after the introduction of harmonic E. M. F. into a circuit with R, L, and C. It is the sum of the two component curves, I. a sine-curve, and II. a sine-curve with an amplitude decreasing according to a logarithmic decrement.

the equation for the current becomes

$$(150)\quad i = .5 \sin\psi - .955\sqrt{-.725\sin^2\psi_1 + .0069\sin 2\psi_1 + 1}\;\varepsilon^{-\frac{t-t_1}{.08}}\sin\left\{955\,(t-t_1) + \chi\right\}.$$

The plot of this equation, when ψ_1 is taken equal to 180° (that is the E. M. F. is introduced when the regular current curve is zero) is shown in Fig. 16. It will be noticed that the initial value of the logarithmic curve has considerable variation according to the particular point of time at which the E. M. F. is introduced. This variation is represented in the curve IV., Fig. 16. The initial value of the logarithmic decrement at 0° or 180° is almost twice as much as the maximum value of the current I, their ratio being $\frac{.955}{.5}$. The equation, when ψ_1 is 180°, reduces to

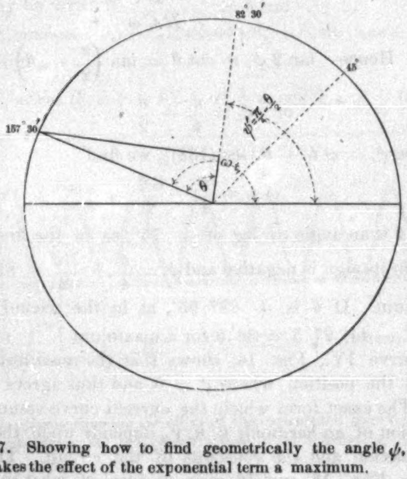


FIG. 17. Showing how to find geometrically the angle ψ, which makes the effect of the exponential term a maximum.

$$(151) \qquad i = .5 \sin \varphi - .955\, \varepsilon^{-\frac{(t-t_1)}{.08}} \sin \left\{ 955\,(t - t_1) + \chi \right\}.$$

In each of the above examples the current follows the sine law in about one-quarter of a second after the periodic E. M. F. is introduced, during which time somewhere in the neighborhood of forty oscillations have been made. It may be interesting to inquire at what point the E. M. F. should be introduced into the circuit to render the effect of the oscillation a maximum. This point may readily be found by referring to equation (147). The coefficient of ε becomes a maximum (for a variation in t_1) when the quantity under the radical sign is a maximum. Differentiating the quantity under the radical, then, with respect to t_1, and equating to zero, we obtain

$$(152) \qquad (L\,C\,\omega^2 - 1) \sin 2\,\psi_1 + R\,C\,\omega \cos 2\,\psi_1 = 0.$$

$$\text{Whence} \quad \tan 2\,\psi_1 = \frac{R\,C\,\omega}{1 - L\,C\,\omega^2}.$$

But it will be remembered that [see equation (130)]

$$\tan\theta = \frac{1 - L\,C\,\omega^2}{R\,C\,\omega}.$$

$$\text{Hence} \quad \tan 2\,\psi_1 = \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right).$$

$$(153) \qquad \text{or} \quad \psi_1 = \frac{\pi}{4} - \frac{\theta}{2}.$$

And since $\psi_1 = \omega\,t_1 + \theta$ [see (138)], we find

$$(154) \qquad \omega\,t_1 = \frac{\pi}{4} - \frac{3\,\theta}{2}.$$

Suppose θ is an angle of lag of $-75°$, as in the first example cited, then its sign is negative and $\psi_1 = \frac{\pi}{4} + \frac{75°}{2} = 82°\ 30'$ for a maximum. If θ is $+ 88°\ 55'$, as in the second example, $\psi_1 = 45° - 44°\ 27'.5 = 32'.5$ for a maximum.

The curve IV., Fig. 16, shows that the maximum point is nearly at the position where $\psi = 0$, and thus agrees with this result. The exact form which the current curve assumes at the introduction of an harmonic E. M. F., depends upon the time of its introduction and the constants of the circuit. The curves shown in Figs. 15 and 16 give an idea of what may be expected in other cases. In all cases, after a very few periods, the current reaches the simple sine form.

The current which flows upon making a circuit which contains resistance and self-induction, but no capacity, is shown in Fig. 18. Its equation is, in general,

$$i = I \sin\psi - I\,\varepsilon^{-\frac{R}{L}(t - t_1)} \sin\psi_1,$$

and is represented in curve III. Curves I. and II. represent the first and second terms of the equation respectively.

It is noticeable that, when there is only resistance and self-induction in the circuit, the E. M. F. may be introduced at such a time that the exponential term will have no effect, whereas if there are all three, resistance, self-induction and capacity, there is no point of time at which the E. M. F. could be introduced so as to make the effect zero.

CASE IV. ANY PERIODIC E. M. F.

If we suppose that the impressed E. M. F. is made up of a number of simple harmonic E. M. F.'s added together, the impressed E. M. F. may be written

$$(155)\quad \varepsilon = E_1 \sin(b_1 \omega t + \theta_1) + E_2 \sin(b_2 \omega t + \theta_2) + E_3 \sin(b_3 \omega t + \theta_3) + \text{etc.}$$

and, therefore,

$$\frac{d\,e}{d\,t} = E_1\, b_1\, \omega \cos(b_1\, \omega\, t + \theta_1) + E_2\, b_2\, \omega \cos(b_2\, \omega\, t + \theta_2) + \text{etc.}$$

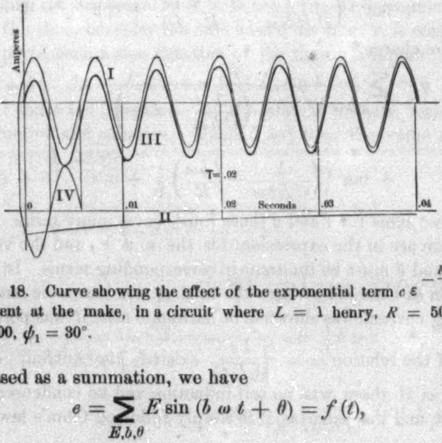


FIG. 18. Curve showing the effect of the exponential term $c\,\varepsilon^{-\frac{R\,t}{L}}$ upon the current at the make, in a circuit where $L = 1$ henry, $R = 50$ ohms, $\omega = 1000$, $\psi_1 = 30°$.

Expressed as a summation, we have

$$(156)\qquad e = \sum_{E,b,\theta} E \sin(b\,\omega\,t + \theta) = f(t),$$

$$(157)\qquad \frac{d\,e}{d\,t} = \omega \sum_{E,b,\theta} E\,b \cos(b\,\omega\,t + \theta) = f'(t).$$

In this summation it is to be understood that E and θ take in succession any values, fractional or integral, but that b may only have positive integral values as the E. M. F. is supposed to be periodic, and consequently the periods of the component sine curves must be commensurable. It was shown by Fourier, in his treatise on the Analytical Theory of Heat, published in 1822, that such an expression as (155) or (156) represents any single-valued periodic function whatever, and is therefore an expression which represents any possible E. M. F. whatever. If (157) is substituted in the general equation for current (17), and (156) in the general equation for charge (18), it will be found, upon integrating, that each component term in the E. M. F. gives a term in the current or charge similar to that given in equations (114) and (115) in Case III., and consequently the resultant current may be expressed as a summation thus

$$(158)\quad i=\sum_{E,b,\theta}\frac{E}{\sqrt{R^2+\left(\frac{1}{Cb\omega}-Lb\omega\right)^2}}\sin\left\{b\omega t+\theta+\tan^{-1}\left(\frac{1}{CRb\omega}-\frac{Lb\omega}{R}\right)\right\}+c_1\varepsilon^{-\frac{t}{T_1}}+c_2\varepsilon^{-\frac{t}{T_2}},$$

and the charge

$$(159)\quad q=\sum_{E,b,\theta}\frac{E}{b\omega\sqrt{R^2+\left(\frac{1}{Cb\omega}-Lb\omega\right)^2}}\cos\left\{b\omega t+\theta+\tan^{-1}\left(\frac{1}{CRb\omega}-\frac{Lb\omega}{R}\right)\right\}+c_3\varepsilon^{-\frac{t}{T_1}}+c_4\varepsilon^{-\frac{t}{T_2}}.$$

In these sums for i and q there must be as many terms in each as there are in the expression for the E. M F., and the values of E, b, and θ must be the same in corresponding terms. In the discussion of Case III., where the E. M. F. was harmonic and the resulting current was shown to be harmonic also, it was pointed out that if the relation $\omega=\frac{1}{\sqrt{LC}}$ existed, the current was the same as if there was no self-induction and no condenser in the circuit, and the same as if it simply followed Ohm's law. This was shown by simply substituting the relation $\omega=\frac{1}{\sqrt{LC}}$, or $\frac{1}{C\omega}-L\omega=0$, in the current or charge equations (114) and (115) and neglecting the complementary function. Those equations, with these substitutions, become

$$(160)\qquad i=\frac{E}{R}\sin\omega t\ .$$

$$q = -\frac{E}{R\omega}\cos\omega t\,. \tag{161}$$

It is seen that current and charge are the same at every point of time as if the self-induction and capacity were absent. Now, since the current is the same at every point of time, the effects of this current will be the same; namely, the quantity which flows in a half period, being $\int_0^{\frac{T}{2}} i\,d\,t = Q$, is the same as when there is no self-induction and capacity, and the energy expended in the circuit in performing work, or in heating effects, is likewise the same, being proportional to $\int i^2\,d\,t$.

In order to ascertain whether some similar relation between self-induction and capacity would cause them to neutralize each other when the impressed E. M. F. is not a simple harmonic function of the time, consider the case where the E. M. F. is composed of two parts, each a sine function of the time. Suppose

$$e = E_1 \sin a\,\omega\,t + E_2 \sin b\,\omega\,t, \tag{162}$$

where a and b are integers. In the circuit there is resistance, self-induction and capacity. Then at any time the value of the current is [see (158)]

$$\begin{aligned} i = {} & \frac{E_1}{\sqrt{R^2 + \left(L\,a\,\omega - \frac{1}{C\,a\,\omega}\right)^2}} \sin\left\{a\,\omega\,t + \tan^{-1}\frac{1}{R}\left(\frac{1}{C\,a\,\omega} - L\,a\,\omega\right)\right\} \\ & + \frac{E_2}{\sqrt{R^2 + \left(L\,b\,\omega - \frac{1}{C\,b\,\omega}\right)^2}} \sin\left\{b\,\omega\,t + \tan^{-1}\frac{1}{R}\left(\frac{1}{C\,b\,\omega} - L\,b\,\omega\right)\right\}. \end{aligned} \tag{163}$$

Suppose the self-induction and capacity have the relation $a\,\omega = \frac{1}{\sqrt{L\,C}}$. Then they will neutralize each other in the first term of the above expression for the instantaneous value of the current. But in the second term the relation $b\,\omega = \frac{1}{\sqrt{L\,C}}$ is necessary to cause the self-induction and capacity to neutralize each other. Now, if one of the above terms is changed by the introduction of self-induction and capacity, while the other term is unaffected, the value of the current which is equal to the sum of the two terms must be changed. It therefore follows that neither the relation $a\,\omega = \frac{1}{\sqrt{L\,C}}$, nor $b\,\omega = \frac{1}{\sqrt{L\,C}}$, will cause the self-induction and capacity to neutralize each other when introduced into a circuit containing an impressed E. M. F. composed of two simple harmonic E. M. F.'s with angular velocities $a\,\omega$ and $b\,\omega$. If $a = b$, the two terms in the expression for the instantaneous value of the current may be written as one, and we have a simple harmonic function of the time. The relation $a\,\omega = b\,\omega = \frac{1}{\sqrt{L\,C}}$ will then cause the self-induction and capacity to neutralize each other.

If $E_1 = 0$, or if $E_2 = 0$, then we have a simple sine function, and the relation $b\,\omega = \frac{1}{\sqrt{L\,C}}$, or $a\,\omega = \frac{1}{\sqrt{L\,C}}$, respectively, will cause the balancing of the self induction and capacity.

In order to ascertain the conditions under which there may be self-induction and capacity in a circuit, just neutralizing each other so that the instantaneous values of the current will be the same as though there were no self-induction and capacity in the circuit, we will consider the general differential equation of E. M. F.'s

$$(164) \qquad e = R\,i + L\frac{d\,i}{d\,t} + \frac{\int i\,d\,t}{C}\,.$$

[See equation (3).] We wish to ascertain the conditions under which the current will be the same as when there is neither self-induction nor capacity, that is, the conditions under which $i = \frac{e}{R}$ and $e = R\,i$, according to Ohm's law. Substituting in the above equation, we have

$$(165) \qquad L\,\frac{d\,i}{d\,t} + \frac{\int i\,d\,t}{C} = 0\,.$$

This is the same as saying that the E. M. F.'s of self-induction and capacity are equal and opposite. By differentiation,

$$\frac{d^2 i}{d\,t} = -\frac{i\,d\,t}{L\,C}\,.$$

Multiplying by $\frac{d\,i}{d\,t}$,

$$\left(\frac{d\,i}{d\,t}\right) d\left(\frac{d\,i}{d\,t}\right) = -\frac{i\,d\,i}{L\,C}.$$

By integrating, we have

$$\left(\frac{d\,i}{d\,t}\right)^2 = -\frac{i^2}{L\,C} + c.$$

$$\frac{d\,i}{d\,t} = \pm\sqrt{c - \frac{i^2}{L\,C}}.$$

The variables may be readily separated, thus

$$(166) \qquad \frac{d\,i}{\sqrt{c - \frac{i^2}{L\,C}}} = d\,t.$$

The integral of (166) is obtained by the formula of integration,

$$\int \frac{d\,x}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}.$$

Upon integration (166) becomes

$$\sin^{-1}\frac{i}{\sqrt{c\,L\,C}} = \frac{t}{\sqrt{L\,C}} + c_1.$$

Taking the sine of each member and writing c' for $\sqrt{c\,L\,C}$,

$$(167) \qquad i = c' \sin\left(\frac{t}{\sqrt{L\,C}} + c_1\right).$$

The only two variables in this equation are i and t, and the current is seen to be a sine-function of the time. When the current is a maximum the sine is unity, and we have

$$I = c'.$$

If the time is reckoned from the point where the current is zero, $t = 0$ when $i = 0$, and we have

$$c_1 = 0.$$

Substituting these values for the constants c' and c_1, in equation (167),

$$(168) \qquad i = I \sin\frac{1}{\sqrt{L\,C}}t.$$

In an harmonic function, as this, the coefficient of the variable t is the angular velocity which we designate by ω. Equation (168) then becomes

$$i = I \sin \omega t. \tag{169}$$

We have, then, the necessary conditions under which the self-induction and capacity will just neutralize each other at every point of time. The current must be a simple sine-function of the time, as expressed in equation (169), and the self-induction and capacity must have such values that $\omega = \frac{1}{\sqrt{LC}}$. Under no other conditions, with self-induction and capacity in a circuit, can the instantaneous values of the current be the same as though the capacity and self-induction were absent.

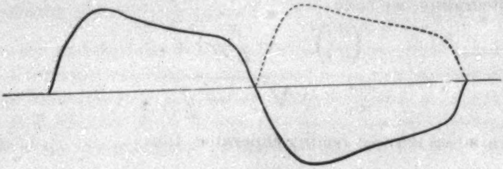


FIG. 19.

Since we have found that there is no possible relation that L and C can have so that the *instantaneous* values of the current are unchanged by their introduction into a circuit with an impressed E. M. F. which is not an harmonic function, it is interesting to inquire whether any relation can be given L and C so that the *energy* spent in the conductor in a given time is the same before as after the introduction of L and C.

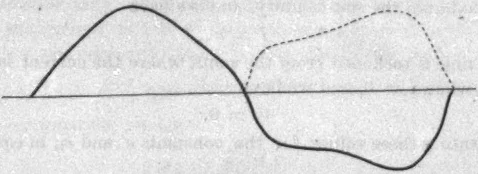


FIG. 20.

Before attempting to investigate such a relation, it will be well to first consider some different classes of current curves, then ascertain the $\int i^2 \, d\, t$ effect for some particular current curves and afterwards consider the energy of any periodic curve whatever.

Fig. 19 represents a curve which has an equal area above and below the axis every period. This means that the integral $\int i\, d\, t$ for one period is zero, that is, the quantity of electricity which flows each period in the positive direction is equal to that which flows in the negative direction. Moreover, if the lower half of the current curve in Fig. 19 is inverted and represented by the dotted line, it is an exact repetition of the first half of the curve. This curve may represent the type of current curves given by alternating generators in circuits with resistance, self-induction and capacity, but no counter E. M. F. For it is evident that as the armature revolves, the number of lines introduced into the circuit every period equals those taken from the circuit. Now, the quantity of current which flows is strictly proportional to the change in the number of lines threading the circuit. This is equivalent to saying that the quantity which flows in the positive direction is exactly equal to the quantity flowing in the negative direction, or the total algebraic quantity per period is zero. Now, if the generator is exactly symmetrical, the current curve in the second half of the period is, if inverted, an exact repetition of the curve in the first half. Any irregularities in the symmetry of the machine might cause slight differences in the two parts of the curve, but hardly enough to prevent this curve from representing the type of curves given by alternating machines. During every complete revolution of the armature, the total algebraic quantity of current flowing must be rigorously equal to zero, no matter how many irregularities there may be in the machine; for, the number of lines introduced into the circuit exactly equals those subtracted from the circuit, because after a complete revolution the number of lines is the same as at the start. It is possible that adjacent positive and negative areas may be unequal in a multipolar machine, due to some irregularity in the machine, but after

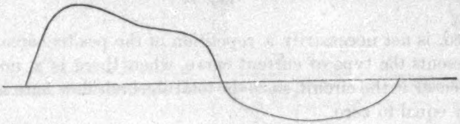


FIG. 21.

a complete revolution of the armature the sum of the positive areas equals the sum of the negative

Fig. 20 represents a current curve which has equal areas above and below the axis every period, but the negative area, when inverted, is not necessarily a repetition of the positive area. This represents the type of current curve when there is a non-leaky condenser in the circuit, since the total algebraic flow here is necessarily equal to zero.

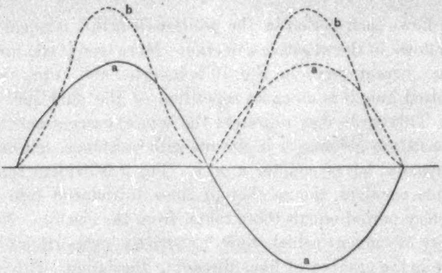


FIG. 22.

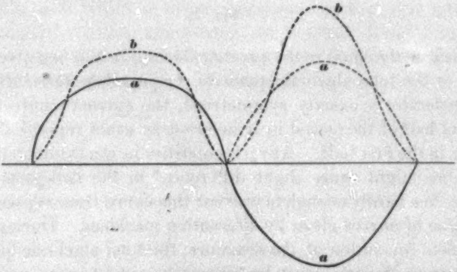


FIG. 23.

Fig. 21 represents a current curve in which the negative area is neither equal to the positive area nor symmetrical with it when inverted.

It is interesting to inquire whether the $\int i^2\, d\, t$ effect is the same in a circuit while the current flows in the positive direction as it is while flowing in the negative direction. We can see that it is the same for a current of the type represented in Fig. 19, for, squaring the ordinate at each point and drawing a new curve, *b*, Fig. 22, the $\int i^2\,d\,t$ effect is proportional to the areas of this new curve. Since the current curves *a*, *a* are exact repetitions, these areas, *b*, *b*, are identical, and the $\int i^2\,d\,t$ effect is the same when the current is positive as it is when negative.

Let us inquire how this is for a current of the type of Fig. 20 where the areas are equal, that is, the $\int i\,d\,t$ is the same for positive as for negative current), but the negative part, when inverted, is not an exact repetition of the positive part. In Fig. 23 the areas between the axis and the current curve *a*, *a* are equal for each half period. The curve *b*, *b* is drawn by squaring each ordinate of the curve *a*. The areas *b*, *b* represent the $\int i^2\,d\,t$ effect, and we wish to find whether they are equal.

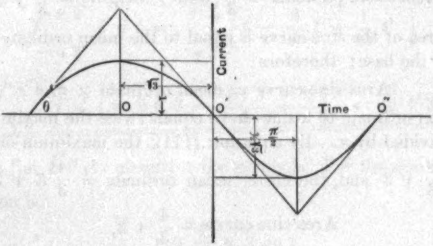


FIG. 24.

To show that this $\int i^2\,d\,t$ effect is not necessarily the same when the current is positive as when it is negative, it will suffice to take one particular case of a current curve. Suppose the positive curve is a parabola [Fig. 24], whose equation, referred to O as origin, is

$$(170) \qquad t^2 = -3\,i + 3.$$

Suppose the negative curve is a sine-curve whose equation, referred to O″ as origin, is

(171) $$i = \frac{2}{3}\sqrt{3}\sin t.$$

It is easily shown that the areas of these curves are equal.

$$\text{Area parabola} = \frac{2}{3}\,[\text{base} \times \text{height}].$$

One-half of the base of the parabola is found by making $i = 0$ in equation (170) and finding the value of t.

$$\text{Therefore,} \qquad \frac{1}{2}\,\text{base} = \sqrt{3}\,,$$

$$\text{Base} = 2\sqrt{3}.$$

The height is found by making $t = 0$ in equation (170) and finding the value of i.

$$\text{Therefore,} \qquad \text{Height} = \text{unity}.$$

(172) $$\text{Hence area parabola} = \frac{2}{3}\,[\text{base} \times \text{height}] = \frac{4}{3}\sqrt{3}.$$

The area of the sine-curve is equal to the mean ordinate multiplied by the base; therefore

$$\text{Area sine-curve} = \text{mean ordinate} \times \pi.$$

The mean ordinate of a sine-curve equals twice the maximum ordinate divided by π. By equation (171), the maximum ordinate equals $\frac{2}{3}\sqrt{3}$ and, therefore, mean ordinate $= \frac{4}{3}\pi\sqrt{3}$, and

(173) $$\text{Area sine-curve} = \frac{4}{3}\sqrt{3},$$

which is identical with (172) above. Moreover, the tangents of the angles, which these two curves make at the point O with the axis, are equal, and the curves consequently blend into one another without any abrupt change in continuity. This is easily shown as follows: Differentiating (170) and (171) respectively, we have

(174) $$\frac{di}{dt} = -\frac{2}{3}\,t = \tan\theta.$$

(175) $$\frac{di}{dt} = \frac{2}{3}\sqrt{3}\cos t = \tan\theta'.$$

Making $t = \sqrt{3}$ in (174), we have the tangent of the inclination of the parabola at the point O. Making $t = -\pi$ in (175), we have the tangent of the inclination of the sine-curve at the point O. These values, it is noticed, reduce (174) and (175) respectively to $\tan\theta = \tan\theta' = -\frac{2}{3}\sqrt{3}$, which is the value of the tangent of inclination of either curve at the point O.

It remains to find the $\int i^2 d\,t$ for each of these curves respectively. By transposing, the equation of the parabola (170) is

$$i = 1 - \frac{t^2}{3}\,.$$

By squaring,

$$i^2 = 1 - \frac{2}{3}t^2 + \frac{t^4}{9}\,.$$

$$\int i^2 d\,t = \int d\,t - \frac{2}{3}\int t^2\,d\,t + \frac{1}{9}\int t^4\,dt\,.$$

Integrating between the limits $-\sqrt{3}$ and $\sqrt{3}$, we have

$$\int_{-\sqrt{3}}^{\sqrt{3}} i^2\,d\,t = {}_{-\sqrt{3}}^{\sqrt{3}}\left[t - \frac{2}{9}t^3 + \frac{1}{45}t^5\right].$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} i^2\,d\,t = 2\sqrt{3} - \frac{4\sqrt{3^3}}{9} + \frac{2\sqrt{3^5}}{45} = \frac{16}{15}\sqrt{3}\,.$$

This is the $\int i^2\,d\,t$ effect for the parabola. For the sine-curve the equation is

$$i = \frac{2}{3}\sqrt{3}\sin \mathrm{t}$$

$$\int i^2\,d\,t = \frac{4}{3}\int \sin^2 t\,.$$

Integrating between the limits 0 and π,

$$\int_0^\pi i^2 d\,t = \frac{4}{3} \times {}_0^\pi\left[\frac{t}{2} - \frac{1}{2}\sin t\cos t\right] = \frac{4}{3}\times\frac{\pi}{2} = \frac{2}{3}\pi\,.$$

This gives the $\int i^2\,d\,t$ effect for the sine curve. Hence we find that, although the area of the current curve is the same for the positive and negative current—that is, the total algebraic quantity of flow is zero—yet the $\int i^2\,d\,t$ effect is different in the positive and negative directions. In the case supposed, the ratio of the two effects is

$$\frac{\frac{2}{3}\pi}{\frac{16}{15}\sqrt{3}} = 1.135.$$

This may afford an explanation for the fact that in many cases one carbon of an alternating current arc lamp is consumed more rapidly than the other, depending upon the way it is connected up.

Let us now return to the consideration of the energy in a conductor when any periodic E. M. F. is applied, and ascertain whether there is any condition under which self-induction and capacity may be introduced into the circuit without changing the energy or $\int i^2\,dt$ effect.

The energy expended in a conductor is proportional to $\int i^2 dt$. When the E. M. F. in the circuit is

$$(176)\qquad e = \sum_{E,b,\theta} E \sin (b\,\omega\,t + \theta), \qquad \text{[see (156)]},$$

which represents any periodic E. M. F., it has been shown that the current is

$$(177)\quad i = \sum_{E,b,\theta} \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\,b\,\omega} - L b \omega\right)^2}} \sin \left\{ b\,\omega\,t + \theta + \tan^{-1}\left(\frac{1}{C\,R\,b\,\omega} - \frac{L\,b\,\omega}{R}\right) \right\},$$

neglecting the complementary function [see (158)]. And when there is neither self-induction nor capacity the current is

$$(178)\qquad i_0 = \sum \frac{E}{R} \sin (b\,\omega\,t + \theta).$$

Substituting

$$(179)\qquad I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\,b\,\omega} - L\,b\,\omega\right)^2}},$$

$$(180)\qquad I_0 = \frac{E}{R},$$

$$\text{and}\qquad a = \theta + \tan^{-1}\left(\frac{1}{C R b \omega} - \frac{L\,b\,\omega}{R}\right),$$

we may abbreviate (177) and (178) as follows:

(181) $$i = \sum I \sin (b\,\omega\,t + a).$$

(182) $$i_0 = \sum I_0 \sin (b\,\omega\,t + \theta).$$

Remembering that the energy is proportional to $\int i^2\,d\,t$, we have

(183) $$W = \int i^2 d\,t = \int \left[\sum I \sin (b\,\omega\,t + a)\right]^2 d\,t,$$

and

(184) $$W_0 = \int i_0^2\,d\,t = \int \left[\sum I_0 \sin (b\,\omega\,t + \theta)\right]^2 d\,t,$$

where W is proportional to the energy expended in the circuit with L and C, and W_0 bears the same relation to the energy when they are absent. In order to find what relation must exist between L and C to cause the energy expended during a certain time to be the same in both cases, we must integrate (183) and (184) between the same limits of time, and equate them. In order to simplify (183) and (184), express as follows:

(185) $$W = \int \left[I' \sin (b_1\omega\,t + a_1) + I'' \sin (b_2\omega\,t + a_2) + I''' \text{etc.}\right]^2 d\,t.$$

(186) $$W_0 = \int \left[I_0' \sin(b_1\omega t + \theta_1) + I_0'' \sin (b_2\omega t + \theta_2) + I_0''' \text{etc.}\right]^2 d\,t.$$

Since the square of any polynomial is equal to the sum of the squares of each term separately plus twice the product of each term by every other term, we have as a result to find the integrals of only two forms, thus

(187) $$\int \sin^2 (b\,\omega\,t + a)\,d\,t, \text{ and}$$

(188) $$\int \sin (b_1\,\omega\,t + a_1) \sin (b_2\,\omega\,t + a_2).$$

If the limits are taken from $t = 0$ to $t = T$, a complete period, —the E.M.F. being periodic with a period $T = \frac{2\pi}{\omega}$ —it can be shown that all the integrals of the form of (188) vanish. For, expressing the sine of the sum of two angles in terms of the angles themselves,

(189) $$\sin (b_1\,\omega\,t + a_1) = \sin b_1\,\omega\,t \cos a_1 + \sin a_1 \cos b_1\,\omega\,t,$$

and

(190) $$\sin (b_2\,\omega\,t + a_2) = \sin b_2\,\omega\,t \cos a_2 + \sin a_2 \cos b_2\,\omega\,t.$$

Multiplying (189) and (190) we obtain terms of the following forms:

(191) $$\int \sin b_1 \omega t \cos b_2 \omega t \, d t,$$

(192) $$\int \cos b_1 \omega t \cos b_2 \omega t \, d t,$$

(193) $$\int \sin b_1 \omega t \sin b_2 \omega t \, d t,$$

which are to be integrated between the limits 0 and T, or $\frac{2\pi}{\omega}$. Substituting for $b_1 \omega t$, $a x$, and for $b_2 \omega t$, $b x$, we have made the integral in (188) depend upon the three forms,

(194) $$\int_0^{2\pi} \sin a x \cos b x \, d x,$$

(195) $$\int_0^{2\pi} \cos a x \cos b x \, d x,$$

(196) $$\int_0^{2\pi} \sin a x \sin b x \, d x.$$

To show that each of these three forms vanishes between the limits zero and 2π, we can reduce as follows:

(197) $$\int_0^{2\pi} \sin a x \cos b x \, d x = \frac{1}{2}\int_0^{2\pi} \sin (a+b) x \, d x + \frac{1}{2}\int_0^{2\pi} \sin (a-b) x \, d x = -\frac{1}{2} \, {}_0^{2\pi}\left[\frac{\cos (a+b) x}{a+b} + \frac{\cos (a-b) x}{a-b}\right] = 0.$$

(198) $$\int_0^{2\pi} \cos a x \cos b x \, d x = \frac{1}{2}\int_0^{2\pi} \cos (a+b) x \, d x + \frac{1}{2}\int_0^{2\pi} \cos (a-b) x \, d x = \frac{1}{2} \, {}_0^{2\pi}\left[\frac{\sin (a-b) x}{a-b} + \frac{\sin (a+b)}{a+b}\right] = 0.$$

(199) $$\int_0^{2\pi} \sin a x \sin b x \, d x = \frac{1}{2}\int_0^{2\pi} \cos (a-b) x \, d x - \frac{1}{2}\int_0^{2\pi} \cos (a+b) x \, d x = \frac{1}{2} \, {}_0^{2\pi}\left[\frac{\sin (a-b) x}{a-b} - \frac{\sin (a+b) x}{a+b}\right] = 0.$$

Since, therefore, the integral in (188) is zero in every case, we only have to find the integral expressed in (187). This is

$$(200) \int_0^{\frac{2\pi}{\omega}} \sin^2 (b\,\omega\,t + a)\,d\,t = \Big[_0^{\frac{2\pi}{\omega}} \frac{b\,\omega\,t + a}{2\,b\,\omega}$$

$$- \frac{1}{4\,b\,\omega} \sin 2\ (b\,\omega\,t + a)\Big] = \frac{\pi}{\omega} = \frac{T}{2},$$

which is obtained by the formula

$$(201) \qquad \int \sin^2 x\,d\,x = \frac{x}{2} - \frac{1}{4} \sin 2\,x,$$

upon replacing x by $b\,\omega\,t + a$, and $d\,x$ by $b\,\omega$. Returning to equations (185) and (186), and replacing the value of the integral in (200) by $\frac{T}{2}$, as determined, we have now found the values of W and W_0 to be

$$W = \left[I'^2 + I''^2 + I'''^2 + \text{etc.} \right] \frac{T}{2},$$

and

$$W_0 = \left[I_0'^2 + I_0''^2 + I_0'''^2 + \text{etc.} \right] \frac{T}{2},$$

or

$$W = \frac{T}{2} \sum I^2,$$

$$W_0 = \frac{T}{2} \sum I_0^2.$$

Equating W and W_0, as before explained, to determine the condition necessary to make the energy the same, we obtain

$$(202) \qquad \sum I^2 = \sum I_0^2,$$

which, written in full, is

$$(203) \qquad \sum \frac{E^2}{R^2 + \left(\frac{1}{C\,b\,\omega} - L\,b\,\omega\right)^2} = \sum \frac{E^2}{R^2}.$$

[See (179) and (180).] This equation expresses the relation which must be true if the $\int i^2\,d\,t$ effect is the same when the self-induction and capacity are present as it is when they are absent. This equation expressed without the sign of summation is

$$(204)\quad \frac{E_1^2}{R^2+\left(\frac{1}{C\,b_1\,\omega}-L\,b_1\,\omega\right)^2}+\frac{E_2^2}{R^2+\left(\frac{1}{C\,b_2\,\omega}-L\,b_2\,\omega\right)^2}+\text{etc.}$$

$$=\frac{E_1^2}{R^2}+\frac{E_2^2}{R^2}+\text{etc.}$$

It is evident that the parenthesis in the denominator of each term of the first member, being squared, is always positive no matter what values L and C may have. Each term, then, of the first member is less than the corresponding term in the second member unless the expression in the parenthesis is zero. And in order that the first member shall be as large as the second member, each parenthesis must be separately equal to zero. That is, we must have

$$b_1\,\omega=\frac{1}{\sqrt{L\,C}},$$

$$b_2\,\omega=\frac{1}{\sqrt{L\,C}},$$

$$\text{and}\quad b_3\,\omega=\frac{1}{\sqrt{L\,C}},\quad \text{etc.}$$

Therefore $b_1=b_2=b_3=$ etc. But this condition is equivalent to saying that the impressed E. M. F. can only be an harmonic E. M. F., and that we must have the relation $\omega=\frac{1}{\sqrt{L\,C}}$ in order to have the $\int i^2\,d\,t$ effect the same in a circuit when the self-induction and capacity are present as when they are absent. There is, then, no relation between the self-induction and capacity which can be given that will make the $\int i^2\,d\,t$ effect the same in a circuit when they are present as when they are absent, if the impressed E. M. F. is not an harmonic E. M. F.

A paper read at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 7th, 1892. President Sprague in the Chair.

ELECTRICITY IN BITUMINOUS MINING.

BY ELMER A. SPERRY.

Many a traveling Englishman on this side turns his eyeglass upon a swift succession of passing trains laden with barely a day's product from some unheard-of western colliery, and thinks that he instinctively recognizes pit coals from Northumberland or Lancashire. He supposes that "all coals come from Newcastle;" nor is this surprising. The average eastern man may be said to be almost unaware of the existence of bituminous fuel. To him anthracite is coal, and Pennsylvania is the place of its nativity. Yet, last year 150,000,000 tons of bituminous coal were mined in the United States, principally in the west.

Of the total anthracite product 78.29 per cent. was shipped to Pennsylvania, New York, New Jersey and New England. To one whose attention has not been directed to the coal industry, it is a matter of surprise that extensive mining operations in coal are going forward in the Mississippi valley. On one of the affluents of the Mississippi river alone, in 1888, was shipped nearly as much coal as was in that year mined in all the continental countries of Europe together. In this valley there are 152,000 square miles of coal area. Illinois alone has 36,000 square miles. In this State $18,000,000 is invested, $11,000,000 is annually disbursed, and about 40,000 miners employed in bituminous mines. These areas of the West are large when compared with those of the Eastern States. The coal area of Great Britain is the same as that of the State of Pennsylvania—9,000 square miles. The coal area in the Mississippi valley is 30,000 square miles in excess of the total superficial area of England, Ireland, Scotland and Wales.

England, doubtless, has the honor of being the first to give up this, the most important of the mineral products. Abundant evidence exists that some of the English outcroppings were worked by the Latins during the Roman occupation. Heaps of debris and cinders from the ancient iron furnaces have been found which were worked at that period, so great in extent that modern furnaces are being supplied from them. While it is not clear that coal was used, yet the fact that the Latins introduced the powerful artifical blast renders its use probable, and may explain the large coal workings of the period.

As to implements and machinery, it is true of the various regions and countries of the old world that if you would know the tools used at an early period you have but to study the tools of to-day. The excavations in the most ancient ruins of civilization show implements used by ancient and contemporary artisans to be the same. Moulds sunk in slabs of limestone used by the most ancient Egyptian silversmiths are identical with those used to-day and sunk in the same limestone. The iron and bronze implements shown in the ancient traceries have been exhumed, together with the drawings, and compare favorably with the picks and shovels used in their discovery. In Nineveh nearly two tons of iron implements were found, among which were bars, shovels, saws, picks, etc. The picks evince excellent workmanship; they are single and double ended, the double in some instances provided with an edge at one end and a point at the other, the eye well formed with extended lips upon the inside for more firmly securing the wooden handle.

There is no doubt but that the implements and tools seen to-day in use in the Lancashire mines, 3,000 feet below the surface, are the same as used by the Latins and Saxons in the ancient coal workings. The motive power has remained unchanged during the centuries; the instrument is good; the method of attack is simply perfect, and cannot be improved. The system, however, has radical defects in point of expense, controllability, protracted effort, want of uniformity, deficient and small power units. The tendency of the times is toward centralization, and a large unit will be the integer of the future. While others among the older industries have adapted improved methods, the mining of coal has remained almost stationary. Ore mining has been revolutionized in many respects; extensive plants of machinery being adopted have been found to increase the capacity and cheapen the product. In isolated cases coal mines have been equipped with labor saving machinery, and the enterprising operator has been richly rewarded.

Veins that were operated at a loss have by the use of machinery been enabled to yield profitable results. In this line doubtless the next decade will witness great advancement. The problem of replacing manual labor in coal mining has engrossed the labors and thought of the ablest engineers of the age. Some of the earliest patents granted by the English monarchs were for mechanical mining devices, with an avowed object of superseding manual labor. These machines were driven by animal or hydraulic power, and more recently by steam and compressed air. They sawed, scraped, cut and plowed, but principally struck a blow. Many, even to a recent date, swung a pick provided with the ordinary transverse handle or power connection. Few, if any of these machines, have been successful under the severe duties required, and none has withstood the test of time. Machines generally employing cutting or scraping devices are of limited application. They are worked with success where the vein is clear; but this, however, is the exception. The presence of iron pyrites or other foreign substances in the strata quickly dulls the knives and requires such heavy driving powers as to preclude commercial use. I have witnessed the operation of this type of machine in strata excellently adapted for machine mining except for a frequent deposit of sulphur about the thickness of a silver dime. The cutters being made of the finest Mushet steel, the entire set required changing every 40 square feet undercut. The current readings would show a power consumption on an average of 27 horse power, whereas in clear coal 10.5 horse power only were required, and one change of cutters for each 250 square feet undercut.

With a more or less friable material, and especially where a cubical or angular crystal exists, great dislodgement and penetration is accomplished by the percussive blow. Its amount is proportional to the stored energy in the projectile. If the energizing motor is deficient, the strokes or impulses must of necessity be feeble; but now that almost unlimited power is at our disposal, the force and work possible is only limited by the rigidity and durability of the immediately attacking medium, namely the tool or cutter. With this form of machine, and especially if organized to be manually directable, the penetration may be pushed at a rapid pace, the foreign substances may be cut about, dislodged and removed without the necessity of directly attacking the substance itself.

These foreign substances are the bane of mechanical coal cutting. They exist in the form of thin layers, thick layers, balls and boulder. The various forms are known to the miner as sulphur balls, lime balls, boulders, bone coal, etc. They are of extreme density, and their hardness is so great that even with the diamond drill they are considered the most difficult mineral substance encountered.

In some instances, especially in the deeper mines and often the thinner veins, it is possible to undermine the coal by excavating in the underlying strata, consisting principally of fire clay and sometimes of a dense slaty and rather heterogenous mass known as "blackjack." In both these materials foreign substances often exist in the form of mineral deposits.

With this foreign matter occurring in irregular forms and in masses of irregular distribution, it will at once be recognized that breakers are ahead of the designer, and that good engineers have been baffled in this field is not a matter of so much surprise.

Successful excavating machinery for bituminous coal is destined to be the product of an evolution, the result of accumulated experiences from various sources and districts, derived principally from the carefully observed action of machines in operation as well as the no less perfect miner, to whose tender mercies the result sometimes of months of work and patience is to be entrusted. This cannot be done by proxy. The environments and conditions are often far from inviting, nor are they entirely free from danger. There seems, however, to be no shorter cut to success. The widely varying systems of mining employed in the different districts render it still farther difficult to construct any single machine which is equally well adapted to all. Such a universal machine is, however, none the less desirable and is doubtless approached most closely by the direct blow or reciprocating action, imitating in this feature the time-honored pick, placing a force behind it never before obtainable. Such a machine may be perfectly guided and directed while in operation. It is the only machine that may be used under almost any condition or quality of coal, foreign substances, underlying strata, or system of mining. It may be used for entry driving, under-cutting, digging, holing or shearing. It may be well here to give a brief statement of the different systems of operation usually met with in the bituminous mines.

After coal is reached, the slope or shaft is divided for ventilation, the air reaching the vein through one division and being exhausted through the other. At this point two or three parallel entries are driven with "break-throughs" at intervals so that the air may be conducted to or near the heading, and at intervals, laterals or "butt entries" are driven, usually at right angles to the main entrance. These butt entries are also driven in pairs, and "overcasts" or "undercasts" are used to connect from one of these entries to the remote main entry. Connections are made through the pillar between these entries in the same manner as in the main entries, multiple distribution being employed throughout. From the butt entries, rooms are turned narrow at first, but widening out from 20 to 60 feet, according to the nature of the roof, leaving a solid pillar of coal between the adjacent rooms approximately equal to the width of the room. As the rooms grow deeper the ribs are broken through for the purpose of ventilation, and curtains are hung in the butt entries which force a small amount of air down through one room, and out through the next, etc. Rooms are usually driven to a depth of 100 yards. It is in these rooms that the mining operations go forward. The product is loaded into pit cars and thus reaches the surface. In rare instances the roof will allow of a number of these rooms being thrown into one, giving a long face of several hundred feet. This is called a "modified longwall" system, as a part of the mine is usually worked ordinary "room and pillar," and in a few places the extended facings are found. In some instances the measure is so dense as to render undercutting out of the question. In these mines the coal is blasted "from the solid," as in anthracite mining. This, however, is not considered good practice in bituminous mines as the product becomes "powder-shot." The slack in this method will run as high as 50 per cent. of the product, and the lump will not bear shipment or the weather without crumbling.

The system of mining employed at the breasts, consists of excavations in the coal or in the underlying strata to a depth about equal to the height of the vein, then by boring near the roof, holes 7 to 15 feet apart, in which light powder charges are exploded for the purpose of tipping out and bringing down the entire breast, which is still farther broken by picks and loaded into cars. After being undermined, in some instances the coal is difficult of dislodgement owing to adherence to the ribs or lateral pillars of the room. To provide against this, machines mounted on high wheels are used to shear in at each side, a depth of 4 or 4½ feet from the floor to the ceiling, so as to entirely separate the portion undermined from the pillars at each side. This operation of shearing is sometimes exclusively employed when the sulphurs are too numerous in the lower strata. The rooms in this instance are sheared in at either end and sometimes in the middle, and the coal is brought down by drilling in an angle and blasting.

Another system of mining prevalent in the northern district of Illinois, and in isolated cases in Iowa, Missouri and Western Pennsylvania is the "longwall" proper. This is among the modern methods of mining. From the shaft "bottom" single entries are driven in four directions through the vein to a distance of about 100 feet, then laterals are taken off each way and driven until they meet. This leaves a pillar of coal to support the top works about 200 feet square. This pillar is never interfered with. After the circuit is entirely completed around this pillar the work is pushed forward on all parts of the facing simultaneously, all the coal being removed; no coal being left, the roof is meanwhile supported by props until at last it gives way and gradually settles down crushing the props, no matter how numerous or how strong they may be. At the same time the floor of the mine "heaves" until the roof and floor practically meet. The rock is taken down overhead in the entries, so that the entries practically consist of tunnels excavated in the "top rock." The long wall facing is pushed forward at all points equally distant from the "bottom," the solid vein of coal ahead supporting the roof in the immediate vicinity, but back from the face a few feet the roof gradually descends until the roof and floor meet, crushing as it would seem the forest of props that is used at the face, entirely out of existence.

The ventilation in this system consists in dividing the air in multiple, taking it up alternate entries and after traveling the face bringing it back through others which are brought together near the "bottom" to the upcast, at which point large furnaces or mechanical ventilators are employed for keeping the air in constant circulation.

Roadways driven off from the main entry like the veinlets of a leaf reach the facing about every 50 feet. The distance between any two of these face entries is called a room, and is worked by two miners. The rock taken from the roof to gain extra height in the entries, is blocked between the floor and the roof near the face, leaving a space for the miners of but from two to three feet from the face. The extreme smallness of this working space may be well understood when it is considered that the thickness or height of the vein is sometimes as low as 18 or 20 inches. Wherever a point of sight may be obtained from these facings back toward the main entries the roof can easily be seen bending downward. The "top rock" is fractured and must be supported at numerous places by props and cap pieces driven in place by heavy sledges. Back a few feet from the face the crushing effect of the enormous weight above can be seen upon these props. The bodies of oak trees six to eight inches in diameter may be seen forcing themselves into the solid rock and curling over at the extremities, exhibiting graphically the enormous pressure at work upon them. Falls of rock are frequent, and constant vigilance is the price of safety at the working face, in a mine of this character.

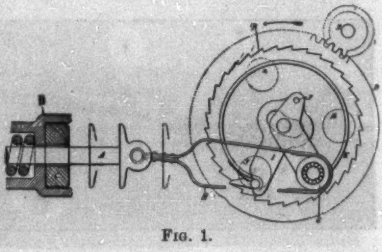


FIG. 1.

The excavating process consists in undercutting in the coal or underlying strata of clay to about the depth of the thickness of the vein, and allowing the constantly descending super-strata to break off the projecting portions of the undermined coal; this occurs from 30 minutes to two hours after the undermining process is completed. This is still further broken up by picks, and loaded as previously described. Drilling or blasting is very seldom resorted to in this system of mining.

I have thus briefly pointed out the methods employed that the conditions may be better understood in the discussion now to follow of electrical machinery as adapted for use in such mines. A diagrammatic outline of the direct blow machine is shown in Fig. 1, the completed machine and parts being shown in Figs. 2 and 3. In this form of machine, a bi-wheeled truck furnished with handles at the rear is provided with a continuously operating electric motor of about four horse power, connected by gearing to a catch or retractor; a longitudinally moving heavy projectile with a pick secured to its outer extremity, is surrounded by a driving spring of about 125 pounds' compression per inch. The line of movement is practically through the centre of inertia of the machine. These springs are preferably made in sections, and an initial pressure is given of about 500 pounds. A retraction then takes place from 6½ to 7½ inches to a pressure of 1,375 pounds whereupon it is released and allowed to make a vigorous forward stroke. This process is repeated from 160 to 225 times per minute. The mean effective of this type of machine is 935 pounds when allowed to strike the cushion, and when striking the work at a distance of one inch from such cushion giving 1,000 pounds mean effective. This is shown in the spring curve diagram, Fig. 4.

FIG. 2.

The mean effective in air machinery employed for the same purpose is between 400 and 450 pounds, a 3¾ inch cylinder being the largest employed. The current required for the operation of this machine is between 10 and 10.2 amperes at 220 volts. The projectile weighs between 100 and 150 pounds in the different sizes of machines. Early in the development of this machine it was found that under the action of the intense concussion resulting from the percussive blows and shattering strains throughout the machine, a motor to last under such conditions must be essentially different in its organization from the ordinary commercial machine. After working along various lines it was at last found that by mounting the driving pinion loosely upon the armature shaft, providing it with wing-like projections between which and the armature shaft elastic cushions such as blocks of rubber could be inserted, the trouble could be averted, and when adopted it was found that the repairs on armatures were reduced to a small percentage of their former value. An elastic medium was also employed in the commutator to account for the difference in expansion of the insulated copper segments and the steel commutator shell. Numerous driving keys or fins, consisting of rawhide, were set in the periphery of the armature and allowed to protude for the purpose of securing the coils against circumferential displacement. The fields were not found to give trouble. With this construction of armature a large number of machines have been made and run successfully in mines in Illinois, Indiana, Ohio and Kentucky. In most instances these machines are run constantly day and night.

A detailed report of each day's work of these machines has been kept from the very first, stating the exact nature of any repairs, the name of the "machine runner," the number of feet undercut, hours of work, etc. These reports are kept on file, and have been of great value in aiding to eliminate the troubles developed. In Ohio alone these reports show that upward of 550,000 square feet of coal strata have been undercut by this type of machine. A somewhat larger amount than this has been excavated in Illinois. In the States of Indiana and Kentucky the operations have not been of so long duration, but at present the daily cutting in the latter States exceeds those in the former.

FIG. 3.

Beside the elasticity introduced in the armature gear, elastic cushions have also been introduced between the retracting device and the main gear, which is found to still farther reduce the shock to the machine. At an early date it was found that a machine capable of storing in each stroke of the heavy projectile as great an effort as is required for rapid and effective cutting, must of necessity be heavy to be capable of backing up the blow without serious recoil, yet the machine on the other hand should be light for easy handling and direction by a single operator. With a light machine it is found impossible to allow the cutter to strike the work sufficiently back in the stroke for its required penetration, without serious effects from recoil. The operator would complain bitterly of the machine running back upon him, jerking him about upon the floor of the mine, making accuracy of direction impossible, and yet lightness of the machine is important. To combine both qualities I devised a cycle of operation shown in the diagram, the object being to immediately start upon the act of retraction at the moment the projectile had been arrested by the work, no matter at what point in the stroke, eliminating thereby the time element necessary for overcoming the inertia of the body of the machine, and starting it backward in recoil by the intense backward spring pressures. The details are simple and readily understood. [Fig. 1.] The projectile *A* is forced forward by the spring *B;* a cushion *C* relieves the shock when the projectile is not otherwise intercepted. A gear wheel *D* is constantly driven in the direction of the arrow by the motor pinion *E;* this wheel is provided with internal ratchet teeth *R*. A crank *G* is mounted entirely free from the wheel and co-operates with a long slotted pitman *H* connected with the projectile for retracting the same. A light steel crank arm is pivoted, entirely free

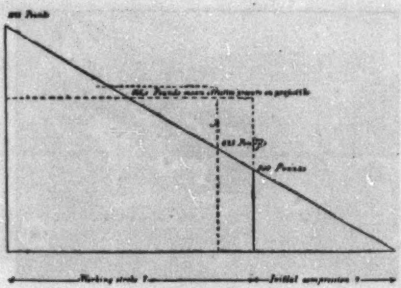


FIG. 4.

of the wheel *D*, and an auxiliary arm *I* provided with a dog coöperating with the ratchet teeth *R*, is pivoted to the main crank arm at point *J* and connected with the crank arm through the circular steel spring *K K*. When the crank *G*, reaches the back dead centre at point *M* a reversal of the strains and a free forward movement occurs in the direction of the rotation of the wheel, but very much faster under the impetus of the heavy compression of the driving spring *B*, the pitman and both portions of the crank moving forward together. Suppose now that the forward stroke was intercepted at the point shown in dotted line near the letter *A*; at this moment the crank *G* would be found in the position *x*, also shown in dotted lines, but its momentum and the slot in the pitman would instantly allow it to take up position *y*, also shown in dotted lines, being at an equal angle on the opposite side of the centre. Its further progress would be intercepted by the remote end of the pitman, and at this instant the dog would instantly fall in the next tooth in the revolving wheel *D*, immediately commencing its work of retraction without the necessity of the full stroke of the pitman being accomplished. The work of the percussive blow is fully accomplished before full retardation, and while being intercepted. Nothing is accomplished by standing and pushing against the face, so that its immediate retraction is not found to interfere in the least with its effective operation, but on the other hand is found to eliminate entirely the serious effects of recoil upon the operator, allowing a light machine to be easily and completely directed while at work. Modifications of the details have been employed, but the general principle has been found to work extremely well in practice. The machine effi-

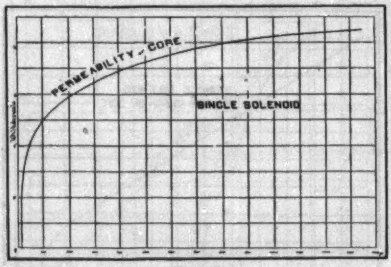


Fig. 5.

ciency from the current, on the one hand, and the blow of the pick upon the other has been found to be in excess of 70 per cent.

Another form of the direct blow machine has been constructed, operating upon the solenoid plan; two and three solenoids have been employed in various combinations. It has been found that the return of energy was extremely small until an approximately close magnet circuit was reached. Another point found was that the effectiveness of the blow was materially decreased if the magnetic circuit was closed beyond a certain point, the reciprocation of the solenoid core being dependent upon the non-closure of the magnetic circuit. Diagrams are submitted herewith, showing the various mathematical functions of the machine.

In the Figs. 5 to 8 the ordinates show different withdrawals of the core, and the abscissæ show other functions named in the diagrams. The pressures obtained in case of both single and double coils shown in diagram, Fig, 9. Fig. 10 indicates the general arrangement of the solenoid machine. The total weight was about the same as the weight of the spring-actuated machine, and the energy absorbed somewhat in excess. The mean effective pressures, however, are not nearly as great, as diagram 9 indicates; especially is this the case when the work was intercepted, say one inch from the end of the stroke, as at point *A*, in this diagram; the mean effective pressure falls from 259 to 210, decreasing, instead of increasing, as in the case of the spring machine. It was possible to recover in the solenoid machine nearly 10 per cent. of the energy expended during the first few minutes, but the

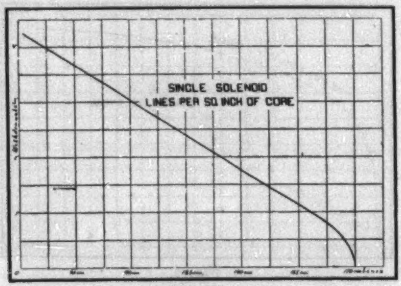


FIG. 6.

heating effect was found to seriously interfere with its protracted operation.

In drilling holes for blasting, an auger machine is used, shown in Fig. 11. In this case the auger is driven by a single reduction motor, the rod holding the auger being provided with a featherway for driving, and a screw-thread for the feed. An automatic attachment is placed upon the nut of this screw feed, by means of which, when sulphurs or other foreign substances are encountered, the feed is automatically relieved, the progress being inversely to the density of the strata. The machine is made light, and is usually fastened from roof to floor. One machine will drill about 50 holes six feet deep per shift of nine hours. The speed of its penetration in ordinary coal is about 3½ feet per minute. It requires about 3½ amperes at 220 volts. Both this and the pick machine are provided with a starting rheostat combined with, or located interior to a cable reel which accompanies each machine. The flexible cable reaches back from the machine to the wires in the entries. The core of this cable is small, and although insulated clamps secured to the ends of the cable for its attachments to the entry wires, have been persistently shipped, it has been found that the miners almost invariably cut them off and prefer to simply bend the end of the cable in the form of a hook, and hang it upon the entry wire. This allows of their being readily pulled off upon the completion of the work, without the necessity of going back

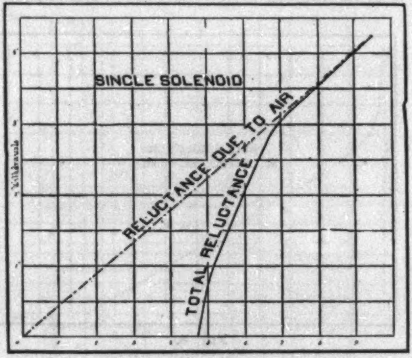


FIG. 7.

to the mouth of the room, and at the same time seems to provide all the contact required.

As to other machines it may be stated that they are divisible into two general classes—the revolving cutter, and the chain machine. In the former the cutters are secured to a revolving bar, which is forced against the substance to be cut. In the latter the cutters are inserted in a chain running over suitable sprocket wheels fastened to a frame, which extend under the coal supporting the entire cutting organization. The chain as it passes out from the coal, goes around a driving sprocket from which the power is obtained. Many attempts at the construction of the former class of machines have been made, which have resulted in entire failure. From my own observation and experience I have found the following to be true with reference to this class of cutters. In Figs. 12 and 13, an outline view of such machine is shown. *A* indicates a rotating cutter bar; *B* the journals of such bar; *C C* the knives or cutters secured to such bar; and *D D* is the coal strata in which the cut is being made. It will be noticed that immediately before the journals, where it is impossible for any cutters to be located upon the bar, there is a solid pillar of coal left. This must be broken up and dislodged by sheer forward pressure of the end of the journal. Furthermore if foreign substances are encountered by the bar, the cutters are usually entirely stripped off at this point. This leaves a pillar uncut as illustrated in the figure, which makes further progress entirely impossible until the machine is withdrawn, new cutters inserted and the

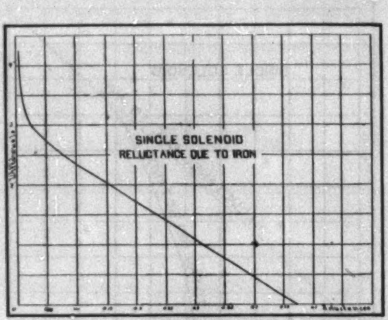


FIG. 8.

pieces of the old ones carefully removed from the kerf. If the machine is not of great strength other portions sometimes break on these occasions, and the machine is laid up for a number of hours and even days for repairs. I have found that in using a chain as shown in the cuts of the long-wall machine now to be described, if some of the cutters are broken, others immediately take their place, and the cutters upon the chain will continue to attack and finally dislodge the foreign substances ; in any event the chain will continue to cut so long as any teeth are left upon it, and their absence is easily observed by the operator, inasmuch as all the points in the chain are constantly passing any point of observation outside of the kerf.

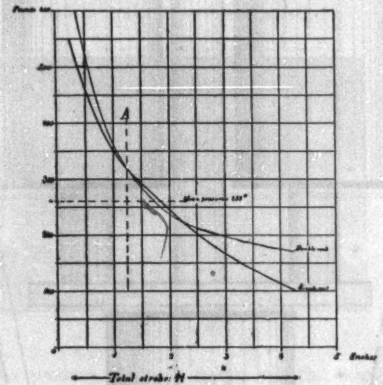


FIG. 9.

The extremely small amount of room between face and props in the long-wall mines sometimes precludes the possibility of even using a very short reciprocating machine. I have therefore resorted to another form of machine to be used in connection with this style of mining, consisting of a mechanism about five feet in length, about 20 inches wide and 18 inches high, from the side of which protrudes a cutting device operated by an electric motor forming part of the machine. This is organized to travel along the face, and undermine to a depth of three feet usually working in the strata below the coal known as "fire-clay."

A number of these machines are in daily operation in the northern district of Illinois. A single machine has repeatedly undermined as high as 1,500 feet per nine hours. The machine

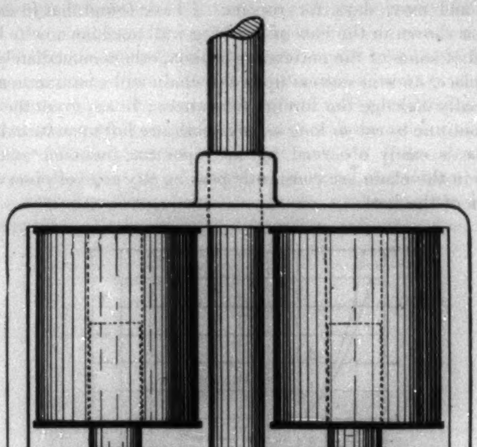


FIG. 10.

weighs somewhat over a ton and travels upon a rail located at one side, which at once provides support for the machine, means for providing against the twisting strains of the lateral cutting device and providing at the same time for feeding in either direction. These rails are in sections, and are taken up after the machine has gone over them, are passed to the front over the machine on rollers provided for the purpose and are laid ahead. The pintles used to connect the rails, form with the other pintles, members of a series by means of which the feeding along the rail is effected in a simple manner, and one that has proven successful in actual work extending over a considerable period of time. The machines are shown in photographs presented herewith and have been in use upward of two years. They require 22 to 35 amperes for their operation at 220 volts. Flexible cable of 150 feet is provided, which, being attached to the live wires ahead, allows the machine to cut 300 feet before readjustment is required.

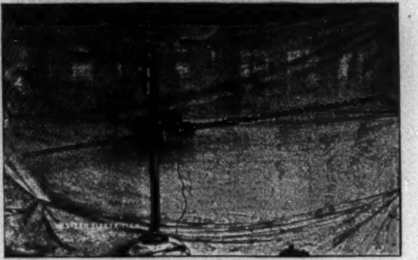


FIG. 11.

After the product has been loaded and the cars massed in trains, their transit to the surface is a problem that has received considerable attention from the mining engineer. There are three prominent systems of mechanical haulage in operation. One in quite extensive use is the tail-rope system, where a main traction rope leads out from a drum over numerous pulleys and rollers to the front of the train. A tail-rope of lighter weight leads from another drum on the same shaft over numerous grooved shives at the side of the track, or suspended from the roof over a "bull wheel" at the farther end of the track, coming back upon the track-rollers and being secured to the rear of the train. Each of the drums is detachably secured to the driving shaft of a powerful engine. The train is only under control of the distant engineer. A trip rider usually goes with the train, and a rather complex system of signals is employed, by means of which he may possibly communicate with the engineer.

Another system of growing importance is the endless rope, which consists of a slowly moving wire cable usually requiring double track and double-width entries, single loaded cars or trains being coupled to the rope by grips at the inner end, which are disengaged at the receiving end and attached to the "empties" standing upon the other track, and are thus gripped to the outgoing cable and sent on their way back. A large number of extra cars are required, owing to the slow movement of the cable. In some instances trip riders are employed to accompany the train. In some mines grip cars are used. In both of these systems a large amount of power is expended compared to the work performed, and the speed is of necessity limited. Another drawback is the excessive complication necessary in track, innumerable rope sheaves, timbering and entry to provide for turn-outs, crooks, cross-overs, ventilating traps, etc.

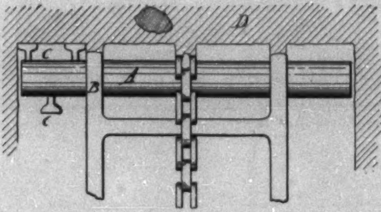


FIG. 12.

Electric haulage has been looked upon by the foremost operators as the coming system, and one which is destined to solve the problem of economical transfer of the heavy product in mines. In studying the problem of electric haulage for mines one is met with the fact that the tracks are poor and light, the roadbed is crooked and irregular as to levels and generally of light construction. The curves, owing to the necessity of leaving pillars, and having connected sight entries, admit only of small radius curves, and excessive grades are often met with in the dip of the vein, which cannot be obviated by cuts or fills as on surface roads without great expense, especially if the entries are timbered, which is usually the case in western collieries. Yet a service of the highest efficiency and greatest capacity in speed and tractive power is required. The motor must work double shifts if necessary, and be at all times under perfect control of a motor man receiving $2.25 per day, and never fail for an instant.

After considering the problem, I have designed a motor shown in the accompanying photographs. A number of plants containing this motor are now in daily operation in Ohio, Indiana, Kentucky and in this state (Illinois). Two of these plants have been installed in one mine in Illinois. The grades are as high as seven per cent., five per cent. being not unusual.

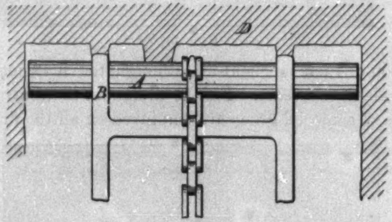


FIG. 13.

The matter of stability of roadbed is a relative one. There are conditions under which a 10 pound rail would be as stable for a light locomotive as the New York Central's 100 pound rail is to the tremendous load per driver in the high speed locomotives recently placed in service on that road. Pounds' pull at the draw-bar is only obtainable by pounds' weights at the rail, yet for a heavy draw-bar pull it is not necessary to have the weight upon any one driver excessive. In designing the locomotive I have employed eight drivers grouped in two swiveling trucks free to turn on the shortest radius curves—as short, in fact, as nine feet radius, measured from the centre of the track. A single powerful motor constituting the frame of the car is coupled to all eight of these drivers, the coupling being simple and of such character as not to be interfered with in the least by the change of the alignment between the truck and the car body, upon irregular track or in making curves. This, while being rather a difficult problem, has been successfully worked out, as shown by the fact that none of the locomotives have ever required repairs as to

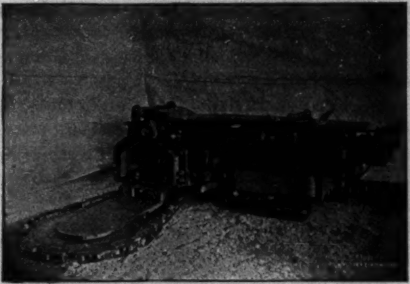


FIG. 14.

this portion of their mechanism, although the first has been in operation about two years. Fig. 15 illustrates the central feature. Steel tires of a special grade are employed on all the drivers, and

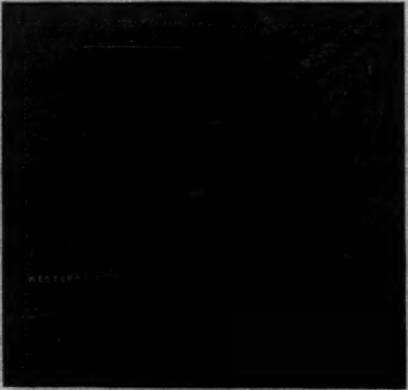


FIG. 15.

the fact that they are driven from a single motor insures absolute uniformity and harmony in their peripheral speed. It is to this fact, as recently stated in a paper before the Chicago Electric

Club, that I attribute the extremely satisfactory results in traction which have been observed, it being demonstrated both in shop tests and under conditions of actual service that two motors without mechanical coupling do not show results that are at all comparable. In regular service as high as 25 per cent. of the weight of the entire machine may be depended upon for draw-bar pull, and considerably over one-half of such weight has repeatedly been shown on the dynamometer. The wheel base is entirely

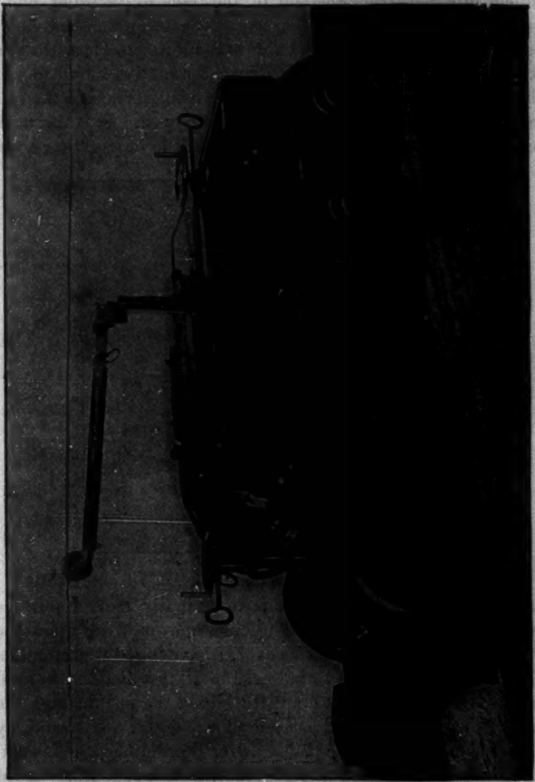


Fig. 16.

flexible, distributing equal portions of the total weight upon each driver under all conditions of irregular track. The trains hauled by these locomotives frequently have as high as 200 wheels upon the track. The pit car wheels are constantly oiled, distributing an excellent lubricant from one end of the track to the other, and I have found for rapid acceleration work on heavy grades and quick action of the brakes that sand is almost as indispensable as the current itself. In using the rails as return the sand is found to seriously interfere with the proper electrical connection to the rail, especially where four wheels only are employed. It is the usual practice in this case to use sand upon one side only giving a torsional working strain which tends to seriously rack the frame increasing the flange friction and giving uneven wear upon the drivers. With eight wheels, however, I have found it preferable, and in fact this practice is followed in all installations, to sand upon both sides and obtain the "bite" upon the rail through the silicates from at least six drivers out of the eight. The sand, by the time three wheels have passeed over it on either side is sufficiently broken up to allow the fourth pair to get a fairly good contact with the rail for electrical conductivity. This is an accidental feature in the use of the eight wheel car which is of great value. The machines themselves are low and narrow, the height being between 34 inches and 36 inches, accommodating them-

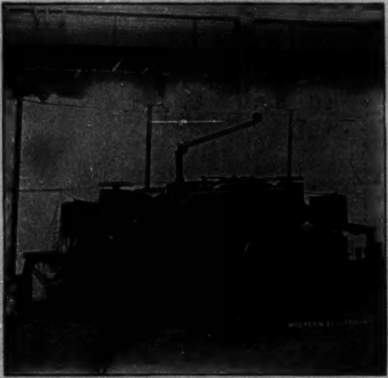


FIG. 17.

selves thereby to the lowest entries. Three sizes have been built, namely, eight, ten and twelve tons, Figs. 16, 17 and 18. The motor of the largest of these is of 125 horse power capacity ; 250 volts is the highest that has been used in connection with the machine. The temperature of the machine is very low, not exceeding 60 degrees above the atmosphere on a 10 hour run.

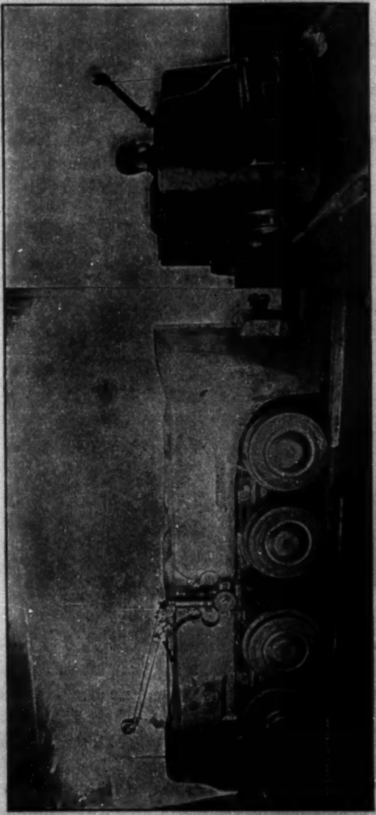


Fig. 18.

These machines have been operated on as low a rail section as 10 pounds with fairly good results. The heaviest rail that has ever been employed in connection with them is 20 pounds. This makes an extremely inexpensive track, not only eliminating entirely the light track argument used against electricity by advocates of rope haulage but "goes them one better," in that a single track with the most abrupt curves serves better purpose than double track with rope. In Streater No. 3 shaft of the Chicago, Wilmington and Vermillion Coal Company a round trip on a haul of 7.000 feet, including switching at both ends, has been repeatedly run in 8½ minutes, part of the entry being a three per cent. grade against the loads. Thirty seven pit cars of about two tons gross weight have been handled on this grade. The controlling mechanism is extremely simple, a single hand wheel serving the purpose of start, stop, acceleration and reverse. A rheostat and series fields are used. The rheostat is of most durable construction consisting of iron and asbestos. The trolley wire is upon the roof and follows the line of the track at a distance of about six inches outside of the gauge line of one of the rails. Excellent results have been obtained by the use of an elastic medium which is located between the trolley wire and solid mine roof, one form used being among the exhibits. The elasticity prevents concussion of the rapidly moving trolley, obviating thereby the serious

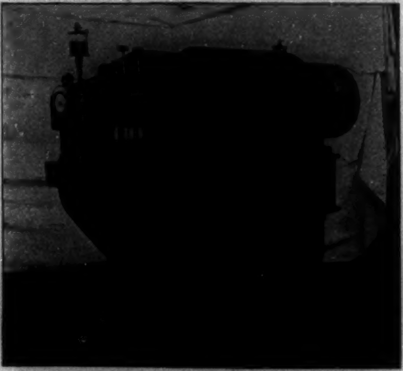


FIG. 19.

sparking often, and most easily observed in mine entries, where nothing but the most intense darkness prevails. To my mind the extreme facility with which electric power can readily be transmitted by the mere contact between conductors has no parallel in any branch of physics. I have often observed the simple contact between trolley wire and wheel while 160 horse power, namely, 480 amperes, was being transmitted to the motor, each of the eight drivers slipping vigorously on the sanded track throwing fire which illuminated the surrounding mine walls, while the trolley, with the exception of slight infrequent scintillations, was perfectly dark and of extremely innocent appearance.

A number of special and automatic devices have been employed in connection with the system with excellent results. Powerful electric headlights are used on each end of the locomotive, and in the 12 ton type Fig. 18, the end is carried out, providing a cab which entirely surrounds the motorman with high walls of metal two to three inches thick, entirely protecting him against collision, piling up of cars, falls of rock, etc. I have operated this motor on tracks which if seen by daylight would be adjudged unsuitable entirely for even the roughest mule service, handling heavy trains at a rate of speed of 2,700 feet per minute, being somewhat over 30 miles an hour. The perfect flexibility of wheel base of the motor is doubtless responsible for these excellent results. Compound-wound dynamos of about seven tons weight are used to furnish current for the machine, Fig. 19. These are designed for sufficient excess of capacity to operate other devices in connection with the mine. In one mine at the south we have coupled to a single dynamo and engine one of our eight ton haulage cars, electric lights at all partings and switches, and on top in engine room and at a store, electric mining machines of the direct blow type, and an electric drill for blasting, by means of which machines, the entire product of the mine is taken out. The mine, which was formerly ventilated by a furnace, at present has its air moved by a fan driven by a powerful electric motor, and an electric pump is at present being installed, all of which are operated from the same feeders. In Kansas a mine pump raising water about 800 feet and two very extensive endless rope hauls are operated from the same dynamo as our electric excavating machinery. The pumps are of simple construction, the motor being geared to the plungers by spur gearing. It has been found with the pumps, long-wall machines and drills, as well as in the case of the percussion machines, that mounting the pinion loosely upon the armature shaft and driving it with cushions, located between it and the armature, has worked a decided economy in repairs, showing its adoption to be along the line of good engineering.

Discussion.

The President:—I am sure the members of this Institute have heard with a great deal of pleasure the interesting and exhaustive paper which Mr. Sperry has read. I wish to express my personal thanks for the information which I have received from it.

The operation of mines by electricity has been a bugbear for all the large electric companies, and it has been almost at a standstill, until some one person has had the energy, the courage and the ability to go into a mine, deal with those who are operating the mines, gain experience from them, learn their present practical or impractical methods of getting out the coal, and then set to work to solve the problems that are presented. I think we may accord to Mr. Sperry the credit of being one of the most successful men who has thus far essayed to operate mines by electricity. [Applause.] I must confess that I am astounded at the care and the amount of work which have been shown in his operations. I did not realize that the business of electric mining had reached the successful condition which it seems to have attained. I think I made an error last night when I did not speak of electric mining as one of the developments marking the vigorous manhood of electricity. To those who have been in mines, whether silver or coal, who are used to going into long headways or down long shafts, the dangers which one has to encounter in dealing with this particular problem must be apparent. It requires on the part of the man who undertakes its solution, more than ordinary courage, and patience to spend as much time, and to run the risks that are necessary to overcome the difficulties that must be met in that way and that way only. The ingenuity which Mr. Sperry has shown deserves special commendation. Personally, I know of no other means which have been more successful in accomplishing the results sought.

In looking forward to the railway work which is to be done in the near future, I take a good deal of comfort in knowing that a single trolley wheel has carried, in actual practice in mines, under the conditions which are there met, half as much current as is required on the very largest class of passenger locomotives when operating at 800 or 1,000 volts, and as much current as would be required by a locomotive in ordinary practice. I confess that there has been a question in my mind whether the contacts at the high speeds operated, and for the current necessary to be carried, would be made without a good deal of difficulty and without a great deal of experimental work. We are under obligations to Mr. Sperry for having demonstrated that large currents at high speeds have been successfully handled without danger to the trolley wire or to the trolley itself.

The paper is open for discussion, and I would be very glad if anybody here, who has had experience in mining, would either criticise the paper which Mr. Sperry has brought forth or advance any arguments for or against this method of operating mines, or if they have any other methods with which they are familiar, that they will bring them forward for criticism and discussion.

DR. EMERY :—I have been very much pleased with the manner in which the mechanical difficulties have been overcome in this work, and it seems to me that with such an adaptation already made underground, where the conditions are much more difficult than upon the surface, there should be no hesitation in pushing forward any work that is desirable, on any scale for which the capital can be secured.

MR. NELSON W. PERRY:—Mr. President, that paper has been very interesting to me. I think it very timely and very fitting that Mr. Sperry should speak of the survival of old methods in the mines. We find that the mining communities are very conservative. They do not change from one method to another readily. For instance, we can illustrate that in our own country here. Our coal miners are fond of using the little coffee pot lamp for lighting, fastened in their hats. They won't use a candle if you furnish it free. You take the gold and silver miner out in the West, and he will take a little piece of candle and daub it up against the wall in any way with mud. You know, in some sections of the country they use a double hand-drill and in other sections you can't hire them to do it—at any rate, where they are doing contract work. And so it goes. When I was mining, one time, in the Republic of Mexico, I received by mistake a shipment of a lot of American picks. Our miners down there use a straight, sharpened bar for doing the same work that we do in this country with the pick. Our foreman looked them over and asked me what they were for. I told him, and he shook his head rather doubtfully. Then I showed him how to use them. They were left there for the men to use if they chose, and I tried to get them to use them, but they wouldn't. They went back to the old bar and I am satisfied did better work with that than they would have done with the pick.

The President has spoken of how much at a standstill we have been in the introduction of electricity in mining. I think that is easily explained. As I said before, the mining communities are very conservative. They have acquired what knowledge they have by hard knocks. They have also had a good deal of experience in this country, experimenting with new devices for treating ores and getting ores out of the earth, and new processes generally; and, as a rule, they have paid very dearly for it. Then, again, electricity being so new, those of us who have lived above ground have been, until recent years, somewhat suspicious of it, and those underground have been still more so. Not to increase their confidence in it, our manufacturers have frequently sent men out to their mines, to tell them how to get their material out, who have never been inside of a mine before. They have sent men out there, to tell them how to place the product in marketable condition, who never saw the product in its crude condition. They have gone further; they have taken coal miners and sent them to the silver fields, and it is a notorious fact that the gold and silver miner has been receiving three to four dollars a day for his work, whereas at the same time the coal miner has been getting 60 or 75 cents, or perhaps a dollar a day for his work.

There has been another drawback to the introduction of electricity in mining operations; that if we come across a mine which is already equipped, we will say, with a compressed air plant for the operation of drills, and we try to induce them to put in electricity, they ask us:—"Well, what can it do? It can't run our drills." Until comparatively recently, we had no electric drill of any kind. Now we have drills to offer them, but the question is one of efficiency. It would be difficult to convince a miner today that electric drills are on a par with compressed air drills. So that since we had nothing to offer them in place of compressed air, the miner could not see where it was to his advantage to put in an additional plant which would do only one operation, an operation which has heretofore worked satisfactorily—the oil lamp and the candle. He can detect the quality of his ore with a very dim light. If you offer him an electric light he regards it as a luxury, and we had nothing to offer in the shape of a drill. As it is now, with electric reciprocating drills, they are all too heavy. It requires in operating them about 75 per cent. of the time in moving them from one place to another, leaving about 25 per cent. for actual operation. If we could get a drill which would give sufficient power but only half the weight, which would not eat up the energy in that way, it would be a desirable thing. I regard the attaining of an efficient electrical drill of such a character as being the key to the introduction of electricity into mining operations generally. If we could mine our ore by electricity, it needs no argument to get an introduction of electricity—if we can carry the current as far as may be necessary and do all of the operations from the same plant.

I was a little surprised at Mr. Sperry's statement of the pressure which he found to be most efficient. If I followed him correctly, in the coal drill it was something like a thousand pounds per square inch.

MR. SPERRY:—The projectile is operated by a spring which takes 125 pounds per inch to retract it. It could not be considered in area, or so much per square inch.

Mr. Perry:—In the softer coal it won't do to strike too hard a blow. A more rapid and lighter blow seems to be more efficient. And, in looking at this bit, it occurred to me—and you were speaking of its striking these nodules of various kinds—it occurred to me that it would be better to have a differently shaped bit in coal and other deposits where you will strike those. A drill naturally assumes a certain shape, and we find that the drill is more efficient if we put it, in the first place, in about the shape that it naturally assumes from wear and tear. A drill of this character, lodged in the soft rock, might bind—get in too far. Another difficulty with these drills—the ordinary material that we may strike is not even. It is almost as bad to strike something that is softer than the average material as that which is harder than the average material. It destroys the drill in a different way. It would be difficult to remedy that in a percussion drill.

In this mining locomotive I was very much interested, as I would have anticipated great difficulty in getting around some of the smaller, shorter radius curves which we are liable to meet any time in the mines with an eight-wheels base, and I think your method of coupling the locomotive to make those sharp turns is ingenious. But doesn't the dirt get into that universal joint and affect it at times?

Mr. Sperry:—No, we experience no difficulty in that direction.

The President:—We were all pleased to hear the interesting remarks of Mr. Perry, in which he illustrated the inertia of the mining class and described the character of the men who have been sent out to instruct mine operators how best to handle their products. It is a familiar inquiry, with which we are invariably met, "How much of my system can you operate? Can you operate only part of it, or can you operate the whole?"

There must be sombody else here who has some remarks to make on this paper.

I am somewhat familiar with the inertia of the companies as well, because some time past, when associated with organizations with which I am no longer connected, we made attempts to get into the mining field. We learned at that time that there was not a man in our employ who was sufficiently familiar with mining and electrical practice to discuss questions with the mine operators and those who were interested from a technical standpoint. Every one had altogether too much to do to devote special attention to such matters. It has been necessary for some one to forsake other fields, leave the arc and the incandescent lamp, get away from the ordinary street railway problems, and devote himself individually to the varions problems that arise in the solution of mining operations.

Mr. Sperry:—Mr. President, among the points that have been brought up, the one raised by Mr. Sprague is of greatest interest. The fact that we can handle from 300 to 400 amperes through a single contact is one that not only gratifies but astonishes me when I think of the small actual bearing through which such a quantity of current is transmitted. Before we adopted the elastic support we found it quite difficult to do this successfully, because, as the upward pressure exerted upon the wire was brought near the trolley wire support, the trolley wire would assume this shape (illustrating). That is, a series of inverted scallops. This affected only the particular section of the wire, of course, upon which the pressure was exerted at the time, but you will readily see by following the wheel along the line that, as it gets to the point of passing the support, it jumps off, because its momentum has a tendency to make it follow on the curved line, very nearly like the line I have drawn. But on using a flexible trolley wire support, this was largely obviated. That, I may say, was the only trouble we encountered in reference to this point.

Now, as to the wear of the trolley wire under these extreme uses, we have found that the trolley wire which we put in about two years ago is in perfect condition. You cannot feel a difference in section in that wire, one way or the other, with a pair of calipers. I found by a micrometer that on a No. 00 wire that the wear was not in excess of 2½ thousandths of an inch. In some places it was difficult to detect any wear. I suppose that the reason that I could not detect wear, and in some places I could not, was owing to this fact:—Where dirt or oxide covers the wire (in the mine we have an enormous amount of sulphur gas, coming from blasts that take place daily), these fumes corrode the wire, and the trolley, in flashing, leaves protuberances on portions of the wire, and as the micrometer contacts with the top of those protuberances, the wire appears in some cases to be even larger than the original.

As to the use of a pick, I suppose we have tried upwards of a hundred shapes of picks, and we have found that the double-pointed pick was the best. We also find that in even the very softest coal, Indiana block coal, etc., nothing can give too heavy a blow. The heavier the better.

THE PRESIDENT:—Hearing Mr. Sperry's description of the action of the trolley wire, recalls to my mind a principle which in early street railroad days was insisted upon at one time. The same difficulty was then experienced, that the trolley wire clamps held by the span wires protruded somewhat, and when the trolley came by, it was thrown slightly from the trolley wire, and left it perhaps for the distance of an inch, and then rebounded. This, after a while, in some cases would melt the solder which was used in the early days to fasten the clamps to the trolley wire. After a while we noticed that the trolley wire showed a very decided wear, and naturally its diameter was increased or diminished in the most varied sort of fashion. So instructions were given to always have the dynamo so connected that the current would pass from the trolley wheel to the wire. We thought if there was going to be any copper deposition, we would have the wheel the sufferer rather than the trolley wire ; but the contact is so perfect that nobody pays any attention to which end is grounded and which goes to the trolley wire.

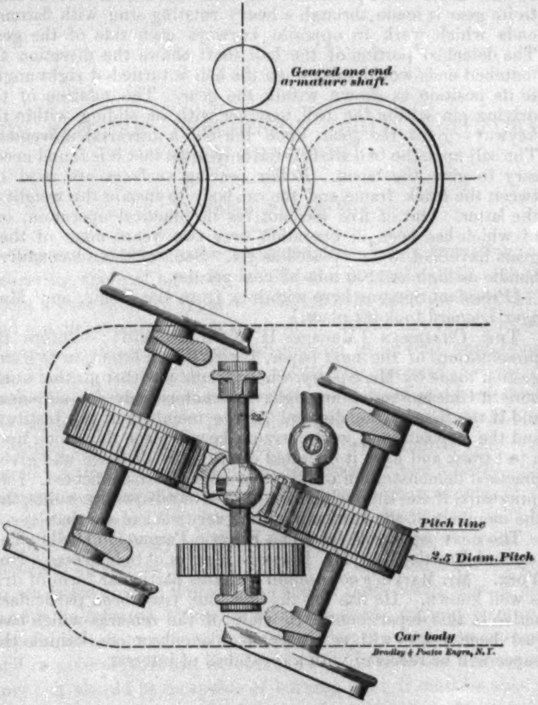


Fig. 20.—Truck of Sperry Mining Locomotive.

Dr. Emery :—Mr. President, I would suggest that Mr. Sperry give us a sketch which will show the way in which the wheels are connected. We would like to see a plan of the whole apparatus grouped together.

Mr. Sperry :—Fig. 20 shows one end of car body with truck turned at an angle of 22 degrees. Gears are placed in the middle of each axle with shrouds on each side. A similar gear is placed in the centre of auxiliary axle which at all times maintains its alignment with the car body and therefore with the motor gearing thereon. The last gear wheel, however, by the abutment of its shroud at all times maintains its alignment with the gears of the truck. The power driving connection with the ball shaft to its gear is made through a heavy rotating stud with flattened ends which work in opposite keyways each side of the gear. The detached portion of the ball shaft shows the direction the flattened ends would take when the ball is turned at right angles to its position as shown within the gear. The rotation of the driving pin within the ball, together with its sliding within the keyways gives the gear upon the ball a universal movement. The ball upon the ball shaft is the only pivot that it is found necessary to give the truck. Roller bearing surfaces are used between the truck frame and the car body to sustain the weight of the latter. Out of five locomotives in practical operation, one of which has been in operation over two years, none of these gears have had to be replaced as yet. Some of these locomotives handle as high as 1100 tons of coal per day.

[President Sprague here withdrew from the Chair, and Manager Leonard took his place.]

THE CHAIRMAN [Manager H. Ward Leonard]:—Before the presentation of the next paper, I wish to call attention to a suggestion made by Mr. Sperry, who informs me that at the works here in Chicago he has an eight ton motor ready for shipment, and if the time at the disposal of the members of the Institute, and the disposition as well, warrants, he will place this motor upon a test track and have it operated over a short distance and give a practical demonstration of how these wheels are connected. I am quite sure, if the time which we have at our disposal permits, that the members of the Institute will be very glad to see that.

The next paper will be "The Electric Percussion Drill in Theory and Practice," by Mr. Harry N. Marvin, of Schenectady, New York. Mr. Marvin's connection with this particular form of drill is well known. He has for the last four years been particularly active in this department. In view of the remarks which have just been made with reference to solenoid motors, I think this paper will be received with a great deal of interest.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 7, 1892, Manager Leonard in the Chair.

THE ELECTRIC PERCUSSION DRILL IN THEORY AND PRACTICE.

BY HARRY N. MARVIN.

Wherever rock is to be excavated, in mining, tunneling or quarrying, there of necessity we find the percussion drill. Until about a year ago, two types of drills held the field, the hand drill and the direct-acting steam or air drill. With the extension of electrical methods of power transmission to mining operations, came the urgent demand for an electrically operated percussion drill. The mechanical and electrical requirements of this machine are particularly severe. It is called upon to continuously endure an action that is, in almost all other machinery, studiously avoided, namely, a practically uncushioned reciprocation and a blow upon substances harder than cast-iron. The general requirements of an electric percussion drill for mining work are that it shall be light enough to be quickly handled by two or three men; it must be powerful enough to compete with an air drill that absorbs eight or ten horse power; it must be so simple in its design that it can readily be repaired by an ordinary mechanic without electrical experience; it must be so constructed that the complete machine and its several parts can be soaked in mud and water without injury; it should be incapable of burning out; it must be able to endure almost any amount of the roughest handling without injury, and the materials employed should be as little subject to crystallization as possible. My experience suggests the following elements of construction to be avoided:—Traveling conductors, commutating or current shifting devices, sliding contacts, spring contacts, cotton insulated wire and all insulating material capable of carbonization, switches, lamination of the moving parts, close fittings and joints liable to be impaired by considerable wear, all constructions liable to be impaired by careless or unskilled attention. For the benefit of those unfamiliar with the actions of the air drill, I will state that the general practice at present is to give the machine a stroke of from five to seven inches, to strike as nearly as possible an uncushioned blow upon the rock and to make as many blows per minute as the power applied will develop, usually from two to four hundred per minute. It is found that a stroke of about five inches is necessary to properly free the hole from mud.

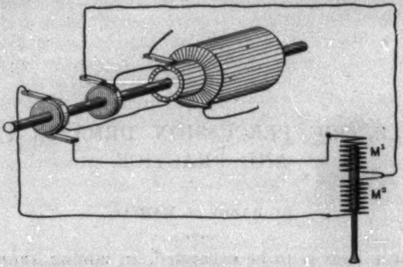


FIG. 1.

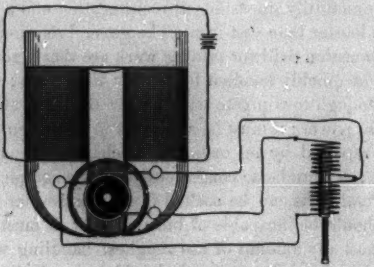


FIG. 2.

In the electrical solution of the problem, the solenoid and plunger affords the simplest method of developing this action, and most of the work in this field has been confined to this line. After much experimental work with divided solenoids and commutating devices mounted on the drill, I abandoned these methods, and in 1888 devised the system of operation that is at present rather well known, but for clearness this may be illustrated.

Fig. 1 shows an ordinary continuous dynamo having two opposite points of its armature connected to a solid collecting ring and a half ring respectively. The outer terminals of the drill coils, M¹ and M², are connected by line wires to the two brushes alternately bearing upon the half ring. The inner terminals of the coil are joined and led to the solid ring. It appears that, as the armature revolves, current impulses are directed into the drill coils in alternation, causing a reciprocation of the plunger in synchronism with the dynamo armature. It will be noticed that the action here closely resembles the action of a steam engine, force

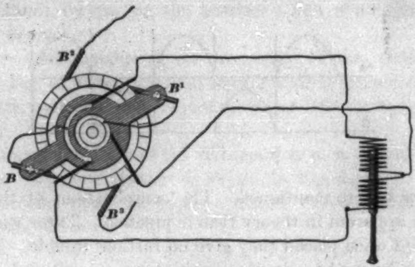


FIG. 3.

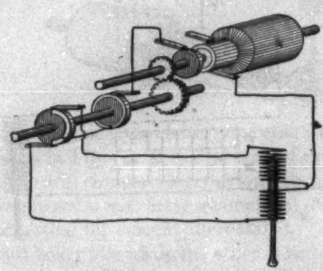


FIG. 4.

being exerted first in one direction and then in the other, one end of the cylinder being idle while the other end is active. Of course, connections are such that the polarity of the plunger is never reversed. This system simplifies to the utmost the construction of the drill and enables it to be so designed as to satisfy all of the conditions before mentioned. The complications of this system are more apparent in theory than in practice. Three wires are required, but once placed they give no further trouble. The generator can be used to develop continuous current and pulsating currents at the same time if desired, but I believe that it is better practice to use a separately excited drill generator without regular commutator. The machine then becomes extremely simple, as shown by Fig. 2. I have found it desirable to run these drills at a speed of about 400 per minute, a rather slow speed for small dynamos. This difficulty may be avoided at the expense of simplicity in several ways. One method is shown in Fig. 3. The regular commutator of a continuous current armature is equipped with a pair of revolving brushes, B, B^1, one of which connects to the solid ring of the drill circuit commutator and the other to the

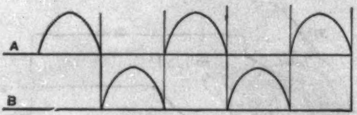


FIG. 5.

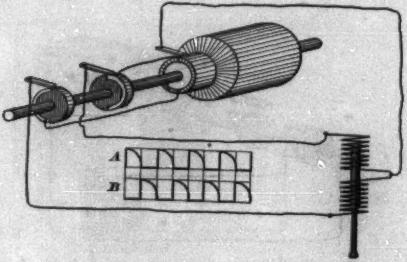


FIG. 6.

half ring. In this manner pulsations are developed in the drill circuit as the brushes B-B^1 are rotated, and with a frequency entirely independent of the armature speed. With this arrangement we are also able to take off 100 volt alternating currents from a commutator that is also supplying a 500 volt continuous current circuit, by spacing the brushes B and B^1 an amount less than 180 degrees.

Another method reducing the frequency of the pulsations is shown in Fig. 4, where the drill commutator is geared to the dynamo shaft, reducing the frequency to one half the speed of the generator.

The general character of the impressed E. M. F. in the two-drill circuit in the above systems is shown in Fig. 5, curve A showing the voltage over the upper coil and curve B showing the voltage over the lower coil.

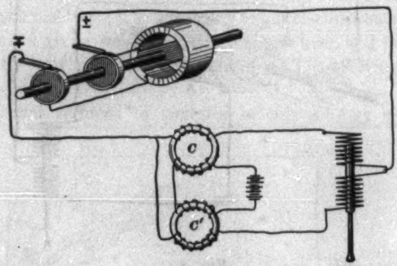


FIG. 7.

A method of operation by which a somewhat different curve is obtained is shown in Fig. 6. It will be noticed that here current is admitted to the drill circuits when the voltage is at a maximum the curve of impressed E. M. F being generally as shown. The general plan of admitting the current alternately to the two drill circuits, one circuit being idle while the other is active, characterize all the above methods. This may be called Plan A.

I have devised a method for effecting this without commutation, as shown in Fig. 7. Here one wire from a simple alternator is led to the drill, while the other wire passes through the secondaries of two converters C, C′. The cores of these converters are continuously saturated by continuous currents in the primary coils, as shown, the windings of the converters being opposite. When current endeavors to pass through these, it is checked considerably in the converter whose magnetism it tends to reduce, and, to a much lesser degree, by the converter whose magnetism it tends to increase. A reverse current is opposed by the other converter, and thus pulsations of one polarity are strong in one coil, while those of the reverse polarity are strong in the other drill coil. This would not probably be a very efficient method.

Another general plan of drill operation, which I will call Plan B, was developed to some extent by the late Mr. Van Depoele. This plan is based on a difference of phase of current in the two coils, and is shown in its simplest form in Fig. 8. The two outer drill wires are connected to the two regular brushes of the armature A and the middle wire connects to the solid ring connected to one point of the armature. As the armature revolves, current rises in one drill coil and falls in the other, producing the reciprocation of the plunger. The voltage curves for this type of machine are shown by Fig. 9. An armature giving two alternating currents of 90 degrees difference of phase may also be used, but with the disadvantage of reversing the polarity of the drill plunger. The above appears to be less efficient than Plan A, for the reason that the drill coils are constantly working against each other and being heated up by current, opposing the action of the drill.

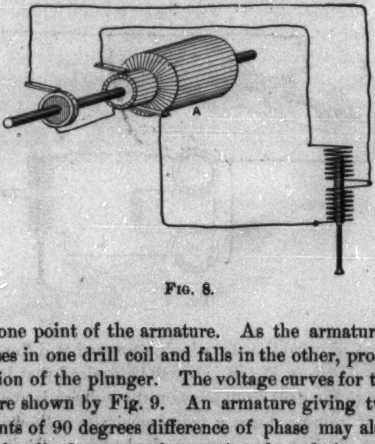


FIG. 8.

A third plan of operation, which we may call Plan C, is that devised by Messrs. Siemens and Halske, and shown in Fig. 10. Here three coils are used in the drill, the middle coil being excited by a continuous current and the two end coils by an alternating current of proper frequency. The function of the middle coil is to maintain a constant polarity in the bar, and the motion is effected by the effort of the two end coils. In this system both motor coils are continuously excited as in Plan B, but there

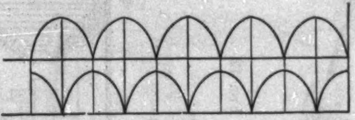


FIG. 9.

appears to be less waste of energy. Aside from the extra complication of drill construction, it is questionable if as much power can be obtained from a given weight of wire with this arrangement as can be realized on Plan A. It must be borne in mind that in these machines it is necessary to get the utmost power possible out of a given weight of copper and iron, and the heating limit is the only practical consideration. The efficiency of the machine is secondary.

Plan C was reduced to practice in this country by the late

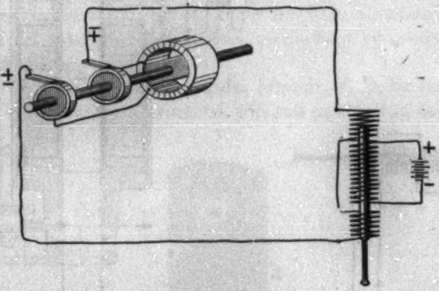


FIG. 10.

Mr. Van Depoele, who avoided the use of a fourth wire in the system by an ingenious method, shown in Fig. 11. The regular commutator A is supplied with rotating brushes B^2, B^3 in addition to the regular brushes B, B^1. It appears that as the rotating brushes revolve, a simple alternating current flows through the

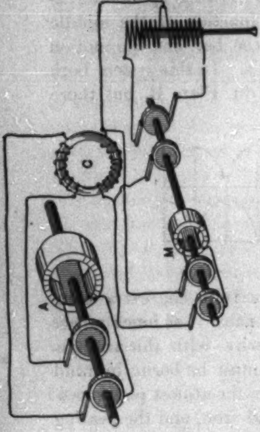


FIG. 12.

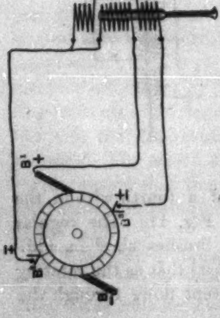


FIG. 11.

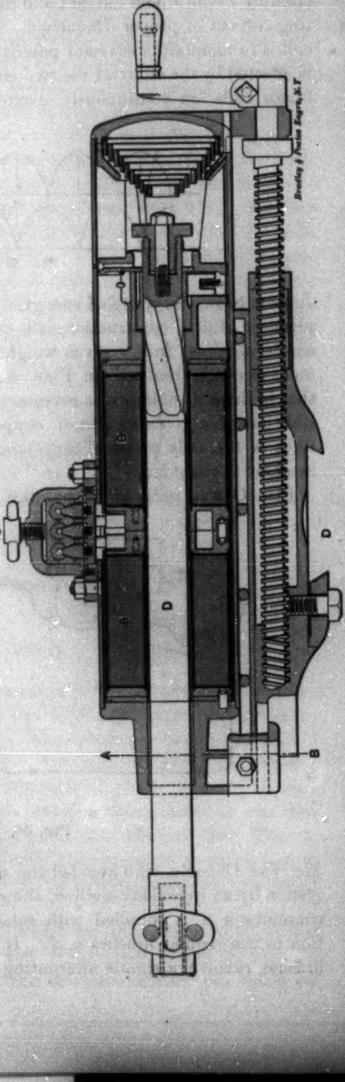
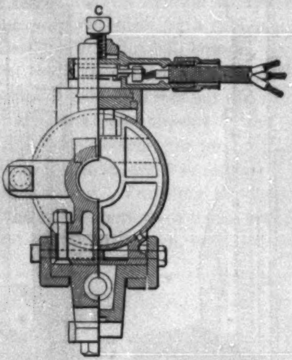


FIG. 13.

end coils of the drill, while a unidirectional pulsating current of one-half the frequency circulates in the middle coil.

The requirements of construction and service have thus far induced me to employ a rather low voltage in the operation of drills, our present practice being to use about 100 volts alternating upon the drill. I have employed several methods of operating drills from high voltage circuits.



SECTION A B. SECTION C D.

FIG. 13A.

Referring to Fig. 3, if the regular brushes, B^2, B^3, be connected to a 500 volt circuit, the armature will run as a motor, and if the rotating brushes B, B^1 be spaced by a suitable amount, less than 180 degrees, they will take off the pulsating current for the drill circuit at, say, 100 volts.

FIG. 14.

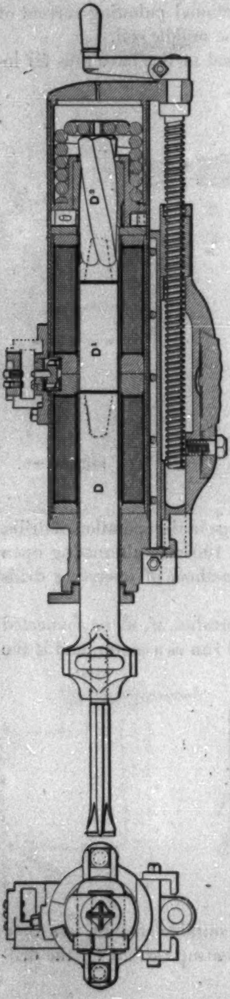


FIG. 15.

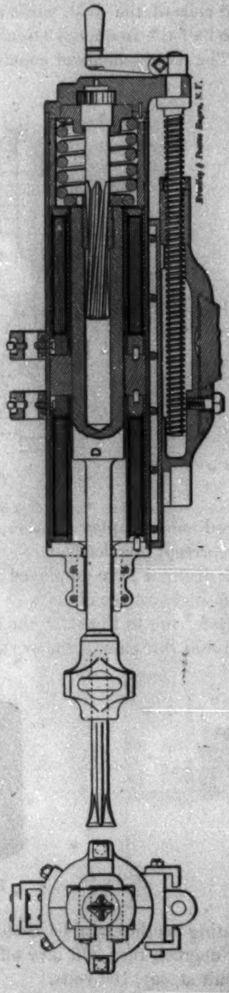


FIG. 16.

Another method is shown in Fig. 12, where a 1,000 volt alternating circuit is led from the prime generator A to the vicinity of the mine, where it passes through the primary of a converter, C, in which the voltage is reduced to 100 volts. A small motor, M, running in synchronism with the generator A, directs the alternate pulsations into the drill circuits.

Turning now to the drill construction, Fig. 13 shows the first form in which the percussion drill was put upon the market. This I call a Type A machine. As appears in the sketch, the machine consists of a 7 inch boiler tube provided with a suitable mounting. Within the tube are two solenoids, B, B^1, a rotating device on a cushion spring. Traveling through the coils is a 2 inch plunger, D, the middle portion of which is of iron; the upper end is of brass and is rifled, as shown, to fit the rotating device; the lower end is also of brass and is enlarged at the end into a chuck to hold the bit. The three sections of the plunger are welded together. The weight of this plunger was 38 lbs. This machine was designed to make 600 blows per minute with a 3 inch to 4 inch stroke. The first of these machines to be put into a mine was that started in May, 1891, in the Black Bear Mine, at Gem, Idaho. This was, I believe, the first commercial electric percussion drill plant ever operated. During the next few months several other mines and quarries were equipped with these drills, with the general result that where the rock was not excessively hard, the work was fairly satisfactory, but in extremely hard rock several defects developed. The brass portion of the plunger would crystallize and break, a short distance above the chuck. Under the severe strain of continuous work in hard rock, the coils

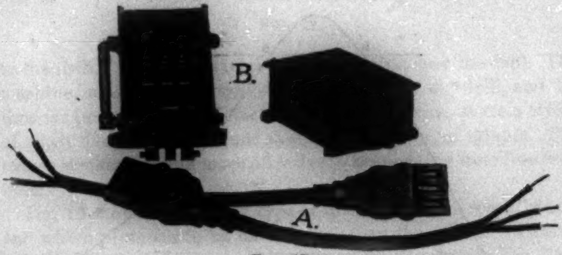


FIG. 17.

would become so heated as to ground, and the spring connections between the cable and the coils would give out. The construction of these drill coils deserves special description. Fig. 14 shows a coil complete, ready to be put into the drill. The coil is constructed as follows:—Upon a brass spool, the body and heads of which are insulated with mica, is wound a bare copper wire of square section. This wire is insulated as it is wound, with mica. The complete coil is wrapped with mica and a metal tube slipped over the whole and soldered to the coil heads, making the completed coil water-proof and heat-proof, as it is insulated with mica alone, nothing but mica and metal being used in its construction. The trouble with our first coils was that they were soldered up with a comparatively soft solder, and when the coils got very hot this solder softened, the joints opened, and oil and dirt getting in carbonized and developed a ground. As an example of the treatment these drills received, I will state that while visiting one of our drill plants that was used for railroad grading on the Great Northern Railroad, in Idaho, I saw the men dump one of these drills bodily off from a flat-car onto a dump of broken stone, down which the drill rolled some twenty feet. I might add that this was one of our most successful plants, 18 foot holes being regularly drilled in granite rock. In view of the defects mentioned, the design of the machine was altered, as shown in Fig. 15, which shows a Type B machine. The plunger has been enlarged to 2¾ inches and is what I call the double magnet type. The forward part, D, is now made of steel; the rifled part, D^2, is also of steel, and the connecting piece, D^1, is of brass. This design

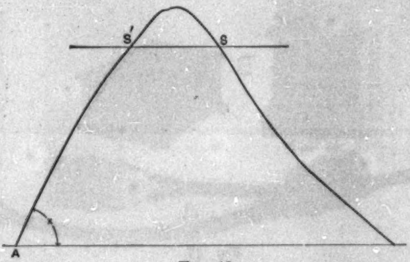


FIG. 18.

gives a solid steel chuck and brings the brass part of the plunger within the drill body. The coils are now incased in boiler tube jackets brazed up with hard spelter, and are heated to a bright red in process of manufacture. The spring connections have been abandoned, and solid set screw connections now connect the cable to the drill. A switch is inserted in the cable near the drill. This machine seems to overcome the defect of the old drill, and has thus far proved very satisfactory. This machine takes a stroke of about five inches and runs at about 380 blows per minute. This longer stroke is found more effective in keeping the hole free from mud.

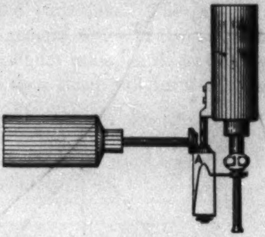


FIG. 19.

Fig. 16 shows a still later type of drill that I have recently built and which promises to be much better than the others. The outside dimensions of this machine are about the same as the Type A machines, but the plunger of this machine is a solid steel forging and four inches in diameter. No brass is used in the plunger, the metal being reduced in section where it passes through the front head. The ratchet device is similar to that used in air drills and, in fact, the whole design of the machine

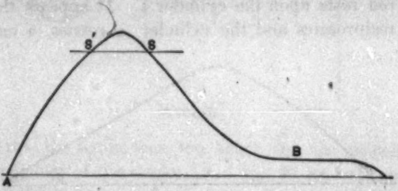


FIG. 20.

quite closely follows the air drill construction. This machine takes a stroke of from 6½ to 7½ inches, and makes about 380 blows per minute. The plunger weighs 80 lbs.

Fig. 17 shows the cable connection and switch A, and the cable fuse box B, through which connection is made from the mains to the cable. A study of the operations of these machines presents many interesting features. In Fig. 18 is shown what I term an energy curve of the machine. This curve is obtained in a manner similar to an indicator card of a steam engine. Referring to Fig. 19, A is a cylinder mounted parallel to the drill rod. This cylinder is geared to the armature shaft in such a manner that it rotates in synchronism with it. A pencil carried by the drill rod rests upon the cylinder A. It appears that as the drill rod reciprocates and the cylinder A rotates, a curve will be traced by the pencil. Fig. 18 shows such a curve. The direction of motion of the paper being from left to right, it appears that the angle x of the curve at any point indicates the velocity of the plunger at the instant; A is the point where the plunger strikes the rock, and the sharp rebound is show by the angle. The

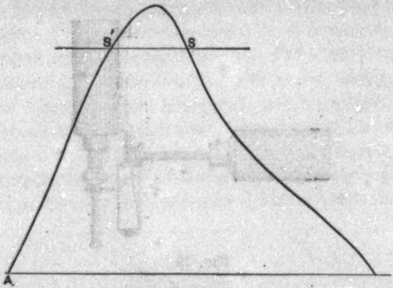


Fig. 21.

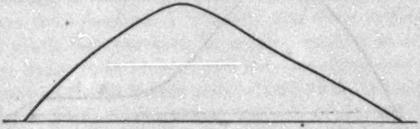


Fig. 22.

concave character of the curve shows the backward acceleration of the plunger. The buffing spring is encountered at s, and the angles at s and s′ show that there is little loss in the spring. The angle x gives the final velocity of the plunger when it strikes the rock. A simple calculation gives the energy of the blow in foot pounds. This affords a good method of comparison between different machines and conditions. The general character of the curve reveals the general action of the forces during the stroke. The curve in Fig. 20, for example, shows that the drill rebounded sharply at A, and then stood still for a considerable time at B, indicating that the stroke was too short for the power applied and the frequency of the stroke. On Fig. 21 we have shown the reverse conditions, where the concave character of the curve at B indicates a loss of velocity, and the length of stroke is evidently too great for the power applied. Fig. 22 shows a curve taken on a Type A drill with an 18 foot bit, drilling downward in slate rock.

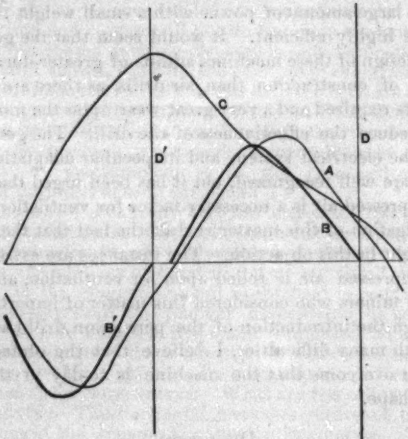


Fig. 23.

The energy of the blow being determined, it can then be decided whether the drill delivers the greater amount of energy upon the rock by a few long strokes, or by a greater number of shorter strokes.

Some other interesting curves are shown in Fig. 23. Curve A shows the voltage on the drill circuit. Curve B and B′ the current in the two coils. Curve C is the energy curve. The lines D, D′ indicate where the brushes make contact with the half ring.

The efficiency of the above machines, compared with other form of electro-motors, is of course low, but this factor is of comparatively little moment in percussion drill work. It is far more important that the machine should be able to endure the development of a large amount of power with a small weight than that it should be highly efficient. It would seem that the general mechanical design of these machines admits of greater durability and simplicity of construction than air drills, as there are no steam-tight joints required and a very great wear upon the moving parts does not reduce the effectiveness of the drill. The general flexibility of the electrical system and its peculiar adaptation to mining work are well recognized, but it has been urged that for drill work compressed air is a necessary factor for ventilation. A careful investigation of this matter reveals the fact that there is little or no weight in this objection. The instances are extremely rare where compressed air is relied upon for ventilation, and I have found few miners who considered this matter of importance.

Although the introduction of the percussion drill has been attended with many difficulties, I believe that the obstacles have been so far overcome that the machine is to-day in thoroughly practical shape.

DISCUSSION.

MR. SPERRY :—Mr. Chairman, I have been deeply interested in the paper that has been read, and we all know Mr. Marvin's enterprise in this line for some years past, and we feel very much indebted to him for being the first one, so far as I know, to prove that a solenoid had the capacity to do the work. I remember quite a long trip I made to see one of the machines in operation. The first part of the paper is interesting as giving accurate data as to the earliest methods of circuit connection.

I also think that, among the others, the last chart presented is one of great interest to us, as presenting for the first time a curve which has been made by the machine itself. Of course, the area of the curve cannot possibly be figured in the same sense as an indicator card, but, as I take it, the deviation from a straight line of the upward or downward curve is the feature that is to be studied.

I would like to hear Mr. Marvin's statement with reference to the comparative merits of the three-coil and two-coil systems of operation in practice. This would be interesting and would add to our knowledge upon this point.

Mr. Marvin:—Mr. Chairman, I would say that in operating a machine with an intermittent current in the end coils, I have found very little advantage to be derived from a constant energizing coil placed between the two end coils. Of course, upon the system shown in the drawing, Fig. 9, the third or central coil is necessary to give any motion at all. You must polarize the plunger, and then one coil is supposed to pull, and the other coil to push, but my experience has led me to believe that the same amount of iron and copper will not do the same amount of work when thus distributed as when disposed in only two coils. In the three-coil drill shown, the coils are constantly energized and the heating is continuous. In the two-coil drill the coils are energized intermittently and the heating occurs only half the time.

Mr. Sperry:—Mr. Chairman, I notice that the tendency in perfecting the machine, was to make a space between the coils. Have you any measurement as to the increased reluctance incurred in thus distributing the coils ?

Mr. Marvin:—I would say that the increased space between the coils in the last form of drill shown, is not an especial feature of the magnetic circuit. It is a mechanical element to afford a bearing for the plunger of the drill. I do not think that a sufficient gain in the magnetic circuit is obtained to counterbalance the increased weight of the plunger necessitated by such a construction. In fact, I have a drill about completed, with no space between the coils, but I think it will give better results.

Mr. Perry:—Mr. Chairman, I would like to ask Mr. Marvin a little further about those curves. What are the ordinates ?

Mr. Marvin:—The horizontal distances represent the successive positions of the dynamo armature in its rotation. The vertical distances represent the successive positions of the plunger in its stroke.

Mr. Perry:—Then it is not a power curve ?

Mr. Marvin:—No, it does not show directly the power of the stroke. It simply shows the velocity of the plunger at any instant. The shape of that curve shows the acceleration of the plunger under the force of the coils. If you had a straight line there, it would indicate a constant velocity without acceleration, and the more the curve departs from a straight line, the more acceleration or retardation is shown.

Mr. Perry:—One feature of Mr. Marvin's drill he did not refer to, which is exceedingly interesting. It is quite as bad sometimes to strike something which has less resistance than the average, as it is to strike something harder than the average. Now, Mr. Marvin's drill has a peculiar property. If it strikes nothing the drill does nothing but tremble a little in space. I presume the explanation of that lies in the sluggishness of the magnetization, does it not? When it is driven in one direction the other coil holds it.

Mr. Marvin:—The coil starts to pull it back, and you really have there an electric cushion.

Mr. Bradley:—How much power does it take, and how many inches of hard rock can you go through per minute?

Mr. Marvin:—With that last drill we absorb on the machine about 6½ H. P., that is, the large plunger drill, and we have made a record in very hard blue lime rock of drilling 56 feet in six hours, using a 2⅝ in. starter and drilling the holes 10 feet deep.

The following paper on "Long Distance Transmission for Lighting and Power" was then read by Mr. Charles F. Scott, of Pittsburg:

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 7th, 1892, Manager Leonard in the Chair.

LONG DISTANCE TRANSMISSION FOR LIGHTING AND POWER.

BY CHARLES F. SCOTT.

The public interest in electrical subjects is continually sustained by the announcement of discoveries and projects which border upon the wonderful and extreme. In power transmission, one of the most inviting and promising fields, popular expectation often rises to operation over distances and at pressures which exceed the limits of commercial engineering. But many schemes which are readily planned with glowing outlines to accomplish wonderful results are defeated by difficulties encountered in details and unforseen obstacles.

The test of practical operation in long distance transmission has been applied in but few cases, and the severe test of continued operation over a considerable length of time is of rare occurrence. The latter is the crucial test, and its commercial significance gives it the highest importance. The continued and successful operation of one plant for a year, under extreme conditions of situation and service, is of higher value in testimony to the practical development and possibilities of electrical work than many elaborate projects, or the operation of novel apparatus for a short time.

It is with this idea in mind that a description is here to be given of two plants, one for lighting and the other for the transmission of power. The conditions which have been met include a very considerable distance, extreme difficulties of climate and roughness of country, exacting requirements in continuity of service, and a pressure above that ordinarily used in the class of machines employed. The plants to be described are the first of their type installed in this country, and the apparatus in the power plant is of a kind that has not been heretofore used. The type and construction of the machines, and the arrangement of apparatus are new in many particulars, and as they have contributed largely to successful operation they will be described with some minuteness. Alternating current machinery is employed, constructed by the Westinghouse Electric and Manufacturing Co., the pioneer company in alternating work in this country.

The lighting plant was first installed. It is operated by the Willamette Falls Electric Co., of Portland, Oregon. The general requirements are those which electrical transmission is admirably adapted to meet. Portland, a city marked for its prosperity and rapid industrial growth, has not the great advantage of proximity to developed coal fields, so that power, the fundamental requirement for industrial activity, cannot be obtained cheaply from this source. Another supply of energy, however, is found in rich abundance. The falls of the Willamette river at Oregon City, in the combined points of size, accessibility and nearness to the seaport, are unequaled. These falls, estimated at from 200,000 to 250,000 H. P., are about 13 miles from Portland, and it requires but a moment's thought to

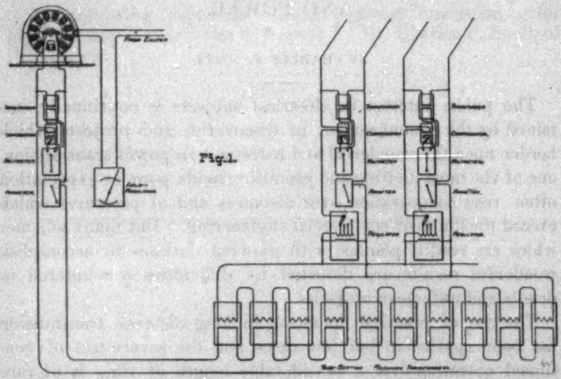


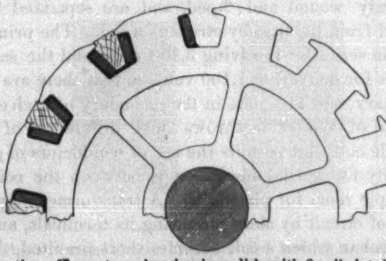
Fig.1.

Lighting plant at Portland. Diagram of apparatus and connections.

appreciate the value of an agent which can make this power available in the city.

The Willamette river is about one-quarter of a mile wide, and the fall is about 40 feet. The present station is located on an island at the middle of the river. Victor wheels of 300 H. P. are geared to horizontal shafts, from which the dynamo belts pass to an upper floor at an angle of 45 degrees. Two alternating current dynamos for incandescent lighting are driven by each wheel. The current, at a pressure of 4,000 volts, passes directly to the line of No. 4 B. & S. wire, which is carried on ordinary double-petticoat glass insulators across the level country to a sub-station in Portland. The current is received at 3,300 volts by transformers in the sub-station and is reduced to 1,100 volts, for distribution by various circuits through the city to ordinary transformers, by which it is reduced to 50 or 100 volts.

Fig. 2.



Section of armature showing iron disk with 3 coils in place.

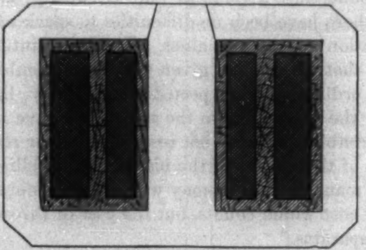
When the apparatus was designed, it was not considered practicable to generate 4,000 volts with the ordinary type of machine, in which the wire is wound upon the surface of the armature, on account of the difficulty of insulating for more than 1,000 or 2,000 volts. The work was undertaken with a new type of armature, which is specially noteworthy, as it has rendered high potentials practicable in a machine of simple construction.

The field of the dynamo is of the ordinary type of alternating current machine in use in this country. The casting is circular in form, with 12 inwardly projecting poles of laminated iron, on which the field coils are placed. This type of machine combines simplicity with rigidity and strength, as both bearings and the lower field are in one casting.

The armature is built up of laminated disks, which are punched with 12 T-shaped teeth. The armature coils are wound in a lathe, are carefully taped and insulated, and are then placed over the teeth and sprung in under the projections. The space between adjacent coils is filled by a block of wood, which holds them in place securely. This form of construction gives all the advantages of machine winding over hand work, allows ample insulation between coils and core, protects the coils from mechanical injury, holds them in position without the use of band wires, and makes the replacing of a damaged coil comparatively simple. The field current is supplied from a direct current machine, and the main current is taken from two collecting rings on the armature shaft. The reducing transformers are placed in a vault in the sub-station at the city. They are arranged in banks, or units of ten. Each bank is supplied by a separate dynamo, and has a capacity of 1,250—16 C. P. lights. The coils of the transformers are separately wound and taped, and are separated from one another and from the iron by strips of wood. The primaries are connected in series for receiving 3,300 volts, and the secondaries are in series for delivering 1,100 volts, so that there are 330 volts in the primary, and 110 volts in the secondary of each converter. This method of connection throws small differences of potential in any single coil and permits the use of conductors of good size. The necessity for special insulation is between the coils, where there is ample room for placing it. A transformer may be readily cut out of circuit by short circuiting its terminals, and in case of an accident in which a coil becomes short-circuited, the E. M. F. on that transformer disappears and the others are called upon to do a larger share of the total work, without interfering with service. The efficiency of the transformer at full load is 96 per cent.

The operation of the system is simple. The voltmeter or potential indicator is supplied by the secondary of a converter reducing in the ratio of 30 to 1. In the voltmeter circuit there is placed a compensator, which is a small transformer with its primary in series with the main circuit, and its secondary in series with the voltmeter circuit, and connected in such a way that the compensator pressure, which is proportional to the main current, is opposed to that of the converter. The compensator is so adjusted as to introduce into the voltmeter circuit a counter E. M. F. proportional to the loss in the conductors, so that the voltmeter indicates the potential at the distant end of the line. The compensator effects the same result as pressure wires run from the point of supply back to the station. The E. M. F. at the dynamo is adjusted for each load by regulating the field current so that the voltmeter shows the proper indication, thus keeping the pressure at the sub-station constant at 3,300 volts. The pressure at the dynamo is slightly in excess of this at light loads, and is increased to 4,000 volts at full load, to cover the loss of transmission. The secondary current from the reducing transformers is treated as if it were received directly from a dynamo. On each supply circuit from the station there is a voltmeter with a compensator set for the loss in this circuit. The exact adjustment of E. M. F. on a circuit is made by a regulator, consisting of a transformer with its primary connected directly across the main terminals, and a secondary divided into small sections, each having an E. M. F. equal to one per cent. of the primary. Any number of these sections may be placed in series with the supply circuit, thus varying the E. M. F. at will. This form of regulator is of high efficiency, as it converts but a small percentage of the total energy, and the loss is a small franction of this percentage.

Fig. 3.



Section of transformer showing iron plate and primary and secondary coils surrounded by wood.

The simplicity of the system, and the perfection of its operation, both from theoretical and practical standpoint, are noteworthy. The electrical system receives mechanical energy, converts it into electrical energy, in a form suitable for transmission over a distance too great for other agents, and by the simplest form of conductor, and transforms it to a pressure suitable for commercial lighting. And yet, in the whole system, there is but one element in which there is mechanical motion, and that motion is simple revolution. The only wear and friction is in the dynamo, and its bearings are large and easily lubricated. The one point at which the high-tension conductors need be exposed is at the dynamo, where the brushes and fittings are of the simplest sort. The line does not deteriorate by carrying current, and as a piece of apparatus the transformer, in simplicity of construction and operation, is unrivaled.

The plant was first installed with two incandescent machines and started nearly two years ago. Since that time, five additional machines have been added, so that there are now seven, each with a capacity for supplying 1,250 16 C. P. lights, in Portland. The total capacity is 8,750 lights. The dynamos run admirably. There was one night when several armature coils were burned out, which was attributed to an iron wire falling across the main line and connecting several of the circuits, grounding them. Otherwise there have been no difficulties to speak of with regard to the operation of the machines. The superintendent of the plant states that the line has given very little trouble, much less than would ordinarily be expected from a city line. He also says, that "the converters in the sub-station have not given one minute of trouble, and have not cost one cent for repairs." One explanation of the success of the plant is the intelligent policy of the general manager, in harmony with his statement that, "It is not the first cost which counts, but the cost of throwing out and replacing apparatus."

The same policy has happily governed the installation of the second plant to be described—the power plant. This is located near Telluride, Colorado, and is owned by Mr. L. L. Nunn. The Gold King mill requires power for operating its crushers and stamps, and fuel can come only from long distances at enormous costs. A few miles from the mill there is a water power, but the country between the two points is steep and rough, and for many months in the year is covered with snow. Electricity is the one means of getting the power from its source to the mill. The conditions are of the most favorable character for demonstrating the value and possibility of electrical transmission.

In this plant a Pelton wheel receiving water through a two foot steel pipe, under a head of 320 feet, drives an alternating current generator. The current is carried over a line of bare wire to the mill, which is nearly three miles distant, and drives an alternating current synchronous motor of 100 H. P. The generator and motor are machines of the same size and form of construction as the dynamos at Portland, already described, and differ from these only in some minor modifications.

The generator is provided with a composite field winding. A part of the magnets are excited by direct current from a separate machine, and the rest are excited by a current from the generator armature, which is proportional to the main current and is commutated by the equivalent of a two-part commutator. The adjustment is such that the E. M. F. on the main terminals rises as the current delivered by the machine increases, compensating for line losses and keeping the pressure at the motor 3,000 volts. The speed is 833 revolutions, giving 10,000 alternations per minute. The switchboard and regulating appliances are similar to those in the station at Portland. Ordinarily no adjustment is required after the machine is started. and the attendant has little to do besides looking after the mechanical running of the apparatus.

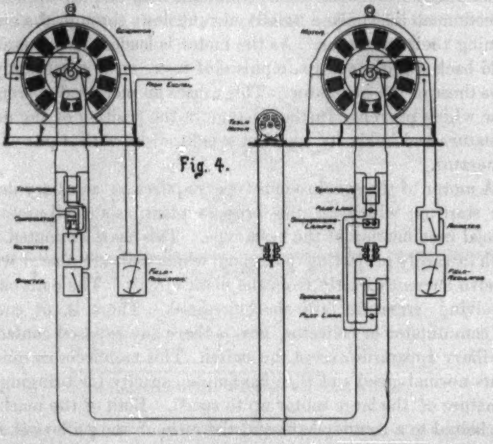


Fig. 4.

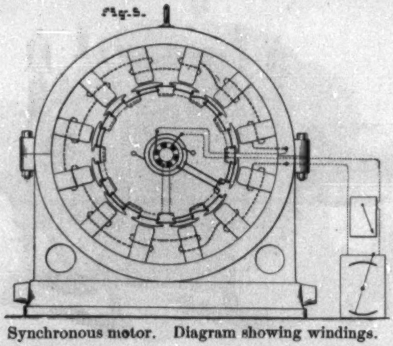
Synchronous motor plant at Telluride. Diagram showing apparatus and connections.

The motor is similar to the generator except in the manner of exciting the fields. The field current is obtained, not from a separate machine, but from a second winding parallel with the main coils around the teeth of the armature. This current is commutated and passes through the field coils. The motor, as already stated, runs in synchronism with the generator, and although the general theory of the synchronous motor is well known, it may be briefly outlined. If the terminals of two alternating current machines giving the same number of alternations per minute be connected together, and one be driven by an engine or other source of power, the second will run as a motor and will do work. The impulses of E. M. F. which are generated in each machine occur simultaneously and are opposed in direction, so that only a small current flows through the circuit joining their armatures. As the motor is loaded, its armature is held back slightly and its impulses of E. M. F. do not exactly oppose those of the generator. This causes an increased current to flow which prevents further change in the position of the motor armature and holds its speed in synchronism with that of the generator.

A motor of the synchronous type requires an auxiliary device for starting, which, in the present plant, is a small motor—a special modification of the Tesla type. This has a laminated field with inwardly projecting poles, on which coils are placed which receive current directly from the main circuit. The coils on the revolving armature are short-circuited. There is, of course, no commutator or collector, nor is there any exposed contact or auxiliary apparatus except the switch. This motor comes quickly to its normal speed and then has ample capacity for bringing the armature of the large motor up to speed. Both of the machines are belted to a countershaft, and the ratio of the pulleys is such that the speed of the large motor is a little greater than than that of the generator. The large motor is then excited and runs as a self-exciting alternating current dynamo at the normal E. M. F. of the circuit. The small motor is then switched off and the speed of the large machine gradually falls until it is approximately equal to that of the generator. The relation between the speeds of the two machines is indicated by the fluctuation in the intensity of lamps, which are connected in series with the secondaries of two converters, whose primaries are connected with the generator and motor respectively. When the E. M. F's. of the two machines are in the same direction, current will pass through the lamps, and when they are opposed there will be no current.

If the motor is running a little faster than the generator its E. M. F. will be in the same direction as that of the generator at one moment, and will be opposed at the next moment, causing a corresponding variation in the intensity of the lights. At the time of slow variation and proper phase relation the machines are connected by closing the switch. The small motor is disconnected by its friction clutch and comes to rest, and the load is then thrown upon the large motor by a second clutch. The operation of starting requires but one man, and is accomplished in about two minutes. The rheostat in the field circuit is adjusted to give the proper field current as indicated by the ammeter. This is done when the motor is started and it may then run indefinitely with varying loads without requiring any adjustment whatever.

Fig. 5.



Synchronous motor. Diagram showing windings.

When the motor is running, the only things to be cared for are the brushes and the bearings. The high tension brushes—the only point besides the switches where the high tension is exposed—will run for a week without adjustment, the exciter brushes run without sparking and the lubrication of the bearings is well provided for. The construction and operation of the motor is strikingly simple in comparison with the steam engine, which it replaces, with its many moving parts and intricate motions.

A few points illustrating the characteristics of the synchronous motor may be mentioned, as they are of both theoretical and practical interest. The connection of the motor to the generator is not a delicate operation. If the motor is running above synchronous speed at the time of connection with the generator it instantly adapts itself to the proper speed. If the motor speed is slightly lower than that of the generator it may fall into step when the switch is closed, but if it be running considerably slower it will not come into synchronism, but will further decrease in speed. When this occurs, the switch of the large motor is opened and that of the starting motor is closed, bringing the machine up to full speed again without any injury to the apparatus. If the E. M. F. upon each of the machines before connecting them be 3,000 volts it will remain unchanged when they are connected. If the field current of either machine be increased the E. M. F. will be raised, but the field current of the other machine may be lowered and the resulting E. M. F. made equal to 3,000 volts. The current flowing between the machines depends upon the relative field charges, and is least, whatever the load may be, when the two machines are equally, or very near equally excited. The field current of either machine may be made zero, and the motor will still run, but with greatly reduced capacity. In a test with machines of a smaller size the E. M. F. was 2,000 volts when the two field charges were equal. When the field charge of either machine was cut out it fell to 1,200 volts, and the current increased very considerably.

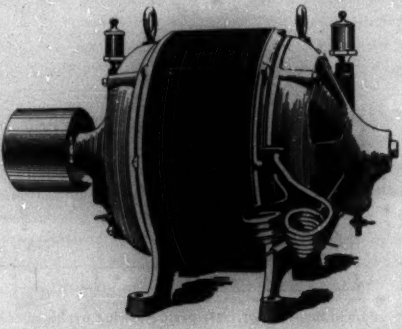


FIG. 6.—Tesla starting motor.

The explanation of the fact that the two machines may run together with widely differing field charges is easily understood when the extra flow of current at that time is taken into account. If the field charges are different the E. M. F. tends to be different also, and a current flows between the generator and motor, which is of such phase that it tends to strengthen the weaker field and to weaken the stronger one. Thus, when the tooth of the motor is in front of a field pole, the current flowing at that time is in such direction as to weaken or strengthen the field as may be required to make the E. M. F. equal to that of the generator. Difference of phase between E. M. F. and current has an effect upon the regulation of a dynamo which is easily shown by changing the character of the load. If a dynamo, with a constant field charge, is used for carrying an ordinary load of incandescent lamps, a certain drop in E. M. F. will result. But if the same current be delivered to an inductive resistance, such as a coil of wire, the starting motor, or a synchronous motor, when its armature is at rest or its field is not charged, the drop in E. M. F. will be considerably greater than with lamps. On the other hand, when the generator is supplying a synchronous motor running under normal conditions, the drop in one test was about half that which resulted from the same current to incandescent lamps. The current to the motor when running without load is quite small and increases in proportion as the load is added. With any definite load the current between the machines, as already pointed

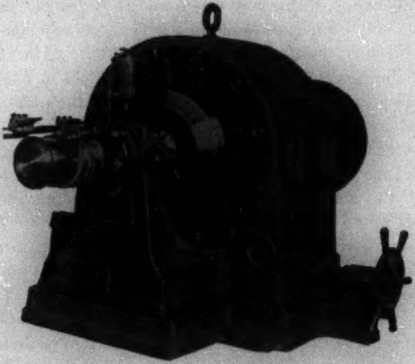


FIG. 7.—Machine used as generator and motor at Telluride.

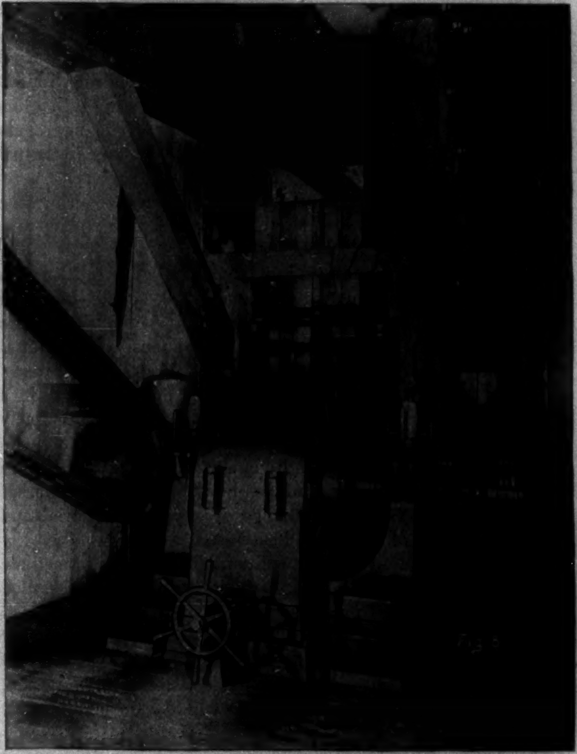
out, depends upon the relative field charges of the machines as well as upon the E. M. F.; but there is, however, a considerable range through which the field charges may be varied without materially affecting the current. If the load on the motor be increased to about twice its rated capacity it will fall out of synchronism and come to rest. The current, however, is held back by the self-induction of the motor armature and is not sufficient to cause immediate injury.

An efficiency test of the motor was made before shipment by driving a dynamo which was loaded. The current and E. M. F. received by the motor and delivered by the dynamo were measured and an allowance made for the loss in the dynamo. The full load efficiency in this way was found to be 93 per cent. In a test upon smaller machines of similar type the efficiency of the motor at full load was found to be 91.5 per cent. The efficiency of the synchronous motor system, leaving out loss in conductors, but including losses in generator and motor, in the plant for the delivery of 50 H. P. was found to be 83½ per cent. at full load, and 74 per cent. at half load.

Full load may be thrown on the motor suddenly. In the Gold King mill the stamps, which are operated by the motor, are usually left raised when the plant is stopped, in order to avoid the extra strain of lifting them when the plant is started. It sometimes happens that the stamps are left down and the motor is required to raise them all at once. When the clutch is thrown in, the current indicates that the load is considerably above the normal capacity of the motor, and yet it is started without difficulty or apparent strain.

The excellent current regulation with different loads, the tendency of the machines to normal adjustment when there is ordinary variation in the field currents, the small liabilty to injury when the motor is greatly over-loaded, the high efficiency and ease of attendance, are points of great value in the practical operation of the system.

The pole line runs from the power station up the mountain to a height of 2,500 feet, and then crosses a rough but comparatively level country to the mill. The line at some places is at an angle of 45 degrees, and many of the poles had to be set in solid rock. The surface of the snow in winter is occasionally at a level with the tops of the poles, and parts of the pole line are practically inaccessible during some months of the year. This region is peculiarly subjected to lightning discharges, and special precautions are necessary to protect the apparatus In one instance there were 42 discharges of the lightning arresters in as many minutes.



The Synchronous motor in the Gold King Mill.

The plant was started for regular work in June of last year. An accurate record was kept from the middle of July to the first of May, showing the actual number and the length of the delays caused by electrical machinery. During these nine and a half months the system was in regular continuous operation six and a half days each week, with but few intermissions. The difficulties which were encountered were insignificant in amount and have resulted, not from any fundamental difficulty in the system, but have been caused by incidental defects or accidents which usually indicated their own remedy. The stops due to the electrical machinery resulted from a variety of causes and comprised the replacing of an armature coil damaged by lightning, renewing of fuses, fixing loose contacts, the examination of the line after a storm, and sundry other slight mishaps. The aggregate time lost on account of the electrical apparatus was, by actual count, less than 48 hours during three-fourths of a year. A recent report from the superintendent of the plant covering the time from December 13th to May 1st, shows that the plant was running 127 days with a loss of 19¾ hours, or, as he puts it, an average of about nine minutes in a day of 24 hours. Although the plant was generally shut down each week for 12 hours on Sunday this was not practicable during a part of the winter, and the motor on one occasion was run continuously for 27 days without any stop whatever.

Such a record as this, with a new type of machinary, in a country where line construction and maintenance are peculiarly difficult, with practically continuous service, with attendants who were not electricians, with a high voltage, a considerable distance and large power, places transmission by the alternating current synchronous system beyond the stage of experimental trial and gives it the stamp of commercial success.

This success is confirmed in a substantial way by the immediate extension of the plant. A 50 H. P. motor is now being installed at a mill a few miles from the Gold King. An order has been entered for a 750 H P. generator to be located in the power station, and a 250 H. P. motor for operating a mill about 10 miles distant. Lighting at Telluride, eight miles from the station, which has been heretofore done on a small scale on a circuit from the power generator is being extended.

The large generator is a new design, and is notable as it has more than three times the capacity of any alternating current dynamo previously made in this country. Two machines of this size have been running for some months for incandescent lighting in St. Louis.

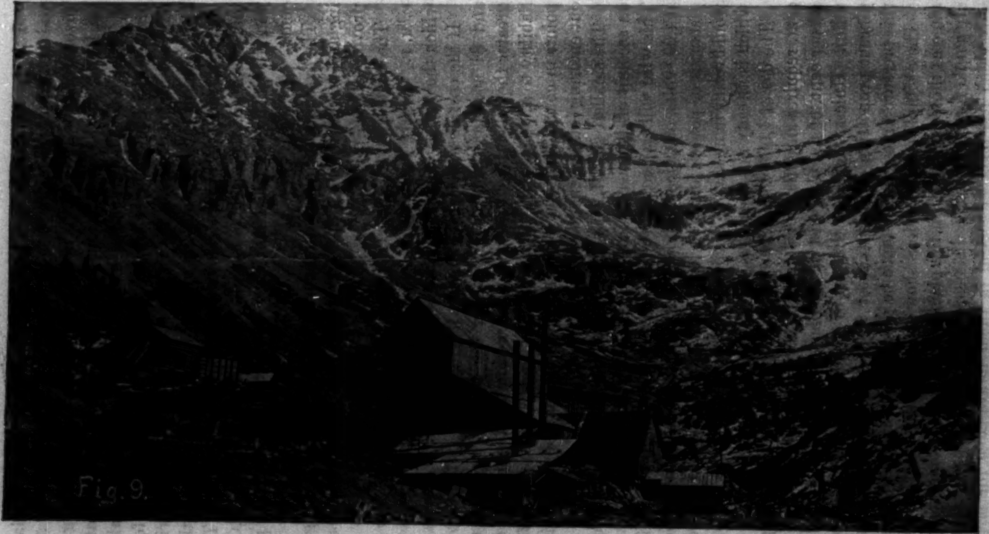


Fig. 9.

View of Gold King Mill illustrating character of country across which pole line is run.

This dynamo is of a type similar to the machines at Portland and Telluride. The field has 28 poles, requiring a speed of 570 revolutions for 16,000 alternations, the conditions of running at St. Louis, and 357 revolutions for 10,000, as it will be operated at Telluride. The armature has T-shaped teeth, as in smaller machines. The diameter of the armature is slightly over four feet and its length is about two feet. There is a third bearing at the end of the shaft outside of the pulley to relieve the other bearings from the severe strains resulting from belt tension. The total height of the machine is eight feet and its weight is 40,000 pounds. The electrical efficiency at full load is over 95 per cent.

The extension of alternating current working, both for lighting and power, by the use of larger machines is therefore already provided for.

The extension to greater distances is largely a question of E. M. F. Nearly everyone is familiar with the rapidity with which the cost of copper diminishes as the voltage is increased.

If the cost be $100 with 500 volts it will be $25 at 1,000 volts, and $1 at 5,000 volts. The higher the tension, however, the greater the difficulty and cost of construction and the greater the liability to accident with apparatus and line. There are a few points in connection with this subject which may be noted without entering into a general consideration of it.

The smallest size of wire that can well be used for line work on account of its mechanical strength, is about No. 6 B. & S. This wire will transmit with 20 per cent. loss 100 H. P. 10 miles at 4,000 volts, or twice the power half the distance. Unless these distances or powers are to be exceeded an increase in pressure would result in no saving in copper but simply in a less line loss, which is already not excessive.

The use of 4,000 volts at the motor and a line loss of 20 per cent. requires an outlay for copper of only about 10 per cent. to 15 per cent. of the total cost of the plant when the distance is 10 miles. Unless, therefore, the cost of copper is to bear an insignificant proportion of the total cost it is unnecessary to exceed this pressure unless the distance be greater than about 10 miles.

These simple considerations show that pressures practically the same as those employed in the plants which have been described are ample for considerable distances. The same type of apparatus which has been successful in them is available for larger capacities. The fundamental elements required for electrical transmission in a very wide range of cases have therefore been tried and their success demonstrated.

For considerably longer distances, where pressures higher than about 5,000 volts are required, good practice indicates the use of transformers for raising the pressure at the generator and reducing it at the motor, similar in general to those employed at Portland. The increased pressure thus available greatly reduces the cost of copper required, and this reduction must, of course, be more than sufficient to cover the cost of the transformers.

The prime requisite in a system for power transmission or distribution is simplicity. In this point electrical appaatus is scarcely to be compared with any other class of machinery. There is a marked difference, however, between the classes of electrical apparatus. In direct current machines the commutator is the characteristic feature. It is expensive to make, to maintain, and to renew. The insulation of the armature is much more difficult, because it is necessary to connect a hundred points in the winding to exposed pieces. The commutator is the part requiring the most careful adjustment and attention both in construction and operation, and yet it is ordinarily the source of more difficulty than all other causes combined. It is, moreover, the principal element which limits the E. M. F. at which this class of apparatus may be operated, and, in order to secure the advantages of a high E. M. F., systems are planned in which the generator and motor is each composed of three or four distinct machines to be run in series, thus avoiding an excessive E. M. F on any single commutator at the expense of many machines and multiplied parts. The alternating current machine substitutes for the commutator the simplest kind of contact—brushes sliding on plain rings.

The alternating current is anomalous. For years it had little place in literature and less in practice. Theoretically, it is intricate and complicated, and the earlier difficulties in its application caused it to give way to direct current. But the direct current is reaching its limitations in conditions which alternating current apparatus readily meets. The alternating current motor has been one of the most complicated and difficult electrical problems, and yet the first large alternating current motor installed in this country excels direct current apparatus in simplicity of construction and operation, in perfection of regulation, and handles with simple sliding contacts, running for a week without adjustment, a pressure which with commutators is impracticable.

There is another class of alternating current apparatus to which no reference has been made. The multiphase, or Tesla system, which has attracted much attention of late, possesses the characteristics of this type of apparatus, which readily adapt it for combining simple construction with the use of high pressures. There are also marked advantages in the characteristics of the motor which are of especial value where distribution of power is to be combined with transmission. Motors of this class are self-starting with load, and may be run either as synchronous motors or with one winding short-circuited and no commutator or brushes. The latter is the ideal motor in point of mechanical construction.

The simplicity, and flexibility, and range of the alternating current system make its possibilities the sole dependence of the largest enterprises, toward which the public and engineers are looking. The records of the plants at both Portland and Telluride, demonstrate that these possibilities are being realized, and that work in this field is fast passing from experimental investigation into practical electrical engineering.

DISCUSSION.

THE CHAIRMAN:—The paper is now open for discussion, which I trust will be free, and that we will pay to this subject the consideration which it certainly deserves. Personally, I am the strongest kind of a believer in the general practicability of the synchronous alternating current motor for long distance transmission, and it is a subject upon which there exists such a difference of opinion that I think a general discussion by the Institute of the features of its use and its present applicability will be of great benefit to users.

MR. HERING:—Mr. Chairman, I would like to ask Mr. Scott whether he has tried starting the motor and generator together. I am told by European engineers, who have put up quite a number of installations with alternating current motors, that if the motor and generator are first electrically connected and then started up together, there is no difficulty and no need of a special starting motor.

MR. SCOTT:—Mr. Chairman, the generator will have to run at a speed sufficient to give some electromotive force in order to send the current through the motor. If the motor be given a slight initial speed, so that the system will really be running in synchronism at the start, it can of course readily be brought up to synchronism, as the speed of the motor will increase as that of the generator increases.

In experimenting with these machines in the factory, it is always very interesting to let the connection between the generator and motor remain when the engine is shut down. As the speed becomes slower, the motor continues to maintain the exact speed of the generator almost to rest, and if the engine is then brought up to speed again, the motor would of course follow.

Mr. Bradley :—I think that the answer to Mr. Hering's question is that the motor must be separately excited in order to accomplish that result. In this case, where the motor has to get the field excitation from itself, it must have a speed in order to generate its own field. I think this paper is a very valuable one, because it is full of facts, and those are what we are all in search of.

I would like to call attention to Fig. 1, which might be of interest as showing one arrangement of a bank of transformers. Now, in this case my criticism does not fully apply, because the electromotive forces are as 3300 to 1100, that is, three to one, and where the transformation is very great, say, from 5,000 down to 100, then the secondary being the low tension and being connected throughout the whole number of transformers, at the end of the bank of transformers it brings primary and secondary very close together, and the insulation must be sufficient between the primary and the secondary to stand the whole voltage. You can readily see that by following it through. We ourselves made the mistake of putting up some twenty in series, and they knocked out the end ones in succession as we ran the voltage up.

Mr. Sperry :—Mr. Chairman, as I recollect, the installations abroad have been run by synchronous motors, and both generator and motor have been excited, so that, on the least turn of the generator armature, sufficient electromotive force can be given to the motor armature to start it and keep it in synchronism. I can voice the sentiments of the last speaker as to the value of the paper. I have been deeply interested in the engineering features. I think it is splendid practice to have a single circuit and some mechanical means of starting the motor. The plan, as shown in the figure in the paper, has an advantage over the method used abroad, due to the fact that no separate excitation is necessary in either motor or generator, and that by the simple addition to the ordinary system of a small motor which must have capacity to generate the field of the motor, besides overcoming the mechanical friction of the generation, the whole system may be set in operation. I think, as one follows through all the details and compares each with the same element existing in the foreign system, we certainly have an advantage in this system. The method by which the two are brought into step exactly and by means of which that step can be observed easily, is as simple as it can be made.

Mr. Bradley :—I would like to ask Mr. Sperry if abroad they run any motors in multiple arc, and operate some of the generators in synchronism.

Mr. Sperry: —I have not seen any installations myself, but I am told by an engineer that he does that right along without any difficulty whatever, with motors of different sizes and capacities. At the Frankfort Exposition, last year, there was a small dynamo coupled with a large dynamo, but I don't remember seeing any synchronous motors run that way abroad.

The Chairman:—I should like to point out one possibility which seems to me to be applicable, and that is that very frequently in a transmission of this nature it may be desirable to have some character of fixed current in the shape of a dynamo of a continuous current type, which may perhaps be operated by an alternating current motor for some other purpose. If so, it would be very readily possible to keep charged a sufficient number of cells of storage battery, such as to enable us to secure the original excitation of the motor field, which would be desirable and sufficient to bring it up gradually into synchronism with the generator in full step as the original power is started. Is there any further discussion on the paper?

Mr Hering:—Mr. Chairman, I agree with the other speakers that this is an exceedingly interesting paper, but I cannot help thinking that the multiphase current system has about as many advantages over this one, as this one has over the continuous current; at least, it seems so to me at present. I have seen a large 100 horse power three-phase motor started without the use of any auxiliary apparatus whatever other than a liquid rheostat. It started beautifully. I also had the pleasure of seeing a small two horse power motor of the same kind, loaded with 100 per cent. overload (that is, with four horse power), reversed as rapidly as the switch could be turned while it was running at full speed. It took but a few seconds before it was up to speed again. It therefore seems to me that a three-phase current has very great advantages in starting as well as in running, because if overloaded the speed will simply diminish slightly, but it will not come to rest suddenly any more than a continuous current motor would.

The Chairman:—We will take up next the paper by Mr. A. H. Bauer, of Chicago, on "Railway Train Lighting." Mr. Bauer being absent, the Secretary will read his paper.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 7, 1892, Manager Leonard in the Chair.

RAILWAY TRAIN LIGHTING.

BY A. H. BAUER.

The first successful system of railway train lighting is that invented by Houghton, and adopted by the London, Brighton and South Coast Railway of England, in 1881–2. Since then the many improvements suggested by other inventors have made it almost automatic.

The system consists of a Brush dynamo carried inside the baggage compartment, and directly connected by link belting to pulleys on the axle of the car. Attached to the dynamo is a ball governor, rotating vertically, whose office is as follows :

1st. To control the movement of a switch, which, when the speed and therefore the E. M. F. of the dynamo reaches a predetermined value closes the circuit, and also opens it again when the E. M. F. approximates that of the secondary battery, which is always in circuit.

2d. The adjustment of a resistance which maintains the current constant, and

3d. To shift the position of the brushes as the neutral point is changed by the varying speed of the armature.

But one set of brushes is used, carried in rectangular holders, at right angles to the commutator, and always in contact with it.

When the rotation of the armature is reversed by a change of direction of the train, the brushes are carried forward or backward by the friction on the commutator and a connection with the governor until the quadrant passes but a fraction beyond the vertical, when it is drawn to the proper position by a spring, the action being similar to that of the snap switch. Limiting stops are provided to prevent the quadrant going too far.

The same movement of the quadrant also changes the position of a switch controlling the connection of the field coils, and therefore the polarity of the dynamo. It therefore follows that the current is always in the same direction, no matter which direction the armature may rotate. One set of accumulators, consisting of 25 cells, is provided for each train, and placed in the baggage compartment. The maximum number of lamps in a train is 70, of 10 candle power each, at 50 volts.

No attendant is necessary, but at certain stations on the line an examination of the plant is made, and required repairs attended to. I am informed the system is so reliable that since the year 1888 but two failures have occurred.

It will be noted that the key of this system is the convenience with which the connection between the armature pulley and that on the axle can be made.

Throughout Europe the rigid wheel base is universally employed under railway baggage cars; consequently, by setting the dynamo across the car, the armature shaft will always be parallel with the axle, and the connecting belt run in practically the same plane.

In this country the bogie truck is exclusively used. Several attempts have been made to drive dynamos by connections with the axle of bogie trucks. All, however, have failed, for the reason that no provision was made for maintaining the armature shaft and axle parallel at all angles of the truck, and because sand, dirt, etc., gathered by a moving truck will soon destroy any mechanism that may be used for the purpose under the car. The problem, however, has not been abandoned, and I am in hopes that within a very short time a solution will be found.

Numerous propositions have been made to use primary batteries for car lighting, but I am not informed of a successful installation of that character.

Secondary or storage batteries have been more or less employed for the purpose, with varying success. The first attempt in this direction was made in 1887–8 on two Pullman limited trains, of six cars each.

Each car was equipped with thirty cells of battery, having a capacity of 150 ampere hours, and weighing 50 lbs. per cell, or 1,500 lbs. per car. Duplicate sets were kept charged at each end of the line, and on arrival of the train, exchanged for those that had been exhausted while on the road. The average number of lamps per car was 26, of 16 candle power, at 60 volts, or a total of 156 in the train. After sixty days' trial, it was clearly demonstrated that, under existing conditions, the system could not be made successful, and it was abandoned.

The reasons for its failure were—1st, that the capacity of the battery was insufficient to maintain the number of lamps required for the proper lighting of Pullman cars on long runs.

For example, the current required for 26 60 volt, .8 ampere lamps, is about 20 amperes. Therefore $\frac{150}{20} = 7\frac{1}{2}$ hours, that is to say, 1,500 lbs. of battery was able to maintain 26 lamps but 7½ hours. These trains departed east-bound at 5 P. M., but the light is required at 4 P. M. for inspection, receiving passengers, etc. Consequently the batteries would become exhausted at 11 P. M.; in fact, it would be earlier, as will be explained. Therefore the oil lamps would have to be used when light was required during the remainder of the trip. West-bound the same reasons held good, although not to the same extent, because the train leaves that end in the morning and passengers will, as a rule, after 12 hours' traveling, retire earlier than those going east, leaving at 5 P. M. Consequently less light is required, and a certain percentage of the charge is not used.

A second reason was the deterioration of the batteries themselves, which were of the pasted grid type. It was found that the constant agitation of the acid solution in the jars, as well as the unavoidable rough usage to which they were subjected when changing, caused a loss of the paste or active material, proportionately reducing the capacity. In fact, close observation indicated the loss during one handling to be equal, if not more, than that caused by 1,000 miles travel on the car. The paste, when separated from the grid, accumulates in the bottom of the jar, or lodges between the plates, forming a circuit through which the cell is discharged, resulting in the buckling and sulphating of the plates, and the destruction of the positive elements within a short time. This was five years ago. Since then, improved methods of manufacture, as well as more intelligent care in handling, has, to a great extent, overcome these objections and thereby increased the life of the battery. It may be asked—why not increase the size and therefore the capacity of the cell? The reply is—that to properly light a train of six Pullman cars, for instance, between Chicago and Jersey City, with the number and kind of lamps above stated, would require 30 cells, each weighing not less than 100 pounds—a total of 3,000 pounds per car, or 18,000 pounds for a six car train. The expense of handling and changing such a weight would make its use almost prohibitory, for each cell would have to be handled singly. The weight on the car would also be objectionable. Since these experiments were made, car lighting for short distances only, from storage batteries, has been made successful. For the past two years four cars, each equipped with 32 cells of 150 ampere hours capacity and 26 16 C. P. 60 volt lamps have been running between Chicago and Cincinnati, and Chicago and Indianapolis. Two of these cars are charged every day at Chicago for about nine hours with sufficient current for a round trip. The batteries, however, are not changed; the cars are always placed on the same side track to which wires from the dynamo are led, and the charging is done without removing them. The average length of time the light is required during a round trip is six hours; the cars leave each end of the line at 8 and 9 P. M. respectively. Repairs are made by removing the defective cell and substituting another if necessary.

The cost of repairs and renewals of all kinds from Sept., 1891, to January, 1892, inclusive, per month, was as follows:

September, 1891	$ 36 44
October, "	108 82
November, "	51 24
December, "	79 21
January, 1892	38 31
Total	$314 02

An average of $62.80 per month.

$\frac{62.80}{30}$ = $2.09 per day for the four cars.

$\frac{2.09}{4}$ = 52.25 cents per car per day.

$\frac{52.25}{26}$ = 2 cents per lamp per day of hours.

$\frac{2}{6}$ = .33 cent per lamp per hour.

As the engine and boiler are used for furnishing power for other purposes while running the charging dynamo, it is difficult to get the cost of the power for charging these batteries, that factor, therefore, is not included in the above statement

In January, 1888, a solution of the problem of lighting six-car trains for long distances was sought for in another direction. Two trains running between Jersey City and Jacksonville, Florida, a distance of 1,100 miles, were selected for the experiment. At first an attempt was made to utilize the power of the axle, but for reasons already stated did not prove successful.

The next step was the substitution of a vertical single-stroke engine, located in the baggage car, for the axle connection. This engine was belted to the dynamo, and was supplied with steam from the locomotive boiler.

While this plan seemed to be a step in the right direction, it still had its objections, due to the fact that the upward and downward movement of the driving-rod acted in the same manner as would a heavy blow on the car body, so that the latter would vibrate in unison with the stroke of the engine.

After a few trials, the vertical engine was removed and a three-cylinder reciprocating engine, with three-inch cylinders and four-inch stroke, was substituted. It was connected direct by a flexible coupling to a 75 volt 60 ampere dynamo of the ordinary open frame type, and both run at 1,000 revolutions per minute.

The working of this combination clearly demonstrated that a solution of the problem was attainable, and it was now possible to guarantee a fairly successful lighting of the two trains A fault soon developed, which could not be anticipated. It is well known that all railway trains, while running, gather more or less dust and dirt, but it was not known until train lighting was attempted, that this dust and dirt contain certain quantities of carbon and metal which adhere to armature and field wires, and being conductive form short-circuits. So frequently did this occur, that it became necessary to carry an extra armature with each plant, and make changes while the train was in motion. Later in the year these dynamos were removed, and the Eickemeyer installed. The Eickemeyer dynamo seems to be perfectly adapted for train lighting. As both the armature and field coils are protected by the iron of the machine, dust and dirt does not reach the wire, except in small quantities, and by removing it every six months, when the car is shopped for repairs, short-circuits have been prevented. Its strong construction is such that it will successfully withstand the general rough usage to which machinery of this character is subjected to on railway trains.

The system which has been used for the past five years on all Pullman trains, consists of a Brotherhood three-cylinder engine and Eickemeyer dynamo, bolted to a cast-iron bed-plate, and located in the forward end of the baggage car, occupying an enclosed space of 6 feet 6 inches by 3 feet 3 inches. The engine and dynamo are connected by a flexible coupling which allows for the irregularity should they not be in perfect line. A two-gallon sight-feed pressure lubricator bolted to the end of the car, furnishes oil to all wearing parts of the engine. The average consumption of oil is about one gallon every 20 hours of actual running of the engine.

Steam is taken from the locomotive boiler, a tap being made at the dome, the pipe passing under the tender and car, between which a flexible rubber hose makes connection. It has been found that with a 1½ inch pipe between the boiler-head and car, there is a loss of 15 pounds pressure due to friction and condensation; therefore, to run the engine with 50 pounds, requires 65 pounds at the supply pipe on the locomotive. With 50 pounds pressure, the dynamo will, at 900 revolutions, generate 80 amperes, at 72 volts, when all the lamps and batteries in a six-car train are in circuit, and 50 to 75 amperes, at 80 to 85 volts, at the same speed when the batteries alone are connected, depending on the counter E. M. F. of the batteries.

On the side of the car is placed an automatic switch, which will break the circuit when the E. M. F. of the dynamo equals that of the batteries. A voltmeter and ammeter are kept constantly in circuit while the plant is in operation. A tachometer, belted to the armature shaft, is also provided for noting the speed of the engine and dynamo. This plant is enclosed by a tongue-and-groove partition surmounted by a wire screen.

The system of wiring adopted is the equi-potential, in which the pressure is practically equal at each lamp and battery. One wire leads from the positive pole of the dynamo to the rear end of the rear car, where it is coupled to a second that returns to the baggage car—but not to the dynamo—being tapped, however by the positive pole of each battery and one side of each lamp, A third wire leads from the negative pole of the dynamo to the rear end of the rear car, where it ends. This third wire is tapped to the negative pole of each battery and the remaining side of each lamp.

All the batteries and lamps are therefore in multiple arc. Double pole, fusible cut-outs are placed in each circuit, and, with the switches, are located in the closets of each car. The wires are always placed on the roof of a car. For this purpose, white-wood moulding, thickly coated with asphalt paint, is nailed from end to end as near the center as possible, usually one and three-quarter inches from the edge of the smoke-jack if the car is equipped with center oil lamps, and from six to 24 inches when gas lamps are applied. Branch mouldings extend to all points on the roof through which it is desired to run wires to the lamps, switches, etc., inside the car. For instance, the drops to the center lamps in the body of the car pass directly through the center of the roof into the air chamber of the oil lamp, from where they are suitably connected. Those leading to the switches, cut-outs and batteries pass through the panels between the ventilators and into the closets where the switches are located, the battery wires continuing on through the floor to the boxes under the car. After being run in the moulding, the wires are covered with wooden capping and the whole tinned over, practically forming part of the roof. The grooves in the moulding are made of such size as to hold the wire rigidly, so that its movement or abrasion is impossible. Wire so applied has, after five years' service, shown no deterioration whatever, and is apparently serviceable for at least five years more. As the tin covering of the moulding is cut and worn away by cinders and the action of the elements, it is found necessary to renew the moulding about once a year. The sizes of wire used are all of B. & S. gauge, and consist of No. 2 for those leading from the dynamo, Nos. 6 and 10 for the lamp mains and No. 14 for the lamp drops. The connection between cars is made by bringing the wires from the roof through the hood to the inside of the vestibule, where they terminate in square brass tubing called connectors, which are permanently attached to the vestibule frame. The connection between these connectors is made with three conductor flexible cables, each conductor of No. 6 gauge. To the ends of each conductor, brass springs are attached which fit closely inside the tubing, giving about eight square inches contact. If cleaned occasionally this connection will carry 100 amperes without sensibly heating. As the connectors are placed horizontally it is evident that should the train from any cause be parted, the connection will be severed without damage to any part of the system, except to throw the load off the engine and cause it to race. Should this occur, which, in fact, is very seldom, the attendant is at hand to cut off steam and stop it until the connector can be replaced. With a No. 2 wire the average fall of potential in a six-car train is nine volts.

When the batteries alone are in circuit, the E. M. F. of the dynamo will be from 67 to 83 volts, which, minus the loss, will overcome the counter E. M. F. of the batteries, viz., 57.6 to 73 6 volts. When both lamps and batteries are in circuit the external resistance is of course reduced and the E. M. F. of the dynamo falls in proportion, and that at the lamps will be from 60 to 66 volts.

It is unnecessary to use a resistance in the lamp circuit, as when the lamps are first turned on, the E. M. F. being say 66, a greater candle power is had just at the time when most needed, which, however, gradually falls and ordinarily reaches 60 to 61 volts in from two to four hours, depending upon the amount of charge in the batteries. It is true that this is at the expense of the life of the lamp, but as the handling of the switches is done by the porters and conductors, the use of rheostats or regulators, no matter how simple, would complicate the system and probably cause the total extinction of the light. Each car is provided with 32 cells of storage battery, placed 16 in each of two boxes, securely fastened under the center of the car, and directly over the truss rod. The weight of batteries and boxes complete, is about 2,000 pounds. As already stated, with the system of wiring adopted the batteries and lamps are permanently connected in multiple arc, so that should the train be parted and connection with the dynamo become broken the batteries will supply current necessary to maintain the light; or should the batteries under one car become disconnected, its lamps will be supplied from batteries under other cars in the train.

The lamp equipment of a six-car Pullman train is as follows:

Combined smoking and baggage car	21 lamps
Dining car	32 "
Three sleeping cars	90 "
Combined observation and sleeping car	29 "
Total	172 lamps.

Of this number, 98 are 16 candle power and 74 are 8 candle power. The latter are placed in hallways, toilet-rooms and vestibules.

During six months of the year, seven ⅛ H. P. fan motors are added, viz.: 3 in the dining, 2 in the smoking and 2 in the observation cars, each absorbing the current of four 16 candle power lamps, or a total of 28, which, added to the 172, makes the total, 200 lamps in the train. To maintain the 200 lamps requires 144 amperes. As the dynamo will generate but 80, the batteries must supply 64 amperes, or 44 per cent. This is effected by charging the batteries during the day and part of the night *en route*, and while the train is standing in the yards.

The following, taken from the January records of a train, will give an idea of the number of lamp hours required:

CHICAGO TO JERSEY CITY.

Time.	Hours.	No of Lamps.	Lamp Hours.	
4 A. M. to 9 P. M.	5	172	860	Leaving Chicago.
9 " 11 "	2	140	280	
11 " 12 "	1	42	42	
4 " 7 "	3	172	516	Arr. at Jersey City.
During day in tunnels and depots ..	1	172	172	
Total	... .	....	1870	

JERSEY CITY TO CHICAGO.

Time.	Hours.	No. of Lamps.	Lamp Hours.	
4 P.M. to 9:30 P.M.	5½	172	946	Leavg. Jersey City.
9:30 " 11 "	1½	140	210	
11 " 12 "	1	42	42	
6 A.M. to 7:30 A.M.	1½	32	48	Arriving at Chicago.
During day in tunnels and depots ..	1	172	172	
Total	... .	....	1418	

Total, 3,288 lamp hours during the round trip.

The cost of an installation for a six-car Pullman train—consisting of one combined baggage and smoking, three sleeping, one dining and one combined sleeping and observation car is as follows:

COMBINED BAGGAGE AND SMOKING CAR.

1 Brotherhood Engine	$500 00
1 Eickemeyer Dynamo	500 00
32 Cells Accumulators	392 00
Wiring	100 00
1 Set Wire Connectors	11 00
Battery Boxes, Crates and Connections	71 00
Steam Piping, Lubricator Gauges, etc	155 00
Voltmeter and Ammeter	77 50
Safety Switch	12 00
Lamps and Sockets	17 00
Switches and Cut Outs	17 50
Shades—Holders and Fixtures	10 80
Partition for Dynamo and Engine	38 00
Two ⅛ H. P. Fan Motors	76 00
Miscellaneous	50 00
	$2,027 80

SLEEPING CAR.

32 Cells Accumulators	$392 00
1 Set Wire Connectors	11 00
Wiring	90 00
Battery Boxes, Crates and Connections	71 00
Lamps	15 00
Sockets, Receptacles, etc	47 00
Switches and Cut Outs	17 50
Shades and Shade Holders	9 00
Berth Lamps	90 00
Miscellaneous	20 00
	$762 50

DINING CAR.

32 Cells Accumulators	$392 00
1 Set Wire Connectors	11 00
Wiring	70 00
Battery Boxes, Crates and Connections	71 00
Lamps	16 00
Receptacles and Fixtures	32 00
Switches and Cut Outs	17 50
Three ⅛ H. P. Fan Motors	114 00
Miscellaneous	20 00
	$743 50

COMBINED SLEEPING AND OBSERVATION CAR.

32 cell accumulators	$392 00
1 set wire connectors	11 00
Wiring	90 00
Battery boxes, crates and connections	71 00
Lamps	14 50
Sockets, receptacles and fixtures	19 50
Shades and holders	10 40
Berth lamps	45 00
Two ⅛ H. P. fan motors	76 00
Miscellaneous	20 00
	$749 40

A total of $5,808.20, or an average of $968.00 per car.

As the fan motors are used but six months in the year, the average number of lamps will be 172 + 14 = 186. Therefore, the cost of equipment per lamp will be $5,808.26 ÷ 186 = $31.22.

The average cost of labor and material for maintaining the lighting of three trains, or 18 cars for 19 months, from August, 1890, to February, 1892, inclusive, was as follows:

Average total cost per month	$1265 95
" " per car per day	1 99
" " of labor per month	712 39
" " " per car per day	1 11
" " of material per month	511 51
" " " per car per day	87
" " per lamp per day	07.17
Average number of lamps in use	558

The item of labor includes the wages of five attendants on the train, at $3.00 per day each, and two men at each terminal station, at $90.00 and $55.00, and $75.00 and $50.00 per month respectively.

That of material includes the cost of renewal of batteries, etc.

The cost of labor is practically constant, while that of material varies from 12 cents to $2.04 per car per day, the average for 19 months being 87 cents. The cost per lamp per day varies from $3\frac{8}{10}$ cents to $12\frac{5}{10}$ cents; the average is 7.17 cents.

The cost of power furnished by the locomotive boiler is not included in the statement, for the reason that a satisfactory measurement of the quantity of coal and water used has never been made.

So long as the dynamo can be run, and the batteries properly charged, the system is reliable.

During the coldest part of the winter, or as sometimes happens, when a poor quality of coal is furnished, it is difficult for the locomotive to supply sufficient steam to run the dynamo. In such cases the batteries are called upon to maintain the light. As a rule, they have sufficient charge to do so for about four hours. Trains so equipped have been run between Jersey City and St. Augustine, Florida, during the months of January, February, March and April of each year since 1888.

The conditions there are decidedly more favorable, for the reason that the trains leave both terminals early in the morning. In consequence, the dynamo is run nearly all day, and the batteries are therefore fully charged and in condition to furnish current for the maximum number of lamp hours. So successful has been the electric lighting of these two trains, that it is rare for other sources of illumination to be used. The system has also been used on other trains, viz., between New Orleans and the city of Mexico, Omaha and San Francisco, Chicago and Portland, Me.

Several specials have also been similarly lighted, notably—the Presidential, from Washington to San Francisco and return; the National Electric Light Association trains, between Jersey City and Chicago, and Jersey City and Kansas City; the National Telephone Exchange Association train to Minneapolis, and that of the Pan-American delegates, which was almost constantly on the road for three weeks.

A duplicate of this system was tried for a short time on the Chicago, Milwaukee, and St. Paul Railway. I must look to others to explain the causes for its being abandoned, and also describe that which has been substituted and which it is understood has proven very satisfactory. One of the most serious sources of trouble met with in train lighting was the effect of the vibration of the car on different members of the plant. At first it was almost a daily occurrence to have lamps drop out of sockets —the shade-holders, which were made of thin sheet brass, were broken at the rate of eight or 10 during one trip. Trains would arrive at terminal stations with two or three batteries out of circuit caused by connections between cells being broken. In the earlier days it was the practice to use a rigid connection between battery cells, as it was considered best to prevent the elements moving in the rubber jar. It was soon discovered that the element would move irrespective of the rigid connection, and in doing so would break the terminal, and in a majority of cases the fracture was next to the plates. As we had no means of burning the lug on again the element was useless. To overcome this difficulty flexible connections were substituted. Contrary to all expectation no damage was done by the movement, and since then should a terminal be broken, which however is seldom, it is in every instance due to a defect in burning on at the factory. Cast brass shade holders were substituted for those of sheet brass. To prevent shades working loose and dropping out, ordinary rubber bands were placed under the lip, against which the holder screws were driven, and they having a soft foundation were prevented turning and held the shade in place. The cause of the lamps falling out was due, in a great measure, to the socket rings working loose. This was overcome by the invention of a tool that would drive the ring into place much tighter than was possible by hand. Many minor causes of trouble that could not be foreseen were eliminated as fast as they developed.

A system of safety cut-outs was devised that protect each wire in the car. They are placed on the same baseboard with the lamp switches, and are located in the locker or closet, convenient for inspection and renewal. Later on portable lamps were demanded that could be placed in the sleeping car sections, either before or after the berths had been made up. For this purpose No. 18 wire was connected to one of the lamp circuits on the roof, and led to a connecting block placed over the mirror between the windows in each section. Connection with the block was made with a plug, to which flexible lamp cord was attached. On the other end of the cord a frosted 16 candle power lamp was connected, the lamp hanging from an arm which was placed in a receptacle in the corner formed by the side of the car and the partition between sections, and immediately over the shoulder of the passenger, the most favorable position for the lamp when used for reading.

It was early seen that the only uncertainty connected with the system is due to the absolute dependence upon the locomotive boiler for power to run the dynamo, for, as explained, there is a constant liability of the failure of the steam supply which, if prolonged beyond the capacity of the battery will result in the use of oil or gas for light. As many other problems connected with the system have been successfully worked out, it is believed that the solution of this one will also be found.

There is no doubt that the ideal way of running the dynamo is by a connection with the axle. Many apparent solutions have been suggested, only to be condemned as uncertain or impracticable. It is, however, thought that within the next year a reliable connection will be had, after which I believe the lighting of railway trains by electricity will be universally adopted.

Discussion.

Mr. C. R. Gilman:—Referring to the cost of installation for a six-car Pullman train, given in Mr. Bauer's paper, for comparison with ours, that is, the Chicago, Milwaukee and St. Paul system, I give the following:—Cost of equipping one ten-car train, consisting of one baggage car, two mail cars, two coaches, three sleeping cars, one dining and one parlor car. Baggage car, including one Westinghouse 6x6½ automatic 15 horse power engine, one No. 4 compound Edison dynamo, switches, meters, etc., $1,183.19. Two mail cars, $119.84 each—$239.68. Two coaches, $147.92 each—$295.84. Three sleeping cars at $299.88 each—$899.64; one dining car, $192.23; one parlor car, $205.50; making a total of $3,016.08, or an average of $301.61

per car, a total cost per lamp of $15.82. Our cost of labor and repairs on four trains, 35 cars for one month, is as follows:—Labor, $650; material, $175.86—total, $825.86. Cost per car per car day, 78.6 cents; cost per car per car day for labor, 61.9 cents. The cost per car per car day for material, 16.7 cents. Cost per lamp per day, 5 cents. Average number of lamps in use, 542.

THE CHAIRMAN:—How much is that per lamp per hour, roughly, Mr. Gilman?

MR. GILMAN:—I will have to figure that out.

THE CHAIRMAN:—While Mr. Gilman is getting some additional figures, is there any discussion on the paper that any gentleman would like to participate in? This is a very important subject, and one which is receiving a great deal of attention from companies which are lighting trains by gas, recognizing the demand for electric lights and the necessity for their having electric lighting systems in order to protect their already established gas lighting industries. The figures which Mr. Gilman has given us would indicate an extremely low rate for the production of light on the trains of the Chicago, Milwaukee and St. Paul Railway, and he will have for us presently the exact figure per lamp hour, which is perhaps the best way of making comparisons as to economy with other systems.

PROF. SHEPARDSON:—I would suggest, Mr. Chairman, that possibly the plan Mr. Sperry has used for coupling together four axles on his mining locomotive could be used for transmitting power to the dynamo on our trucks. I understand that plan has been quite successful in Europe, where the car axles are not swiveled.

MR. WILLYOUNG:—Mr. Chairman, Mr. Bauer was talking about this matter, Sunday, and I asked him whether he had finally secured the results that he was after, and he said he had; he had plans in his office, but he said it was not an original plan. He said it was a plan that has been used in Europe for some time with considerable success, and it may be that it is a similar plan to that to which the gentleman refers.

MR. SPERRY:—Mr. President, I would like to call attention to the fact that it would be impossible in such a system, although on the face it cost about half to install, to detach any of the cars without leaving them in darkness during the time that they are so detached.

MR. HERING:—I would like to ask also what you would do when the cars are in the station?

MR. GILMAN:—Oil lamps are used during stops.

THE CHAIRMAN:—The figure that Mr. Gilman presents is five cents per lamp per day. He says the average burning is about nine hours a day, and it brings the cost of the lamp hour down to approximately a half a cent per lamp hour. That is extremely low, as the cost of the production per lamp hour of the largest central stations to-day is in the neighborhood of a quarter of a cent. But it is natural to suppose that the cost of production per lamp hour by a system such as this, would be much less than in a system which would involve storage batteries, although this system would be applicable only to through trains, on account of inability to take care of breaking up the trains.

Mr. Clement F. Street:—Mr. Chairman, there is one criticism I wish to make regarding this paper. The average total of material per car per day is given as 87 cents. Now, it seems to me that is the cost per day while this plant is in existence. It is not the cost per day of 24 hours, of 12 hours during which the light is being used, and I do not think this is the proper basis on which to figure. If the number of hours which the plant was in use had been taken and the figures derived from that, it seems to me it would have given more information. As I understand it, he has given the cost of the 24 hours during which the plant is in existence, whether it is in use or not.

There is another point, in regard to breaking up the train. Of course, in the system which is in use on the St. Paul road the statement was made that when the train was in the depot it was necessary to use oil lamps This occurs only at points when the engines are changed and when the baggage car plant is used. When the light and heat tender is used in connection with the system, it is never necessary to use the oil lamps at all, because the light and heat tender has an independent engine and boiler. This is used in the winter, when it is necessary to supply steam heat to the cars. In the summer the same engine and dynamo are transferred from the light and heat tender to the baggage car, and steam taken from the locomotive, consequently it is necessary to shut down the plant during the time of taking one engine off and putting another on, which will be every 100 or 150 miles, depending on circumstances, and only in rare cases more than once during a single period when the lights are in use.

The Chairman:—I would like to say in this connection that I have had brought to my attention a system which is likely to be put into commercial use, in which the number of storage batteries which will be used will be very much reduced, so that there will be but two or three batteries per car, although the lamps which are in ordinary service will be lamps of 110 volts, and the main lighting of the train will be direct from the dynamo engine, upon the locomotive probably; although in case of the removal of the locomotive or separation of cars the storage batteries will act automatically, lighting by electric lamps of different voltage from the main lamps in the car, which will obviate the necessity of lighting up some other lamps by hand means, which would be troublesome.

Mr. Gilman:—I would like to say that Mr. Gibbs has already got that system started. We have three cars equipped on the road with small batteries and an automatic switch that throws them in when the train is cut off from the locomotive.

[Recess.]

EVENING SESSION,

At the Chicago Electric Club, June 7th, 1892, at 8.30 P. M. President Sprague in the chair.

THE PRESIDENT:—The first business before the meeting this evening will be the consideration of the report of the committee appointed at the New York meeting, to make some changes in the Rules with reference to the election of officers. It has been concluded, and I think very reasonably, that the method we now have of electing our officers is not a satisfactory one, that is, it does not necessarily represent the choice of the Institute as a whole. At that meeting a committee was appointed, consisting of Dr. Herzog, Mr. Upton and Mr. Martin, which was instructed to submit recommendations for such changes in the Rules as would make the election of officers somewhat more representative of the feeling of the Institute.

The report will be read by Mr. Martin.

REPORT OF COMMITTEE ON REVISION OF THE RULES REGARDING THE ELECTION OF OFFICERS.

V. ELECTION OF OFFICERS.

During the first week in February of each year the Secretary shall mail to each full and associate member of the Institute a list of members; a list of the offices to be filled at the ensuing annual election in May, giving the names of the incumbents, and a copy of this rule, with the request that nominations, propositions and suggestions as to desirable candidates be made promptly and prior to March 1st.

The Secretary shall submit all answers to the Council, who during the month of March shall prepare a complete ticket to be headed "Council Nominees," containing the names of members whom they deem best suited, all circumstances considered, for the offices falling vacant. This ticket shall be printed on the same sheet with a "General Proposal list" containing the names of all eligible members proposed, with a memorandum stating the reason of ineligibility of all other names proposed, and with a statement that the nominations by the Council are in nowise governing, but are intended only to assist members in making a choice. The voting shall be restricted to the names on this sheet This sheet, together with an envelope, on which shall be printed the address of the Secretary, and the words "Voting Envelope—Enclosing a Ballot Only," shall, not later than the 15th of April be mailed by the Secretary to every member in good standing.

Each member may cast three votes for Vice-Presidents and he may cast all three votes for one candidate or two for one and one for a second, or one for each of three. Each member may cast four votes for Managers, which he may in like manner divide between from one to four candidates. This division shall be specified by the appropriate numeral marked opposite the desired name.

All names voted for shall be written, printed or otherwise marked on a single ticket or ballot, which shall be enclosed in a sealed, unmarked and unidentified envelope of any suitable size to be in its turn enclosed in the voting envelope received from the Secretary. On this outer envelope the member shall add his signature, and the postage stamp.

At the annual meeting, these outer "Voting Envelopes" shall be opened in the meeting room by two tellers, then and there appointed by the presiding officer. After all the inner envelopes shall have been thoroughly commingled, they shall be opened and the votes shall be counted by the tellers, who shall report the results in writing. The eligible persons receiving the greatest number of votes for the respective offices shall be declared duly elected. The tellers shall reject all names that do not appear on the "Council Nominee" list or the "General Proposal" list, and they shall also reject that part of a ballot which shall name for any office or offices more candidates than there are vacancies.

This method of election shall apply to the offices of President, Vice-President, Treasurer and Manager. No unsalaried officer except the President and Treasurer shall be eligible to immediate re-election to the same office (as provided in Rule IV). The Secretary and any other salaried officer shall be chosen each year from among the members of the Institute by the Council. The Secretary shall be a member of Council, but shall have no vote in its proceedings.

New York City, June 4, 1892.

Committee to report on Improved Plan of Election and Change of Rules for that purpose.

F. BENEDICT HERZOG, Chairman,
FRANCIS R. UPTON,
T. C. MARTIN, Secretary.

APPENDIX.

The following is the section of the existing Rules, which it is proposed to amend:

V. ELECTIONS.

The annual election shall be held in the city of New York, on the third Tuesday in May of each year. Nominations of officers may be made in writing to the Secretary at any time previous to the meeting, but the votes shall be by ballots deposited only by members actually present. *Provided*, that no member or associate in arrears since the last annual meeting shall be allowed to vote until the said arrears shall have been paid. If for any reason it is deemed proper to postpone the election, it may be adjourned from day to day with the consent of two-thirds present or voting.

The following is the existing Rule regarding Amendments:

VIII. AMENDMENTS.

The Rules may be amended at any regular meeting by a two third vote of the members present, provided that written notice of the proposed amendment shall have been given at a previous meeting.

It was voted that the report be printed and a copy mailed to each member, but that action upon the report be postponed until the first regular meeting in New York City. After considerable discussion an informal vote showed that the sentiment of those present was entirely favorable to the proposed plan.

The following papers were then read by Professor R. B. Owens on "Electro Technical Education," and by Professor Dugald C. Jackson, on "The Technical Education of the Electrical Engineer." They were discussed jointly.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 7th, 1892, President Sprague in the Chair.

ELECTRO-TECHNICAL EDUCATION.

BY R. B. OWENS.

In his presidential address before the American Society of Mechanical Engineers, in 1883, Professor R. H. Thurston, in speaking of the policy and work of the mechanical engineer, claims electrical engineering as "the last established branch" of his profession. And again, in 1887, President Geo. Babcock, referring to electricity, says:—"Yet another great force . . . is yielding its neck to the yoke prepared for it by the mechanical engineer;" and a great number of similar quotations from the writings of other engineers might be made to show that when the importance of the industrial applications of electricity were fully recognized, the mechanical engineers claimed such electrical work as a branch of their profession. But the very existence of this body to-day, evinces the uselessness of such a claim. Again, it is not unusual to hear it asserted by some members of our profession that electrical engineering will, in time, absorb all engineering professions, although I believe those more familiar with the extent and variety of the work which might be called purely electrical are rather of the opinion that life is all too short in which to deal with the electrical problems, presented and are glad to refer the problems of mechanics to the mechanical engineer. Indeed, the field of work seems amply large for both professions, and, as remarked to the writer some years ago by Professor W. P. Trowbridge, "A definite and amicable understanding should be reached between them, so that we do not duplicate each other's work."

My object in presenting this paper at this time is to induce, if possible, a discussion which will throw more light on the relations which should exist between our profession and other engineering professions, as well as to learn by an interchange of opinion the best methods of instruction to be adopted in the training of electrical engineers. The subject of the technical training of mechanical engineers has been very fully treated by the American Society of Mechanical Engineers, and the Transactions of the American Institute of Mining Engineers contain valuable suggestions as to methods of training mining engineers, and so on with other engineering societies. But as yet the subject of the technical training of electrical engineers has, I think, hardly received the attention it deserves from our society, and in view of the fact that it includes managers of manufacturing companies, members of engineering firms and a number of men prominent in educational circles, I shall venture to hope the subject will receive the exhaustive treatment it well merits. We can consider ourselves fortunate in having a society such as this, where engineers engaged in every variety of work may meet on common ground and learn from each other, and I might here suggest, if indeed it has not been already suggested, that subjects be chosen by a suitable committee and appointed for "Topical Discussion" at certain meetings, as the mechanical engineers have done, and in this way still further increase the sphere of our usefulness.

Having stated my reasons for presenting this paper, I shall try to show the necessity of making technical training in general, graduate instruction and electro-technical training in particular.

A moment's reflection will show, that to the man of pure science or to the engineer, we owe all our methods of rapid communication and transportation, our supplies of metals and the necessities of daily life, as well as the utilization and adaptation of the latent and wasting powers of nature, but it is difficult to say to which one we owe most. "To have the application of a science, we must have the science itself," as remarked by Professor Rowland, in his address before Section B of the American Association, in 1883; but how often are the inquiries of such physicists confined to the simple enunciation of a physical or chemical principle which remains practically unproductive of good for centuries, and perhaps has to be discovered again, as witness the work of Lord Henry Cavendish. But I would not depreciate the work of the man who so often foregoes wealth and ease, to devote his life to that highest of human efforts, the study of nature's secrets and the enunciation of her laws. His mission is a high one and rarely appreciated as it should be, or else the "lucky inventor, who is lauded in every newspaper, would not receive more praise than him who perhaps discovered the principle upon which the invention is based, and died in obscurity and want."[1] Not that the trained inventor deserves no credit; the world owes him much, but not more than the man of pure science.

Between the scientist and skilled mechanic or handicraftsman is the engineer—the man who, familiar with the results of scientific investigation and methods, uses both, by the help of the mechanic's skill, to minister the more effectively to the wants and will of an ever increasingly exacting public. To him alone, we look for the solution of the problems of transmitting and distributing the power of waterfalls and fuels, the tunneling of our mountains, the building of our railways and steamships, as well as the more delicate work of flashing our thoughts and words from town to town and from continent to continent. This field of work is so vast that of necessity it must be divided among specialists and sub-specialists; but since the "Vril" of Lord Lytton's "Coming Race" has shown itself so marvelously well fitted to do our every bidding, I believe a great portion of the work hereafter will be done by the electrical engineer. And as so much will be expected of him, the duty devolves upon us to give him such training in his youth as will most effectively enable him to grapple with the problems to confront him in his future work. I have said the engineer uses the mechanic's skill in the accomplishment of his aims, and if so, he must possess a certain familiarity with tools and shop methods, or else he will not always be using such skill to the best advantage. But, evidently, it is not expected of every engineer that he spend sufficient time on shop practice to attain the same degree of efficiency in the use of tools as acquired by the mechanic who devotes his whole life to their use without attaining the highest skill in manipulating them. And experience, both at home and abroad, has shown that systematically conducted elementary schools where the "manual element" has been introduced "are far more efficient in developing and training of youth than the best managed mill or shop,"[2] for in the latter, despite the good intention of the manager or president, the learner is kept months by the foreman stamping out armature disks, if there were need of armature disks, or punching holes, if there were need of holes, while in the school the variety of equipment is greater than in the best shop, and every operation is taught with the object of making the student familiar with the principle involved, and no more time is spent on one operation than is sufficient to insure a thorough understanding of it.

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[1] Professor Rowland.

[2] Professor R. H. Thurston.

Until within the past few years, the value of the manual element in education has been sadly overlooked in elementary instruction in this country. Special and distinct schools, called variously; manual training schools, schools of mechanic arts and developing schools, have arisen, where those elements which underlie all industrial pursuits have been introduced into general elementary education, not because students leaving such schools expect always to enter upon some trade or into some "trade school," but because such instruction is eminently "desirable for the roundness and unity of general education, and valuable no matter what the future of those so educated may be."[1] The Massachusetts Institute of Technology, the Washington University, St. Louis, and others, have attached to themselves schools of mechanic arts and manual training schools, for the reason, I take it, that the elementary public schools do not afford such instruction, and they recognize the great educational value of their training.

But as soon as such instruction is given, as it should be, in the public or preparatory schools, the universities will find it no longer necessary to devote their energies to such elementary work, but will reserve their whole force for the higher instruction for which they are originally designed. Aside from the educational value which instruction in the use of tools, wood turning, pattern making, forging, metal working, etc., has, and which alone would justify it, "simply on the ground of the discipline it gives,"[2] such instruction becomes of double value to a student intending either to learn a trade or become an engineer. If he intends to learn a trade, and perhaps be a shop foreman, he probably knows, due to the manual instruction already received, whether he will become a watchmaker, carpenter or machinist, and enters a trade school where the technique of his chosen trade is taught.

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1. President J. D. Runkle, "Manual Element in Education."
2. President Runkle, "Report in Developing Schools."

If his aspirations are higher, he pursues his studies still further and enters a technical school whose primary province is the training of engineers.

The necessity of trade schools has arisen from two facts which make it particularly difficult to learn a trade to-day. First, the "abandonment of the old apprenticeship system," and, second, "the division of labor occasioned by the general introduction of machinery."[1] Many of these schools are doing excellent work and turn out workmen in two or three years, quite as skilled and possessed of very much more general knowledge than the apprentice of fifty years ago after spending "seven years learning his trade."[2] And all engineers should be glad to see such schools prosper, as they produce another tool which, to them, is of almost inestimable value, the skilled handicraftsman. In nearly every profession which makes applications of the teachings of science "manual operations of exceeding delicacy have to be performed,"[3] and it is but just to the skilled handicraftsman to acknowledge our great indebtedness to him. It is difficult to say whether the science of astronomy owes more for the discovery of the moons of Mars to Asaph Hall, or to Alvin Clark who made the instruments by which the moons were discovered. And just in proportion as the progress of the world requires a superior training of engineers, it will require the better training of the handicraftsman. Engineers should naturally, then, give all the encouragement in their power to apprenticeship and trade schools, since the instruction given in such schools unquestionably produces in time, better and more skilled mechanics than those not receiving such instruction.

Professor Huxley, in an address before the Workman's Club of London, I think, says:—"Technical education" in "good vernacular English would be called the teaching of handicrafts." But I think, in this country at least, we have attached a different meaning to the term. Here we generally mean by technical education the training of engineers, and in that sense I will use it.

"Technical schools are not indigenous to this country."[4] They have generally been copied from European models and cannot claim as yet to equal the schools of the old country. Yet, due to

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1. President Barnard, *Century Magazine*.
2. Mr. Hale, "Report on Developing School."
3. Professor Huxley, *Fortnightly Review*.
4. T. M. Drown, *Journal Franklin Institute*.

the exceptional skill, ingenuity and enterprise of our people, the engineering works of America are, with few exceptions, unexcelled. And I hope that, in order to demonstrate our ability in long distance electric transmission, much of the energy wasting at Niagara will drive the machinery and light the halls of the World's Fair buildings in 1893.

In studying the various courses given in the best technical schools in this country and abroad, and the opinions of educators and engineers, I take it that a technical school is primarily a place for the preparation of men who expect to earn their living as engineers. It is not a school of general culture, nor is it a school of abstract science. It is a device to save time, and teaches the applications of pure science to industrial purposes.

A technical school is also, I claim, a post-graduate school in the ordinary sense, and in this respect should rank with schools of law and medice. If the engineer is a man who uses both the results of science teaching and mechanical skill, he must certainly have great familiarity with both, and the attaining of such familiarity will constitute his undergraduate work. But after such general knowledge is acquired, no time should be lost in fitting each student to be of the greatest immediate value to those who employ him. Unfortunate as it may be, we must reconcile ourselves to the necessity of becoming specialists. It may be argued that "the more a man knew of all the related branches the better he could perform his special work, but this is only arguing the advantage of omniscience."[1] Time was when all known about electricity would not greatly tax the ordinary intellect, but now each student can only hope to become an expert in one very narrow department, and I believe we must shape the courses, as far as possible, in our electro-technical schools to meet the requirements of the times.

Of course, to employ a number of specialists and to give each student the opportunity to choose the work he is best fitted for, will be expensive, but if our country is to keep apace with others in industrial progress she must educate her engineers. When private generosity fails, state aid must be forthcoming, or we will have to witness others surpass us in the race for international position. "It is in consequence" of superior industrial and technical education "that Germany is gaining a better industrial position daily."[1] And the same might be said of several other foreign countries, so that on the score of economy no more profitable expenditure of public money is possible than for the support of technical schools. Education is not a money-making business. "No fact is more firmly established all over the world than that higher education can never be made to pay for itself.[2] No considerable income can be anticipated from student fees. Higher education must look for its support to private munificence or public help, and, as suggested to the English Institution of Electrical Engineers by Professor Ayrton, it would seem to be the duty of such a body as this to influence, as far as possible, public opinion in favor of advanced electro-technical education.

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[1] T. M. Drown, *Journal Franklin Institute.*

"How many of our people," says President C. K. Adams, "know that one of the minor universities of Great Britain has recently completed a college building at a cost of more than £500,000 ($2,430,000), not to speak of the four millions that were put into the Polytechnicum at Charlottenburg? How many have had their attention called to the fact that the little Republic of Switzerland, with a territory not a third as large as the State of New York, has recently, from its public treasury, built a chemical laboratory for the Polytechnic school at Zurich, at a cost of 1,337,000 francs ($267,400)? And of those who suppose that needless sums are expended at Harvard, Yale and Cornell, how many know that the little Kingdom of Saxony, only half as large as Vermont, gives from its public treasury annually $400,000 to its university, although the institution itself has great wealth? . . . Let us not cherish the erroneous supposition that there is a single well-endowed university in America. Let us remember that the richest of our institutions has an income not much larger than that of a single one of the twenty-four colleges of Oxford."

Having seen the importance of manual training as a factor in elementary education, and knowing its special value to those intending to become either handicraftsmen or engineers, I would recommend that a fair knowledge of forging and foundry work, pattern making, wood and metal turning and drawing be required of every student for matriculation into any college course. Such an elementary knowledge can readily be acquired by the average

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1. Professor R. H. Thurston.
2. Professor Rowland.

student by the age of 16 or 17, along with the elements of English education, such as given in our high schools and academies.

The importance of drawing can hardly be overestimated. It is rightly called " the shorthand language of modern science," and every one can learn to draw with a fair degree of accuracy, although the " negative faculty of drawing in some people is almost miraculous."[1] I no not mean artistic drawing, painting and perspective, but the elements of orthogonal projection can be as easily acquired as the art of writing, and for some purposes it is just as useful. Indeed, " drawing is now regarded by many educators as an established factor in elementary education, and destined to work its way into all classes of public schools."[2] At the age of 16 or 17 a boy having had good advantages should be able to enter any of our best American colleges, and here, if he intends becoming an electrical engineer, I would have him study for three years mathematics, chemistry and physics, with German and French. These apparently few subjects will absorb the greater portion of his energy. Nothing is gained by working at excessive pressure in early life, and, as to knowing anything about engineering itself, it would seem best to leave that alone until a thoroughly good scientific education had been first acquired. When electrical engineering is taken up in earnest, the student should have such a command of physical and chemical facts and methods, and be such a master of mathematical analysis as to leave him free to discuss engineering problems without the necessity of first acquiring the facts and methods he may wish to make use of.

A good course in modern geometry, differential and integral calculus, say, according to Williamson, and differential equations, say, Forsyth or Craig, would be sufficient to pursue with advantage nearly any course in electrical engineering given in this country at the present time, and when, in reading Maxwell, it becomes convenient to have quaternions or spherical harmonics, they can be studied in connection with such reading. But to attempt to analyze the action of alternating current apparatus without the use of differential equations, is, to say the least of it, no very easy task. We admit the value of geometrical methods and especially as expressing in convenient form the result of algebraic analysis.

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1. Professor Huxley.
2. President J. D. Runkle.

Indeed, in the hands of Mr. Kapp, Mr. Blakesly and a few others, they have been used with excellent analytic effect, but their general weakness, as compared with modern methods, is too well known to need comment. As regards the amount of general physics required, I should say the student ought to be familiar with the theory of heat measurements, thermo-dynamics, the principles of fluid motion, the dynamics of a particle and of solid and rigid bodies, the theory of sound and physical optics, and the facts and principles of magnetic and electric actions. Above all, he should be able to apply mathematical reasoning to physical problems and be skilled in the art of experiment.

In ordinary electrical work, a knowledge of inorganic chemistry would perhaps suffice, but if one expects to make a specialty of electro-chemistry, then both organic and inorganic chemistry should be required.

Besides mathematics, chemistry and physics, which must be considered as principal subjects of the directly preparatory course, German and French should be acquired, as so large a part of our best electrical literature is in these languages. Sanscrit and psychology might be classed as electives, as affording variety.

Indeed, aside from the three principal subjects of the preparatory course, it would seem best to allow as wide range of choice in the remaining subjects as one is capable of carrying, for engineering is " a narrow life which concerns itself solely with building bridges or making analyses,"[1] building, erecting and testing electric and magnetic machines. Unless his general education is gotten in early life, I believe it is the experience of engineers that such culture is doubly difficult to attain after entering upon strictly professional duties.

Many colleges give, for the completion of the course I have outlined, as preparatory to electrical engineering, the degree of B. Sc.; but, irrespective of degrees, I believe the subjects above named should be thoroughly mastered before electrical engineering can be taken up with the best advantage.

Assuming, however, that a thorough working knowledge of mathematics, physics and chemistry is possessed by the student, then I believe he should spend three years in studying the applications of his acquired facts and principles before attempting to practice his profession.

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1. T. M. Drown.

We must, I think, recognize three classes of electrical engineers in this country at this time. First, the installing engineer, who superintends the equipment of central or isolated electric light or power plants, electric railways, mining plants, etc., and runs the same when completed. In this class of work, more than a knowledge of electricity is required. It becomes essential to have a thorough knowledge of steam engineering, track and pole line construction as well. He should be as well able to choose the best engine and boiler for a given set of conditions, and to install and operate them economically, as he is able to choose the best dynamo, motor, line wire or lightning arrester. A practical knowledge of building foundations, laying out railway curves, running pole lines, is to this engineer a matter of greatest importance.

The next distinct class of electrical engineers I would recognize, are designing engineers, who rather have to do with the manufacture of electric apparatus than its installation. Such men design and superintend the construction of the apparatus put on the market by the several manufacturing firms, but rarely also have to do with steam engines, track construction or pole lines. To this second class, a correct knowledge of machine design is of equal importance with a knowledge of magnetism and electricity, and it was perhaps of such men that Lord Kelvin was thinking when he said an electrical engineer should be nine-tenths mechanical and one-tenth electrical.

The third class of engineers are employed by standardizing bureaus and in laboratories, to make tests of electric and magnetic constants, calibrate instruments, etc. For such work, neither a knowledge of pole lines, foundations or machine design is so important as familiarity with delicate methods of measurement and skill in manipulating sensitive instruments.

It would seem best, then, to shape the course, in any electrotechnical school, as far as possible to meet the requirements of these distinct classes of engineers.

The proportions of the three kinds I have named vary greatly in different parts of the country. With us, at the West, the greater part of electrical work consists in installing and operating the apparatus manufactured by Eastern firms. In some few cases, electric factories started west of the Mississippi have been successful, but their number is not great, and the proportion successful is less. The time will undoubtedly come when the mineral wealth of the

West will be mined and manufactured by Western waterfalls, but so long as the East continues the centre of manufacturing interest, the principal work of the electrical engineer in the West will be installing and operating Eastern manufactured apparatus. Therefore, to meet the immediate home demand for engineers, I believe our Western schools of engineering must make a specialty of training installing engineers. In the East the case is different. More men are employed by manufacturing firms, testing and standardizing bureaus than by construction and engineering companies, so that the course given in the Eastern schools of engineering, and which are largely for the designing engineer, are justified.

After an inspection of the engineering departments of the University of Nebraska some months ago, Professor R. H. Thurston declared the electrical equipment to be "the finest, considering the size of the institution, in the United States,"[1] so that a word about the electrical department may not be without interest.

Our apparatus for instruction and experiment consists briefly of one Sterling water tube boiler, 125 H. P. at 150 pounds pressure; one 100 H. P. tandem compound, high speed, condensing engine, arranged to run either cylinder high pressure; one Worthington independent condenser, one 25 H. P. single cylinder high speed engine, also one 5 H. P. Otto gas engine, a line of countershafting with clutch pulleys and couplings throughout; one 500 light, 1,000 volt alternating current machine with converters and regulating apparatus; two 15 K. W., 120 volt continuous current machines complete; two 25 light, 10 ampere arc machines, and six smaller machines of various types; seventy-five 200 ampere hour storage cells; Prony friction brakes, cradle spring and hydraulic dynamometers, standard electric and magnetic measuring instruments, the greater part of which were imported; steam calorimeters, indicators and other instruments for steam measurement. The equipment, which is entirely new and of the latest and most approved design and construction, is valued at about $45,000. All the heavy machinery is placed in a separate building, 50 by 60 feet, designed for the purpose, while wires lead from it to the physical laboratory, where the more delicate instruments are placed on brick piers, as usual.

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[1] *Nebraska State Journal*, March 19th, 1893.

The requirements for entrance into the college course preparatory to the electrical course are:

English—Spelling, capitalization, punctuation and pronunciation, and thorough, logical and historical study of English grammar, two hours.

History—Mediæval and modern, three hours.

Hygiene and physical training, one hour.

Language—French, five hours. German, five hours; 10 hours.

Mathematics—Algebra, five hours. Algebra and geometry (alternating,) five hours. Plane trigonometry, two hours; 12 hours.

Science—Botany, chemistry, physics, five hours.

Second year:

English—Continuation of the work of the first year, two hours.

History—Mediæval and modern completed. English history, three hours.

Hygiene and physical training, one hour.

Language—French, five hours. German five hours; 10 hours.

Mathematics—Algebra and geometry (alternating), ten hours. Plane trigonometry, two hours; 12 hours.

Science—Botany, chemistry, physics, five hours.

Manual training, three hours.

Each "five hours" represents full time for one subject for one semester or one "course." After completing the matriculation requirements, about six years are required for the degree of "Electrical Engineer."

The first three years are given mainly to a study of mathematics, chemistry and physics, and the last three years to engineering proper.

The course is as follows:

Chemistry—(1) Study of the metallic elements, their compounds and characteristic reactions. (2) The elementary principles of chemical philosophy. Two courses.

Mathematics—(1) Solid geometry and trigonometry. (2) Higher algebra and analytic geometry. (3) Conic sections and calculus. (4) Calculus and differential equations. Four courses.

Physics—(1) Experimental physics; lectures and recitations on mechanics, sound and heat; light, electricity and magnetism;

supplemented with exercises in the laboratory. (2) Advanced physics; application of calculus to the solution of physical problems; thermo-dynamics; physical and geometrical optics; dynamics of solids and fluids; theory of elasticity; problems in sound; theory of electricity and magnetism; supplemented with work in the laboratories. Three three-fifths courses.

Graphics—Descriptive geometry; orthographic projections of points, lines, planes, surfaces and solids. Four-fifths course.

English—(1) Lectures on the principles of rhetoric; practical exercises in criticism and composition; themes and studies in style. (2) Advanced course in rhetoric and oration; studies of the best styles in models, etc. One and three-fifths courses.

English Literature—English literature from Chaucer to the modern period. One and one-fifth courses.

Germanic Languages—Cohn's Ueber Bakterien; Helmholz's Ueber Gœthe Wissenschaftliche Arbeiten. One course.

Romance Languages—Scientific French; reading of scientific authors. Four-fifths course.

Military Science. Two-fifths course.

Physiology and Hygiene. Two-fifths course.

Shop Work. One and one-fifth courses.

Applied Mechanics and Strength of Material and Structures. One one-fifth courses.

Mechanism and Mechanics of Machinery. One one-fifth course.

Machine Design—In connection with mechanical drawing. One one-fifth course.

Steam engineering—Study of types of boilers and engines; their design, construction and erection; boiler and engine trials; fuels. One one-fifth course.

Theory of Prime Movers—Theoretical study of the principles underlying the action of steam and hot air engines; hydraulic and pneumatic machinery. One one-fifth course.

Theory and Practice of Electrical Measurement—Special problems involving exact measurement, supplemented by readings on problems assigned. Four-fifths course.

Electromagnetic Mechanism and Machinery—Includes a discussion of the principles of design and construction of dynamos, motors, transformers, lamps, telegraph instruments, telephones, etc.; the effects of self and mutual induction, capacity and hysteresis are considered.

Electro-Thermal Apparatus and Thermo-Generators—In this course are discussed arc and incandescent lamps, electric welding, thermo-generators, etc. Two-fifths course.

Chemical Generators, Electro-Chemistry and Electro-Metallurgy—Includes a discussion of primary and secondary batteries, electroplating and electrotyping, electric refining of metals, etc. Two-fifths course.

Mathematical Theory of Electricity and Magnetism—Requires a thorough knowledge of the calculus. One three-fifths course.

Principles of Electric Installation.—In this course are discussed the design and equipment, care and management of electric light and power stations, electric railways, telephone and telegraph systems, electricity in mining and hoisting, pole line construction, inside wiring, underground conductors, problems in the transmission and distribution of power by electricity. Two two-fifths courses.

The last year of the course is given up mainly to theses and electives in electrical engineering. Besides the regular course above outlined, leading to the degree of "Electrical Engineer," short and special courses are given to undergraduate and special students, who for various reasons cannot complete the full course for the degree.

No attempt has as yet been make to teach telegraphy practically, and as we have here some members who have had much experience in practical telegraphy, I hope they will express their opinions as to the advisability of introducing it into electro-technical schools. Some declare it properly belongs to engineering, and others contend, with much truth, I think, that it properly belongs to a trade school. Similarly with wiring, electroplating and electrotyping. Is a practical knowledge and skill in them to gotten in or out of the school? I trust the discussion on such points will be full.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 7th, 1892, President Sprague in the Chair.

THE TECHNICAL EDUCATION OF THE ELECTRICAL ENGINEER.

BY DUGALD C. JACKSON.

Perhaps it would be well to call my subject the "College Education of the Electrical Engineer," for it is strictly of the technical college course that I shall speak. We can truly affirm that the technical education of an engineer does not end until his work is ended, and the college course is but the commencement of it. That the college course can be made a very important fundamental part of this education, is becoming more thoroughly appreciated as the work of the technical schools comes into closer harmony with the demands of the profession, and it is now generally agreed that a technical college course of the proper kind forms a valuable aid towards the success of the average young man who wishes to enter the engineering professions. It therefore becomes a matter of no little moment to so arrange the course that its usefulness will be a maximum. A few years back, a college course entirely devoted to the training of electrical engineers was unknown. At the present time there is no dearth of such courses, and their organization is progressing right and left, whence it is well to carefully consider what requirements of the electrical engineer's profession they may be made to meet, in order that no powder be wasted. It is neither possible nor desirable that the courses of study of electrical engineering students in the various schools should be alike, but a certain unity of purpose and treatment should be observed, and all can profit by the suggestions made by the practical man.

With this in view, I present the subject to your attention as it is looked upon at the Engineering School of the University of

Wisconsin. There is no originality claimed for the ideas presented, as they are based upon the recorded experience of some of the country's most successful practical men, and are virtually followed in such other engineering schools as make their courses thoroughly practical, and therefore, in the true sense, professional. I trust, however, that a discussion will arise that is in proportion to the wide importance of the subject to the electrical profession and that must result in a considerable increase in the efficiency of the electrical engineering courses in our various colleges, nearly all of which are still in process of crystallization.

In order to enter the freshman class of the best engineering schools, the applicant must have a thorough common school education, including mathematics through ordinary algebra, a fair knowledge of English, a reading acquaintance with German or French, a little elementary physics and chemistry. This can be gained in the high schools of most of the cities of this country. The high school timber (some of it quite green) the college is required to work, and to work it to the best advantage requires no little careful designing. In order that an engineer may use his abilities and training most advantageously, he should have a good general education, including a fair knowledge of literature, history, economics and certain elements of law. This cannot be expected to come from the high school, and you can readily appreciate that an attempt to give a general education in an engineering course can only result in sacrificing the good of the students by omitting essential fundamentals. Thus, to have an average chance of proving successful, an electrical engineer must be well grounded in three sciences besides those gained in the common schools, and which connot be classed as engineering. These are: Higher mathematics, as far as it may be practically applied in engineering; chemistry and physics (including elementary electricity and magnetism); and manual training. A few students enter college who have been given a fair start in these, but they are the exception. Consequently the subjects must be taught from the ground up, with a common sense view to their practical applications. Unlimited time could be given to these preparatory subjects, but it is necessary to clear them away in the actual time of two college years. With this requirement, it is impossible to give a very thorough knowledge of analytical chemistry, or of physics, but they are taught so as to give the student a good working knowledge and so that he can readily go deeper if he finds it to advantage in his future practical experience. The higher mathematics require all the time that can be afforded, especially in its last division, that of applied mechanics, where the student gets his systematic knowledge of the properties and uses of materials.

With the preparatory studies cleared away, the student must enter into professional studies in earnest, but there is little time for true engineering. The developing electrical engineer must expand his physics and his chemistry and mathematics into the laws of electro-magnetism, alternating currents, electrolysis and electro-metallurgy, and study the conditions of their numerous practical applications in engineering and the arts, each of which may demand months of constant effort before an intelligent mastery is attained. Neither can he confine his attention to these during two full years, for he must gain an elementary but practical knowledge of thermo-dynamics and hydraulics, with an efficient working knowledge of their applications in steam and water power plants. He must also get a *common sense knowledge* of the principles underlying the design, manufacture and selection of machinery.

This is a great deal to expect a student to efficiently absorb in four years, and it requires a most judicious selection in order that nothing unessential be allowed to enter and that nothing essential be omitted. Let us see how the selection is made at the University of Wisconsin. The arrangement of the fundamentals will first claim our attention.

During the first year the student is given a course of four subjects, continuing through the year. These are:—1st, English and rhetoric, with such reference to technical forms as seems desirable so early in the course; 2d, mathematics, beginning with higher algebra, passing through trigonometry and descriptive geometry, and into analytical geometry; 3d, advanced French or German, grammar and reader; 4th, manual training. In the latter, which continues during the following two years, we do not think it necessary or desirable for the student to spend sufficient time during his course to become a carpenter, machinist, blacksmith or foundryman. His future calling will probably not demand that his wages be earned in either of these trades, but they are tributary to his profession, and he must have an intelligent mastery of the tools and an appreciation of shop requirements. In order that some future day he may become a successful designer, or a useful shopman or superintendent, it may be desirable for him to take a properly arranged apprentice course in a first-class commercial shop, after completing his college course. Mathematics are also continued through the second and third year, during which time analytic geometry, calculus and applied mechanics are passed through. All mathematics are taught with especial view to future *practical* applications, and good use is made of the laboratory in applied mechanics. During the second year of the course, elementary chemistry and physics are disposed of, and here again the laboratory is put to good service. At the same time, work in draughting and the elementary designs of machines is begun. The third year is about half, and the fourth year wholly devoted to what may properly be called professional studies. The arrangement of the latter in the electrical engineering course, we will examine later.

Upon completing his technical college course of four years, an average student has spent at least 144 weeks of hard study, much of it of a practical workday nature. During this time he has been called upon to spend upwards of five hours per day in class-room and laboratories, and about as much more time in individual study. No one is likely to go satisfactorily through such a course unless he has a decided taste for engineering work, but many students find themselves capable of doing a considerable amount of extra work and yet have sufficient time for recreation to keep their health and spirits. It is well for an engineering course to stand beyond the reach of students without a taste for the work, for a successful engineer must be pre-eminently an enthusiast, while he is at the same time a candid and careful *thinker*. Those who are not fitted by nature to become engineers, are better placed in a general educational course at college, and they are then more likely to become useful to society and to themselves than if passed through the technical mill.

It may here be asked:—Of what use is the severely specialized education to the successful student in the engineering courses? The graduate does not become an engineer merely because he has successfully met the college examination. College cannot make an engineer, however practical the course of study may be. Practice has made thousands of good ones without the aid of the college, but I venture to say that these would frequently have become more eminent if they had received a thorough technical college course. While theory alone, wherever learned, can not make a practical man, it is the one who can follow the guide of theory along the paths of practical work and experience, who makes the fully developed engineer. In order, however, that neither theory nor practice may lead him astray, he must have a *well-educated common sense.* The eminent and eloquent engineer, Alexander L. Holley, well illustrated this in one of his addresses, when he said:

"Mere familiarity with steam engines is not, indeed, a *cause* of improved steam engineering, but it is a *condition.* The mechanical laws of heat were not developed in an engine house, yet without the mechanism which the knowledge derived through this familiarity has created and adapted, the study of heat would have been an ornamental rather than a useful pursuit. So in other departments. . . . When one in any art can make a diagnosis by looking the patient in the face rather than by reading about similar cases in books, then only may he hope to practically apply such improvements as theory may suggest, or to lead in those original investigations upon which successful theories shall be founded."

The true object of the technical college is here outlined. It is to teach the fundamental theories, with a common sense view to their practical applications, in such a way as to aid in a diagnosis, not by the application of a mathematical formula, but by comparing the accumulated experience of the practical world. Take two young men of equally *good* ability and equal age. Put one through a thorough technical college course and the other through an apprenticeship of the same length of time. Finally, put them side by side in a working position, where they must work out their own salvation, and the college man will usually have more ambition and adaptability, and will outstrip his mate, though perhaps not at once. The college man may fall behind at first, but, having worked through the transition period, he will prove the winner. I venture to say this is the well-nigh universal experience of those who have had the opportunity of dispassionately trying the experiment.

Another illustration of the advantage of the technical college course, lies with the designer. To design good machinery is a natural gift, and to become thoroughly successful requires long experience, in order that the widely varying requirements may each be given due weight. Proper instruction at the technical school may here do much toward stimulating an appreciation of the lessons of experience. The considerations of primary importance to be followed in designing machines, are admirably divided by Professor A. W. Smith into four:—1st, adaptability; 2d, strength and stiffness; 3d, economy; 4th, appearance. In developing the design of a machine, the practical but highly sanguine inventor often forgets all the considerations except the first. A theoretical draughtsman may figure the strength to great precision by formulas that may not fully cover the required conditions, and in the meantime forget the other considerations. When the design reaches the shopman, it must be altered to suit his views of economy, as the prime factor. A machine is thus produced that has lost part of its adaptability as designed, and has neither sufficient stiffness to properly do its work, nor a thoroughly substantial, workmanlike appearance. The economical shopman has been defeated in his object, for the machine is hard to sell or requires costly repairs at the expense of the maker. A proper college course should sufficiently broaden a man, so that he can quickly appreciate the demands of the prime considerations of practice, and will apply his formulas with common sense and moderation. If we replace our three men in the machine transaction, by men of equal experience and a technical college education of the right sort, the work of each should supplement the work of the other, and the product can be predicted, with some confidence at least, to be a satisfactory commercial one. The fault of much of the college training for engineers has been the lack of this *education of the common sense* or judgment. The result has often been graduates with as great a contempt for the practical man as the latter could return. These graduates have, it is needless to say, been a failure in their calling, and it is such men that technical colleges should not turn out. The best engineering schools desire to, and do, turn out men who have a *capacity for practical* work and research, and who are in a fair way to make useful engineers.

It is comparatively easy to properly teach the fundamental theories, hence it is so frequently overdone. It is not so easy to educate the judgment of a student in electrical engineering, whose entire knowledge of his future profession has been acquired from the electric bells in his father's house, and who may never have examined a dynamo or storage battery until he visited the college laboratories. But it is wonderful how rapidly such students, when of good timber, absorb a beginner's information and a thirst for investigation. In this part of the student's education, the manufacturers and large users of electrical apparatus, who have become directly or indirectly interested in the work of the graduates, can assist with little direct inconvenience and much indirect advantage. In a properly organized technical school, as shown above, the student gains his fundamental theory during the first three years, and, if of good timber, he will absorb much of the practical methods of thought required for successful after-work. Moreover, a considerable part of the third year is spent in practical instruction. As the fourth year is wholly spent in practical training, or the education of the common sense, the student must have some acquaintance with the methods of commercial work before entering it, in order that he may properly profit by the instruction. It is impossible for many, and doubtless undesirable for the majority, to take a year from the midst of their college course for outside work. The summer vacation between the third and fourth years should, therefore, be occupied in some such employment as wireman on electric light or telephone construction, or better, in the station and repair room of an electric railway, under the eye of an appreciative superintendent. Three months spent in this work may seem very little, but it will do a deal of good in giving an apt student a fair idea of how far exact formulas will carry him. It is only by the generous co-operation of employers that students can obtain this summer's work. At first thought it appears that the employer gains no advantage from it, but upon careful consideration, an advantage is evident. To begin with, the properly trained student will not prove useless during the summer, and the satisfactory one will usually find employment after graduation with the interests of those who afforded him summer work, and who thus gain the benefit of his greater advancement during his last year at college. In a similar manner the manufacturer gains an advantage from placing his apparatus in the technical school laboratories for *proper* use in instruction.

Suppose a student has completed the prescribed college course, and has done a proper portion of repairing armatures, stringing wires, or similar work, at some interval between his terms at college, what shall we call him? A few of the technical colleges of the front rank call their graduates engineers, but we have already seen that they must pass through a transition period, during which the claim to the title can be proved. To call an untried graduate an engineer does not seem proper respect for himself or the successful workers in his profession. The transition period may never end for some graduates, while its length must always depend upon the man. Until the graduate has been in practical life a sufficient time to show his capacity, and has reached a position of responsibility, he has no right to claim from his college an engineer's degree. Upon this ground, the University of Wisconsin, as do many others, confers degrees in engineering upon graduates of its engineering school of not less than three years' standing, who have held engineering positions of trust for at least one year. The minimum transition period is thus tacitly recognized as three years. Upon completing his college course, the student is given a graduating degree of Bachelor of Science by the engineering school, which is simply an endorsement by the University that he has received a good technical college education and is in a fair way to profit by it.

That the rigid specialization required in the technical school may not diminish the graduate's field of vision, and thus his usefulness to society, is a matter of much concern. With the college left behind, there is little opportunity to gain a broadening culture, except that received by contact with broad men, while we have seen how little opportunity for this can be afforded in the technical course. With this in view, we recommend, at the University of Wisconsin, that all who can afford the time and money complete a four years' undergraduate course in the University School of Arts and Sciences before entering the School of Engineering. By proper elections during the general course, the studies of an engineering course can be completed in two additional years. By this plan a solid educational foundation is laid for the specialized studies of the engineering student, and the best conditions are developed for his ultimate success in professional work. The plan offers two other points of advantage. First, the student comes to his professional studies in the engineering courses with a more matured mind, which is of much importance; second, students without the taste for hard engineering work, which is required for their future success in technical industries, will not often attempt a technical course after having completed a general course.

We can now usefully inquire into the specialized work that should be prescribed for the average electrical engineering student during his last two years at college. Up to this point, students in mechanical and electrical engineering courses have received virtually the same instruction. Here, we hold, with several others, their paths should diverge. The student of mechanical engineering goes into careful study of shop practice, designing and utilizing various types of machinery and similar subjects. The electrical engineering student must receive a good working knowledge of the problems of the mechanical engineer, but he must, above all, be trained in the practical problems of electrical engineering. He, therefore, goes into a study of that which will aid most in making him truly an electrical engineer. His knowledge must all be based on mechanical laws, but he must be much more than one-tenth electrical.

Before reaching his truly professional studies, the student should gain, during his course in physics, a common sense grasp of the elementary notions of electricity and magnetism and of the "all-pervading law of Ohm." The latter can be properly enforced in the laboratory by placing in the student's hands ordinary electrical instruments, such as bridges, galvanometers, amperemeters, voltmeters, etc. Before beginning his specialized work, the student's knowledge of Ohm's law and its common results should have become almost instinctive.

With due regard for his preparation, it seems best to arrange the professional studies for the average electrical engineering student in four divisions, thus:

1st. Electro-magnetism and its application to practical uses, with special reference to dynamos and motors.

2d. Electro-chemistry (including primary and secondary batteries) and electro-metallurgy.

3d. Alternating currents and alternating current machinery, including dynamos, converters, condensers, etc.

4th. The special application of the preceding divisions in electric light, power, railway, mining and other types of plants.

The last division is allotted about twice the amount of time given to each of the others.

While higher mathematics is a useful aid in each of the divisions, its limitations as an agent must be carefully set forth in the class room and laboratory. For the purpose of educating the judgment and fully defining the limitations of theories and mathematical deductions, the laboratory is indispensable. As much as one-half of the total time spent by the student under the direct instruction of the professors of electrical engineering should be devoted to the laboratory. This work, moreover, should as far as possible deal with commercial instruments and machinery, and actually follow the methods of testing and research used in practice. Physics, chemistry, mechanics, steam-engine, hydraulics, dynamos, electrolysis, alternating currents and other subjects should all be properly represented by a commercial laboratory equipment, which is made useful in everyday instruction under the direction of a man who has had experience in similar commercial work. The laboratory method of educating the student is unfortunately too little developed in many of our engineering schools, but a strong movement has begun in most schools to increase it in efficiency and amount. At the University of Wisconsin we carry the laboratory instruction as a part of the required work in every subject in which it is possible.

While the specialized course of the electrical engineering student during the last two years is largely devoted to strictly electrical engineering, he is also given proper class room and laboratory instruction in useful allied subjects, such as the steam-engine, boilers, water-wheels, laws of contracts, etc., as has been already explained.

Students who are mature and show that they can usefully specialize more severely than is done in the regular prescribed course are permitted, by election, to devote a greater proportion of their time to either of the first three divisions already enumerated. Thus, a student may have reason to know that a thorough course in electro-metallurgy will be specially useful to him. In this case, his work in the second division is increased beyond the course requirements, and his work in the first and third divisions is proportionately decreased. Other things being equal, a student who has thus arranged his course may graduate with his classmates who have followed the fixed course as laid down. In the same way, a student of sufficient maturity, who feels assured of special advantages in the field of electric transmission of power or electric railways, may increase his work in the first or third divisions and proportionately decrease it in the second.

The student who satisfactorily completes a *proper* professional course at college, whether laid down in the college catalogue or carefully elected from that prescribed, is not likely to become one who "turns out results like a cornsheller, and never grows wiser or better tho' it grinds a thousand bushels of them." In order that he may have a fair opportunity of growing "wiser and better" in the practice of his art, he should be given reasonable encouragement. As Mr. Holley one time said, an understanding should obtain—

> "among the owners, directors and commercial managers of engineering enterprises that it is not a matter of favor, but a matter of as much interest to themselves as to any class, that young men of suitable ability and of suitable preliminary culture, however acquired, should have an opportunity and encouragement to master the practical features of technical education *in works*, not as mere apprentices, but under reasonable facilities for economy of time and completeness of research."

A legend on the cover of a circular lately issued by the Engineering School of the University of Wisconsin, gives the true object of the technical college, when it says, "We do not aim to produce engineers, but to produce men with great capacity for becoming engineers." If our product is accorded the treatment advised by Mr. Holley (himself an experienced manufacturer), we feel sure the work of our school and of similar technical schools will not be useless.

Madison, Wis., May, 1892.

DISCUSSION.

THE PRESIDENT:—The two papers are before you for discussion, gentlemen. I ask that the discussion be brief, pointed and that those who have anything to say upon these papers will be prompt in responding. I take the liberty of calling upon Professor Crocker to open the discussion.

PROFESSOR FRANCIS B. CROCKER:—Mr. President, these papers have quite fully treated this very important subject, but I do not think it could be exhaustively treated short of a large volume. We are simply experimenting at all the colleges at which electrical engineering courses are given, to find out the best method, just exactly as we are experimenting in the various branches of electrical industry to find out which form of dynamo, or which form of lamp is the best. The work of this investigation can be greatly facilitated, I think, by just such discussions as these. I consider the matter is worthy of as much time as we can give to it this evening.

In regard to what electrical engineering is, strange to say, that point has been the most unsettled of all, because such men as Sir William Thomson and one or two others have stated very definitely—in fact, in mathematical language—that an electrical engineer is *not* an electrical engineer. They say he is nine-tenths mechanical engineer. When I entered into the profession of teaching electrical engineering, I took the stand that that was not true, and could not be true. I have no occasion to regret having taken that position. We all know that an electrical engineer has his hands full to know even a part of electricity. I would turn the figures around. Instead of spending one-tenth of his time learning electricity, I consider that the time has come or will come very shortly when men will have to specialize in electrical engineering; and that entirely disposes of the idea that a man can be nine-tenths mechanical and one-tenth electrical.

No one appreciates more than I do the importance of the mechanical side of electrical engineering. But I claim that it is incidental; it performs exactly the same function and occupies the same position as mathematics or chemistry. We must know these subjects in order to be finished electrical engineers. It is a necessary part of the profession, but the problems that present themselves are primarily electrical. I have only to refer to the papers that have been read before the Institute at this meeting, almost all of which are electrical. Look over our programme, and you will find that nine-tenths of the papers are electrical, which is certainly proof of what I say.

In electrical engineering we do not have complicated mechanics; we have simple rotary motion in most of our apparatus, and we have reason to thank heaven that such is the case. If we had reciprocating motion, the mechanical engineering of which is infinitely more complicated than uniform velocity, our problems would be correspondingly complicated, and it is largely due to the fact that we have simple mechanics, that electrical engineering is so successful. Ours is a sister science to civil engineering, to mining engineering, to mechanical engineering. Each one of these borrows from the others, but each is independent.

Now, in regard to a course of instruction in electrical engineering, I agree with Professor Owens and Professor Jackson very thoroughly, that a man must have a taste for the subject. It is absolutely essential, I think, that a man should be born for it, and the course should be so designed that the man who is not cut out for the profession will find it out, and will give up the attempt to be an electrical engineer.

The preparatory course should be simply a first-class high school education, carried as far as possible. At Columbia College, we are trying to raise our standard, at the lower end as well as the upper. That is one way to get around the difficulty of putting so much in a short time. Instead of beginning at sixteen or seventeen years, as stated in these papers, we do not allow a man to enter until he is eighteen or perhaps nineteen years of age, when he is supposed to have pretty well mastered the elementary subjects in mathematics, and so forth, and to have really begun on higher work. The first two years of the course, as both papers have stated, are largely preparatory. They are spent in going further with mathematics, chemistry, physics, mechanics, particularly laboratory work in chemistry and physics, and bringing the student right up to the point where he can begin the truly professional part of the course which is carried on during the last two years, and can be completed pretty thoroughly in that time, provided the preparation of the first two years has been substantial. That has been my experience. In fact, I will guarantee that in two years I can make an electrical engineer out of a man who has been thoroughly over those preparatory subjects, provided he has the taste for it. That is a short time, but it can be done with first-class preparation. By an engineer I do not mean an experienced engineer, but one who is thoroughly familiar with the subject, and more familiar with it than the practical electrical engineer. His general knowledge is far superior to that of the average electrical engineer in practical work. But, of course, his experience is much less extensive. That is the principal difference between them. After a few years, he has not only the scientific and general knowledge, but the practical experience, the combination of which makes the finished and really valuable electrical engineer.

I agree with Professor Jackson heartily in his strong recommendation of the education of common sense. The mere teaching of principles is not enough. A man must be taught to use his judgment, and he can be taught to do so. You can put a man through one course where the principles are perfectly taught, but very little of the practical or common sense part is given, and the result will be that he is a good abstract thinker, perhaps. You put the same man through a course where at the same time he learns the principles, he also acquires common sense and judgment, and he will be a first-class, useful man. I claim that the college can turn out a man who is useful immediately, but of course not a finished, experienced engineer.

The papers, I say, are so satisfactory that all I can do is to endorse them. The points that I have given are the ones that occurred to me as being particularly important.

The President:—I think that the definition which has been given of the electrical engineer may, from practical experience, perhaps be slightly varied so as to be as follows:—One-tenth electrical, one-tenth mechanical and eight-tenths I don't know what.

I have been in the unfortunate position sometimes of having a great many people come to me for employment. Some of them have been young men fresh from technical schools—not those which pretend to give an electrical engineering education, but which have been sending out great numbers of graduates with no special electrical training. Oftentimes they seek employment in electrical work because it seems to them to be the coming profession. The first question I generally asked them was:—"Why do you want to go into the electrical field? Have you any particular fitness for it?" "Well, I have worked with batteries, I have put up the bells at home and I like to use the telephone." "That is a somewhat restricted training for an electrical engineer. At what school were you educated? Have you had a common school, high school or collegiate education? What was your principal bent? Have you mathematical tastes? Have you studied differential calculus and, if you have, do you still know that two and two make four?" [Laughter.] After I had learned something about a man in that way, I asked him:—"Have you mechanical tastes? What salary do you think you would be able to earn when you start out? Are you aware that the average man, when he enters into the electrical field will do his employer more damage, unless he is reasonably well trained before he starts, than he can possibly earn in salary the first year?" For I oftentimes had in mind an experience which I once had with a technical graduate, now happily successfully filling an important position. He watched a motor with its commutator and brushes going to destruction as rapidly as possible, because he didn't have the knowledge to move the brushes to the neutral point. I watched him about five minutes until my ire began to rise and I had to unbosom myself. That was simply because he had not had that preliminary instruction which one single week in any respectable technical school should have given him. He was not a handicraftsman, in any sense of the word, and he certainly was not an electrical engineer.

After I had learned something about a man, if he looked promising, I would say:—"Now, when you go out on the road, you will run across a number of men, perhaps younger than yourself, who have had more or less of a mechanical training. They have not had the advantages, perhaps, of a technical school such as you have been in, but they have been in the shop; or if they have been in an ordinary school, they have been for six months or a year getting practical experience under a clear-headed superintendent of an electric railway. Are you willing to go in there and put yourself on the same level with, or below, them, take off your coat, get on your back in the pit, underneath a car, and hustle?" When a man said he was ready to do that, was anxious to start in and did not expect an exorbitant salary, and realized that the experience he was going to get in the first six months would probably be quite as valuable to him as the work he would do would be to his employer, I concluded there was something in him.

Now, going back to the training which I had to go through some years ago in the Naval Academy, at Annapolis. I find that there is there a course of instruction, as there is also in the Military Academy at West Point, which, it seems to me, carries out in a practical way some of the results which may be attained in a manual school, in schools for handicraftsmen. A man to know how to teach another man to pull a stroke oar, must get on the stroke oar himself; to be safe on the quarter-deck, to give orders for reefing a topsail in a gale of wind, he must himself have reefed a topsail in a gale of wind. To know how to tell a man to ease a weather sheet or to work the running gear of any part of a ship, he must have had the practical experience on that same gear. He cannot instruct his men properly, he cannot command them safely and efficiently, unless he has been through three or four years of hard, practical experience, hand-in-hand with the men in the forecastle. The same thing is true of electrical engineering. No man is fitted to be superintendent of a road or works, no man is able to carry on large engineering operations, until he has had the practical experience and the necessary education which fits him to pass judgment upon what will be the results of the directions which he may give to others.

Mr. Lockwood:—Mr. President, it has been a source of pleasure to me to see that you yourself appreciate the importance of a full discussion of these very valuable papers. I am especially obliged to you, sir, for your definition of the term "electrical engineer." I may perhaps be pardoned if I add to that definition the reason that it came to be so universally used. After the first race of electrical men had to some extent outlived their usefulness, there sprang up another, a mushroom generation, who advertised themselves as electricians. They were not electricians in any sense of the word; they were not even medical electricians. [Laughter.] They were mechanical bell hangers who had appreciated the necessity of being in the swim, and who had therefore added electricity to their former means of obtaining a livelihood. But they so universally called themselves electricians that the word fell into some disrepute, and therefore it was necessary to coin another and more euphonious one. And thus it came about that before we had any institutions for learning in that line, we had electrical engineers.

With you, sir, I am extremely obliged to the authors of the papers now before us, because I think, in the first place, the subject of technical education has been by this Institute in some measure neglected. It seems to me that education should be the pursuit of every man's life, from its beginning to its end, therefore being a life work. Education, as I think every gentleman present will agree, is not a mere matter of the four or five, or perhaps ten years that we may spend, first at the common school, then at the high school, then at some academy or college, and perhaps by and by in a technical institute. If a man concludes that his education is finished when he leaves the institution where he is being drilled and fitted for practical life by his preceptors, I hardly think that the education of that man as far as it had gone, will be worth anything to him or to any other member of the human race. It does seem to me, therefore, that any paper which calls our attention to the importance of education, and especially such education as is discussed in these papers, deserves our hearty thanks.

Having said so much, perhaps I may be pardoned if I say a few words regarding the papers as they were read, not page by page, because that would take too long, but by reference to certain points to which my attention was especially attracted. In Professor Owens' paper I notice that on the first page he speaks of the rather premature assertions of some mechanical engineers whom he mentions. It seems to me that, even setting aside the fact that those assertions were not founded on fact, mechanical engineers had very much to do with electrical engineering, as we know it now, than had either civil or mining engineers. For if we look at the poles, outside wiring and cable running, or anything of that kind connected with outside construction and the construction of conduits, it seems to me that that is very much more the work of the civil engineer than of the mechanical engineer. If we look at the appliances for the utilization ef electricity in which ingenuity is manifested, there the mechanical engineer plays his part. But when we come to the construction of conduits, it appears quite proper that we might class it as the work of the mining engineer. [Laughter.]

Passing on to page 464, I find that he says, near the top of the page: "Between the scientist and skilled mechanic, or handicraftsman, is the engineer—the man who, familiar with the results of scientific investigation and methods, uses both, by the help of the mechanic's skill, to minister the more effectively to the wants and will of an ever increasingly exacting public. To him alone we look for the solution of the problems of transmitting and distributing the power of waterfalls and fuels, the tunneling of our mountains, the building of our railways and steamships, as well as the more delicate work of flashing out thoughts and words from town to town and from continent to continent. This field of work is so vast that of necessity it must be divided among specialists and sub-specialists." This statement may be true at the present day—no doubt it is partially true—but in this connection it seems to me to be proper to point out that it is not necessarily so. During the early part of my career with my present employers. I had as an assistant a gentleman who had been a professor, or partial professor, at some institute, and the one thing that he seemed to have thoroughly learned the power of saying, was "Not necessarily," when a proposition was put forward by that part of the inferior human race that we call "the practical man." There was not a single thing that was put forward by myself or any other gentleman there, but what he would at once give utterance to the shibboleth "Not necessarily," after which he would proceed to prove why it was not necessarily the case, and then would finish by following my way of doing it.

In this connection, I would say that I do not think it is alone to the skilled engineer, who is most familiar with scientific investigation and methods, that we look for these great achievements. That is partially disproved by the history of the lives of such engineers as Metcalf and MacAdam, the road makers; Stephenson, the railroad builder; James Watt, who made the steam engine what it is; Telford, the bridge and canal builder; and Brindley, who made the first modern canal—these were all self-taught men who did not regard their education as finished when they learned to spell "cat," but who carried on their education all their lives.

Professor Owens says that schools are far more efficient in developing and training youth than the best-managed mill or shop. With that statement I most cordially agree. When I was thirteen years old I had to go to work in a machine shop in Birmingham, where I was to remain for seven years. Fortunately for myself, I left when that term lacked three years of its completion and three years of the four during which I worked there, I was drilling holes in iron plates. It was a foot drill which I had to work, and I think had it not been for that hard work, I might have been three inches at least taller than I am to-day.

At this time in the history of electrical applications, I think we must all agree with the statement that the time was, when it was possible for men to be generalists in electrical science and electrotechnology, but that time has passed. That part of the science which relates to electro metallurgy alone is enough for one man. Telephony is enough for one man; telegraphy is enough for one man, even with but one company in the United States. Although the Western Union Telegraph Company does not now depend largely upon electrically educated engineers, the time is coming and is not far distant, when it and all other companies must employ educated men or must fall behind in the race. [Applause.]

As we heard last night, there is so much to be done and so few people at present to do it, that there is enough in the field for all the first-class men that we can get for at least the next ten years. The way our colleges and schools have been run in the past is, that they usually send out men with such exaggerated ideas of their importance that they want $5,000 a year for their electrical energy, and they haven't got very much of it at that. That leads me to ask what I regard as a pertinent inquiry:—Will it be possible for all these engineers who so far have been educated to this point in the schools, to find sufficiently lucrative employment? And would it not have been better, for many of them at least, to have been educated in pharmaceutical establishments and turned out as druggists? [Laughter.]

Professor Owens says that besides mathematics, chemistry and physics, which must be considered as proper subjects for the course of preparation, German and French must be acquired, "as so large a part of our best electrical literature is in these languages." Yes, I think there is no electro-scientific man who can afford to dispense with German and French. Some of our best periodicals are written in German and French to-day—and some of the others might as well be. The professor says:—"German and French should be acquired, as so large a part of our best electrical literature is in these languages." Then I find a phrase that is rather mystifying to me, namely:—"Sanscrit and psychology might be classed as electives affording variety." At first I thought that was written seriously. I had some conversation this afternoon with the distinguished authors of both of these papers, and in the course of the conversation, one of them, I do not remember which, remarked that Maxwell was a great author and that a great many things which were coming out as new today, were to be found in Maxwell, and it was further stated by one of the party that there were a great many jokes in Maxwell, if people could only find them. And that remark, with what I have heard to-night, convinces me that the gentleman who wrote this paper was having a quiet little joke to himself. I have not the slightest objection in the world to jokes, but would suggest that to these elective studies the gentleman may very properly add the study of Volapuk and Christian Science. [Laughter.]

Upon page 473, in the third line of the page, is something which recalls to my mind an incident that is related by Sir Edwin Arnold, when in Boston, of a visit to the Zoological Garden. He said that while he was visiting the Zoological Garden in London, once, he listened to the conversation of some of the people who were looking at the animals, and he heard a little girl ask her mamma "if she thought it would hurt the elephant if she offered him a chocolate drop." In the same guarded and respectful spirit, I desire to call attention to the third word in that line, on page 473, "capitalization," by which it appears that Professor Owens has not neglected that important part of the electrical engineer's education. Capitalization has, of course, been one of the most successful branches of the profession, and any electrical engineer of the latter part of the nineteenth century who does not know how to capitalize a large company upon a carbon brush, is hardly worthy of notice. [Laughter.]

Briefly referring now to the paper of Professor Jackson, I noticed something which struck me rather forcibly, and that is that the engineer to be successful must be a trained and careful thinker. I believe it is the lack of independent and careful thinking which keeps us—I was going to say as far behind as we are in the attainment of knowledge, but I would rather say—no further advanced than we are in the attainment of knowledge. When I was a very small child, certainly not more than ten years old it was my fortune to fall upon a book entitled "Learning to Think;" and what little success I may have achieved in life, I regard as being largely attributable to my perusal of that book, because it led me to think upon various subjects in many ways, and I think it taught me to think with some degree of vigor and independence; and I hope that the school and college training of our young men, whatever else it neglects, will not fail to teach them to think for themselves—to teach them at all times to put to themselves—no matter what they may be studying—the question represented by the little monosyllabic word "Why?"

It is impossible to put everything in a paper, or even in two papers, but I was a little surprised that there was not any mention of the literature which students of electricity and electrical engineering might profitably peruse. It is not an easy thing to direct a course of reading, because the literature of the subject is changing so rapidly that what is new to-day may be old to-morrow, and certainly what is now before us was not in existence as long as ten years ago. Ninety-nine one-hundredths, at least, of the books which have been written upon electricity, magnetism and electrical science might just as well never have been written, except for this one thing—that while they were new they served the purpose of keeping alive interest in the art and science and thus helped to give us those wonderful discoverers, Ampere, Arago, Oersted, Faraday, and many others too numerous to mention, all of whom were careful thinkers, great readers and intelligent experimenters. And I say to every one who has the care of those who are still studying. " Not only should you teach them to learn to think, but to learn to read, and, above all, to ask questions of nature by experimenting."

Dr. Emery:—Mr. Chairman, I had fully determined not to say anything this evening, until the remarks of my friend Lockwood caused me to change my mind. I think that each of the engineering societies should avoid the assumption of superiority which is evinced at times by some members of particular organizations, the architects, perhaps, being the greatest offenders. I belong to all the principal engineering societies, and have unfortunately heard such remarks in nearly every one of them. I do not think there is really any such feeling among the better class of members, but some are incautious and choose such language as they think will please some of the persons assembled. Evidently, therefore, the language should never be repeated in other circles. There are individuals who would reduce the members of all other special professions than their own to the plane of artisans, because solicited to use their manufactures or brains, while perhaps the very person who thinks himself superior is the most skillful in making himself " the servant of all " among rich families " that he may win the more." An assumption of superiority can only be sustained by an organization like the Institution of Civil Engineers. In England all the engineers of reputation, from all branches of engineering—civil, mechanical, electrical, sanitary and otherwise—must belong to the Institution to have a good standing. That arises from its age, from the high character of its work from the beginning, and especially the high standing of its members of all grades. Full membership is given only to those who have actually done practical work for a considerable period, not to those who have theorized upon it, or who are going to do it, but the latter can generally enter other classes of membership and be heard by all. I was very much pleased with the remark of Dr. Duncan, at our annual dinner, in which he urged that there should be no antagonism between theory and practice. One is a necessity of the other. It is only the ignorance of the one class of the value of the work of the other, that makes it possible for remarks to be made, such as are sometimes heard. We must have those with the brains to conceive, and those with the muscle to execute. Knowledge is power, but its foundation is the practical work of others. It is well to think often of the legend in which King Solomon is said to have assembled all the workmen, at the completion of the temple, and asked which was entitled to greatest honor, when the dirty blacksmith stepped forward and took his position on the right, and when the cutters of the massive but beautiful stonework disputed the propriety of his action, the blacksmith asked what they could have done if he had not made and mended their tools. On the other hand, we may say that those who study principles and develop theories to be applied by others, really aid in the march of civilization. They should, however, claim no superiority for this, as there would have been none to civilize if the brawny sons of toil had not perpetuated the race. In the stone age our ancestors were but animals, using the advantages which reason gave them to conquer other animals on land and ensnare those of the sea, but reason and necessity taught them to work with their hands in cultivating the ground; and in seeking tools, the earth was robbed of its various treasures; and by combined manual and mental labor, civilization was developed and we were brought from the stone age to that of electricity. Now, all must be considered workers and each has his place. He that has the most skill and the most knowledge which can be usefully applied, is strictly superior, but one whose information is entirely in one branch of engineering, may be excelled by one who knows as much of that branch and much also of other branches. Much knowledge or superior skill should, however in all cases bring due modesty. Again, with Dr. Duncan, I say there should be no antagonism between theory and practice; there should be none between the different branches of the engineering profession. Any assumption of superiority is unseemly, and all branches of the profession should co-operate in the general advance to a higher plane of civilization. At the same time it may be entirely proper to believe and state among ourselves that in the field of electrical engineering there promises to be a greater opportunity than in any other, for the utilization of all branches of engineering for the benefit of mankind. [Applause.]

Professor Roberts:—Mr. President, I wish to quote a remark made by Professor Coleman Sellers, whom I had the pleasure of hearing speak on this subject the other day. He said that he had been investigating the technical schools of Europe, when he was over there recently for the Niagara Falls company, and he decided that the technical schools of this country were in some respects better. If I recollect correctly, he was judging from results along the line of independent thinking which Mr. Lockwood spoke of.

I was going to ask our President to speak upon the subject which he has just touched upon. We also have Mr. Upton with us, and he certainly has had to do with a great many of the technical school graduates. I think the graduates should be advised not to go to the large shops, but to go to small first-class shops that do all-around work—go to some small constructing company or small repair or manufacturing company—and get their all-around experience there, and then they will be of more value to the larger companies. I know how it is with one or two of the larger companies, and I think it is the same with a great many others, that it is difficult to find any position where they can even earn anything for the company, until they have gone through some such course either in the works or out of it. They are shoved off to one side to do routine work until they have grit enough to go out and take hold of the dirty work and then come back. The gentleman by my side thinks that Mr. Lockwood left out one branch of engineering—sanitary engineering—which could be utilized in ventilating armatures.

The President:—I do not know but he left out another branch of the profession, that of entomologists, or bug hunters.

Mr. Lockwood:—I have no doubt, Mr. President, that I did omit some of the branches, because I did not undertake to go through all the sciences.

Mr. Upton:—Mr. President, Professor Owens asked me if I would not make a remark on his paper. I, of course can only speak from one point of view at present, and that is from the standpoint of the employer. In the company with which I am engaged, the question has often come up, "What use shall be made of the graduate?" And I have had some little experience with men coming to me and asking for advice. I have given them the advice as suggested by Prof. Roberts, to go into a small shop first—go into some repair shop, in a small town, where they can learn a great many parts of their work, so that they will get experience. The trouble with the large corporations is that the work is subdivided. They have men in the large factories who sit in darkness year after year, repeating one operation. I know one man who for nine years has been doing one operation in darkness. He earns his living there, it is true. It is sad to see a man laboring under those circumstances, with no possible opportunity of learning anything but the one operation that he is kept at, and we have got to look to the technical schools for men trained in different branches of the business. Take Schenectady, for example. Everything there to-day is divided so that each man performs one operation, and of course he becomes very skilful at it. I would say to the technical schools that they should bear in mind one thing, always to exercise and drill their students in the rudiments of the art. I do not think there is anything more useful to a man in ordinary electrical work than to know Ohm's law, when twisted around in every way it can be. I have known men, who came out of technical schools, who could not tell you anything about the multiple arc, nor could they answer two or three simple questions about Ohm's law. I think the students should be so trained in these things, exactly as children are trained in mental arithmetic, that they become part and parcel of their mental make up. I do not think anybody can deal with alternating currents to-day, unless he is well up in mathematics. I have given up the study of alternating currents, though I had an excellent mathematical training, because I haven't had time to follow it up, and I have had to drop it except upon the mere theory of alternating apparatus. It is not in my line. Of course my line is specialized, but I consider myself a sufficiently competent judge of literature to be certain that the technical schools must carry a man well up in mathematical and technical training to make him master of that part of the art.

Another thing which I would like to add, the progress that is now being made in electrical work is being made in the United States, and it is being made by men who are in this Institute of Electrical Engineers, and I think that any man that takes a broad view of progress will say that a meeting like this marks an important advance in electrical knowledge. There are many things brought out here, many things entrusted to your hands, and that are being given to us here, which are ahead of anything in the world. The foreign papers do not compare with ours. Progress in the alternating current is being made in this country.

I have been pleased with these papers, and especially with the purpose that has been evinced by them to put the students in a position to be well-grounded in their chosen profession. I think the students should be given a post-graduate course and should have special instructions to fit them for the requirements of the different sections of the country.

Mr. Perry:—Mr. President, with Mr. Upton's last remark I most fully agree, in regard to the education of the engineer whether he be an electrical engineer or other engineer, that our object should be to give him breadth. School education is to my mind a preparation for future education in practical life. The broader we can make that education the more useful have we been in the instruction of the to-be-engineers. It is not long since, that a man was not considered liberally educated unless he had spent a number of years in the study of classics, but the views of educators have changed very radically during my time. Now for a man who is to enter the engineering profession,—time is limited as we all know,—how can we in that limited time give him the broadest education possible? It seems to me if we could devote our time to but one study that there is none that could give us such a broad foundation for any scientific pursuit as that of chemistry; and in both these papers this evening the study of chemistry is a part of the course recommended to the young engineer. It is practically significant in the electrical profession just now, since in our profession we are turning our attention to molecular action. We have an explanation of molecular magnetism by Mr. Tesla, and of the molecules striking each other to give us light. We hope the time will come when we can produce light without heat, as the fire-fly does.

Then there is another branch of the electrical profession which is very important. I refer to metallurgy. A man without a thorough knowledge of chemistry cannot hope to do much in metallurgy. It seems to me that to make metallurgy even a subordinate study it is necessary to give chemistry as full time in the course as possible.

MR. BRADLEY:—Mr. President, the gentleman speaks of chemistry, but I will say that I should think general physics would cover the subject and cover it better, a good understanding of general physics. Chemistry then comes under that head, and a man must necessarily understand it.

Mr. Upton suggested an idea to me when he said that he had to drop out of the consideration of the alternating current. I am dealing with questions of alternating currents myself, and I am not educated in higher mathematics. I have an assistant who is pretty well educated. I think that I am able to form a pretty correct idea of the actions that take place without the use of any great amount of mathematics. I think I can get it in my imagination and I have done it oftentimes, because I could not do it mathematically, and if I had mathematics it would be in my way. That is the education that makes of it what the general public can use. We think of the wonderful things in connection with electricity, and we think of electricity as being a profession now. In a few years it will be within the knowledge of the general public.

My two boys, one is twelve and the other is eight, and there are some things that they know about it that I haven't thought of. They fooled with the lamps and broke the carbon by scuffing their feet over the floor, and got their bodies charged, and all such things as that they are trying. They know more about electricity now than I did at 25 years of age; they understand it better; they have got the theory more thoroughly in their heads than I had at that time. Now I think that the boys or young men growing up ought to get an education without taking the regular course. Of course the selective studies are a great advantage, they help a great deal. It is a step in the right direction, but it does not go far enough. Our boys ought to have a chance to get a galvanometer and work with it. I actually believe my boy in a year's time will handle a galvanometer, if he is allowed to. Of course there are lots of them that won't do this, unless they take naturally to that direction. It makes all the difference in the world whether a child naturally inclined that way. If he had a chance and takes a great interest in astronomy, for instance, and goes up and watches the professor in the observatory, he would gather a great many ideas. Give him a chance at the observatory. That is what we want. It is a bold idea, I know, but that is what we want. If there is danger of their breaking it, they can be watched the more carefully. I often think if I could take two or three boys and bring them up and attend to their education and pay no attention to anything else whatever, I could fix them in ten year's time so that they could earn so much more than I can, that it would pay in the long run to apply one's time and attention to it. [Laughter.]

PROFESSOR JACKSON:—Mr. President, Mr. Owens states that he does not care to reply. I have no reply to make, except to thank the various gentlemen who have taken part in the discussion. Their recognition of the advantages of an engineering education is very gratifying.

I wish to add a remark to Lieutenant Sprague's definition of an electrical engineer, that he should be one-tenth mechanical, one-tenth electrical and the rest he didn't know what. Dr. Nichols says the rest should be man. I want to add to that, what is left should be good, manly common sense. [Applause.]

I want to especially thank Mr. Lockwood for his remarks, which were evidently very pleasing to us all, and I want to assure Mr. Lockwood that we consider one of the necessities of an electrical engineering course is to teach the students how to read technical journals without wasting too much time, and also how to make references to current books of the day which it is impossible to thoroughly read but which must be used more or less.

I also wish to thank Mr. Upton for his recognition of the necessity of practical instruction in engineering education. I feel that the practical is demanded at the present day. The theoretical side has been developed. I do not think it is necessary for an electrical engineer to be able to read Maxwell through all its parts. I would scarcely claim that distinction myself. In fact, there are very few pages in Maxwell that would be enjoyed as reading matter.

As to the question of shop apprentices, I think it is well, under certain circumstances, for a student to enter a properly arranged apprentice course in a first-class commercial shop. The size of the shop may be of little importance, but if it is not *first-class* the student will probably be wasting his time.

Finally, I want to say a word about our universities. If a man comes to a university with the view of learning something special, and doing good work, and making use of what he can learn, he will receive every attention and fully as much assistance as any man who takes the regular prescribed course.

The following paper on "A New Rheostat" was then read by the author.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 7th, 1892, President Sprague in the Chair.

A NEW RHEOSTAT.

BY CHARLES E. CARPENTER.

It is a well-known fact that an insulated wire suspended in the air, will carry a given amount of current at a lower temperature than the same wire without this covering of insulation under the same conditions. This is obviously due to the fact that while the cross-section of both conductors is the same, the one which is insulated has a much larger radiating surface than the bare wire, and therefore dissipates the heat conducted through the insulation more rapidly. Suppose, for convenience of illustration, we conceive a conductor to be first covered with some insulating substance, and then enclosed within a metallic tube so that its insulated surface shall fit perfectly within and against the inside surface of the tube, as shown in section in Fig. 1. We thus have a comparatively small conductor with a largely increased radiating surface, and the insulation between the two metallic bodies does not materially obstruct the flux of heat from the conductor to the outside metallic shell.

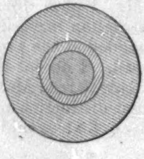
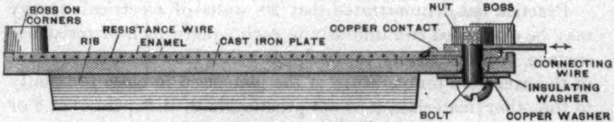


FIG. 1.

One of the early experiments of the writer in electric heating —in fact, his first successful apparatus—consisted of two iron plates, between which was interposed an iron resistance wire of the "reflex" or "zig zag" type insulated from both plates by means of thin sheets of mica or asbestos paper, with screws passing through one plate and engaging the other. By this means the plates could be pressed into close relation to the resistance wire. By connecting this apparatus to a constant potential system, the current at first would be comparatively large, but would gradually decrease as the temperature of the plates increased until a point was reached where the heat was dissipated as fast as generated. By tightening or loosening the screws, the current could then be either increased or decreased. This increase of the current by increasing the mechanical pressure is due to a reduction of temperature effected by the more intimate heat conducting relations between the resistance and plate, facilitating the flux of heat.



Section of Standard Rheostat Plate

Fig. 2.

The better the conduction for heat is made, the less will be the difference of temperature between the wire and the outside radiating surface plate, and it follows that the larger the surface of the radiating plates which are closely impressed upon the conductor, the larger will be the current-carrying capacity of the latter.

These experiments are cited as an illustration leading to a thorough appreciation of the extent to which the current capacity of a resistance wire may be carried, as it is upon this principle, but with modifications of construction, that the new and improved rheostat herein described is based, that is, a resistance wire of small cross-section is used, but a large radiating capacity is secured by placing the wire in such relations to a metal plate as to practically increase its radiating surface to a very great extent.

The results of the experiments described above suggested a modified form of construction, one in which the resistance would be entirely covered by a medium which would protect the wire from oxidation and yet act as a good insulator of electricity and a good conductor of heat, and which will permanently attach the wire to, but insulate it from, a radiating plate. A special enamel was finally found by the writer to be suitable for this purpose. A section of this construction is shown in Fig. 2.

The injurious effect of linear expansion of the resistance wire, due to heat, is avoided in this method by the "reflexed" or "zigzag" form in which the wire is bent, as shown in Fig. 3, and it has been demonstrated by experience that this linear expansion is thereby so distributed as not to injure or crack off the enamel in practice.

On this principle, since we are able to greatly reduce the cross-section of the conductor, it follows that its length is also proportionately shortened for any required resistance, and no consideration of mechanical strength of the wire enters into this construction, since it is so perfectly confined and supported on all sides.

Practice has demonstrated that 20 watts of electrical energy may be dissipated continuously to each square inch of resistance surface, without injury to the apparatus. However, on account of the excessive temperature of the plate used in close proximity to the other materials, it is not practicable to dissipate over 8 or 10 watts to the square inch in continuous service.

By placing one surface of the plate in contact with water, so that its heat is dissipated in this manner, we are able to continuously dissipate about 25 watts to the square inch. From this it will be seen that by means of water we are able to increase the capacity of a certain plate to about three times that of the same plate dissipating its heat directly into the atmosphere, or we may decrease the size of the plate to one-third for the same capacity where water is used.

In many applications, especially in railway work, apparatus is often subjected to a load far beyond its rated capacity. Rheostat plates for such service are made with narrow ribs, between which the resistance wire and enamel are placed. It is found that by means of these ribs, the safe carrying capacity of a wire is greatly increased, while the ribs serve also to protect the insulation and wires from mechanical injury.

In adapting this construction to meet practical requirements in its several applications, it is necessary to alter only in detail the general construction and distribution before explained, and shown in Fig. 3, which shows the usual plan of distributing the wires in a simple rheostat plate. Connection to the resistance wire is made by means of a sheet copper contact strip, one portion of which is folded back upon itself, and after inserting a portion of the wire this sheet copper is crushed about it. The joint so formed can not oxidize, since it is completely sealed by the enamel fused about it.

To divide the resistance so formed into steps, it is only necessary to place contacts for connection to the resistance wire at convenient points where the loops approach the edge of the plate, as at *a*, and so on, in Fig. 3. This particular way of locating the connecting points on one side of the plate is especially convenient in using these plates with small motors, where only few steps or divisions of the resistance are required. A modification of this arrangement is to have the connecting points alternate from one side of the plate to the other, as at *b*, *a*, etc., Fig. 3. A distribution best adapted for stage or field-controlling rheostat, or one in which a great number of steps are required, is secured by arranging the resistance in a similar manner around the entire outer portion of a disk with contact connections near the circumference. Another arrangement is required in a plate of low resistance and high ampere capacity. In this case, several coils are placed in parallel and connected to a common contact piece.

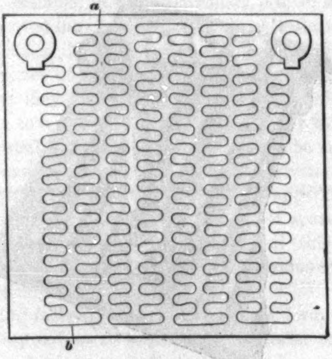


FIG. 3.

The plates, with their coils and contacts distributed as above described, are well suited for use in connection with rheostat switches. They are provided with ribs or blades for extending their radiating surface, and lugs at the corners for convenience in attaching them to the back of the switch. Each contact of the resistance plate is connected by a very short wire or strip, to the corresponding contact piece on the rheostat switch.

A very simple and complete rheostat is made by placing the switch contacts and lever, properly insulated directly upon the radiating plate, on the surface opposite the resistance face. With this arrangement as shown in Fig. 4, a rheostat is obtained which is no larger than the switch parts alone of existing rheostats.

To make a single plate large enough to dissipate the energy wasted in the rheostat in starting a large motor, would make it necessary to have a surface so large as to be objectionable. To avoid this difficulty, a number of plates of, say, 10 inches square, are placed in parallel with each other, but separated by sufficient space to admit of proper ventilation, and connection made from the switch contacts to the contacts upon the various plates as required. With this arrangement, a rheostat of very large watt capacity may be placed within a small cubic space.

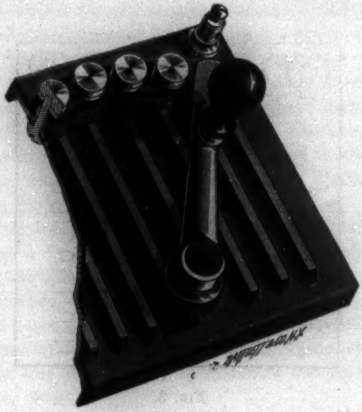


FIG. 4.

This form of rheostat, by reason of its compactness, is well adapted for use with arc lamps on constant potential circuits. In this application, the resistance plate may be attached directly to the outside of the casing, leaving just sufficient space between the casing and the plate for ventilation. Where the casing is cylindrical, the plate may be disk-shaped and placed on top of the cylindrical casing, with holes in the plate for the hooks and chimney to pass through. A valuable feature of this rheostat in this application is that it will endure exposure to the weather while in use. As an example of this, the writer knows of a case where one of these plates has now been in use on a pair of lamps for over seven months, where the plate is in a horizontal position, with the enameled side up and water dripping upon it continually.

In some places the heating effects of rheostats should be avoided. For example, in stage controllers, 5 to 10 kilowatts continuously converted into heat is extremely objectionable in warm weather. This may be remedied by conveying the heat away through the agency of running water. To accomplish this the resistance plate may be hollow, with inlet and outlet pipes connected directly to a source of water supply. By this means and with proper circulation, the plates will always be comparatively cold, and no sensible amount of heat will be given off to the surrounding atmosphere, while the size of the apparatus is so reduced as to occupy no considerable amount of space.

It is evident that numerous modifications of this general design may easily be made to adapt the apparatus to the requirements of practice.

The following facts are true regarding this form of rheostat:

1st. It is fireproof, as no combustible material enters into its construction, and it can never be submitted in normal use to a temperature so high as that towhich it is subjected in the process of manufacture.

2d. It is durable, since the resistance wires cannot deteriorate from oxidation, electrolysis, etc.

3d. It is compact, as it requires less than 5 per cent. of the cubic space required by the ordinary coiled wire rheostat of the same capacity.

4th. It is simple, the three necessary elements, resistance wire, insulation and radiating surface plate, being fused into one integral mass.

5th. It is cheap, for a certain resistance and carrying capacity can be obtained by the use of a wire whose cross-section and length will both be but one-tenth of that required for a wire used in the air. That is, the total weight of wire, and hence the cost of the same, will be but one per cent. of that usually required.

6th. It is unaffected by heat, cold, acids, alkalies, oils and other chemicals, and for this reason is especially adapted for use in mines, breweries, marine vessels, etc., where chemical action rapidly destroys ordinary insulators. It is even practicable to operate these plates submerged in flowing water, in case it is desirable to carry off the heat in this way.

Another and important application of this method of converting electric energy into heat in a compact form is for the operation of various heating devices.

The requirements of a commercially successful electric heating device are:

1st. The concentration upon the working surface of the apparatus, of practically all of the heat produced in the resistance, thus preventing as far as possible the dissipation of heat by radiation or conduction to other bodies or parts than those it is required to heat.

2d. The ability to rapidly supply a large amount of heat energy from the resistance to the working surface, through a medium having a high conductivity for heat, thus keeping the working surface at the proper temperature even when a large amount of heat is rapidly drawn from it.

3d. The protection of the resistance wire from deterioration by chemical and electrical action, so that its life shall be indefinitely prolonged.

All the above requirements are fulfilled in this device.

DISCUSSION.

THE PRESIDENT:—Gentlemen, you have heard Mr. Carpenter's interesting paper. I am sure those who have had anything to do with rheostats will appreciate the very practical and simple form in which this rheostat is presented. I see many applications of it which are of interest, and may be made practical. If there is any discussion on this paper, we shall be glad to hear it.

MR. SPERRY:—Mr. President, I would like to call out some points with reference to it. I would like to know if the rheostat has been designed for heavy currents, and if so what is the heaviest that has been used?

MR. CARPENTER:—We are manufacturing resistance plates of low resistance and high ampere capacity. In these low resistance plates, several wires are in parallel or multiple, and have a capacity depending upon the kind and size of the wire, from 25 to 200 amperes or more. A 50 kilowatt 500 volt motor-starter is made up of a number of these low resistance plates connected in series. Each plate has 4 wires in parallel, and each plate forms a step or division of the rheostat. The plates for this size of motor are 18 inches long and 3 inches wide, and weigh about 4 pounds. Twelve or 14 of each are used for this size of machine.

These series plates have the coils or wires enameled between ribs for increasing the adhesion of the enamel to the plate. It has been found by experience that 50 per cent. more current may be passed through a wire when placed between the ribs in this manner, without breaking the enamel, than where the wire is simply enameled on a plain surface. It is also a fact that these plates used in this way, will endure splashing of water about them without breaking the enamel, when the temperature is as high as will be reached in normal use.

Mr. Upton:—Mr. President, I would like to add one word, that I believe that at the Exposition at the Crystal Palace in London the invention of Mr. Carpenter's is one of the leading features That is, it is most talked about and most noticed by far. There is a large exhibition there, and it ~~is meeting~~ with a great deal of success.

[Adjourned to June 8th.]

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 8, 1892, Manager Webb in the Chair.

PRACTICAL NOTES ON THE ELECTROLYTIC REFINING OF COPPER.

BY LIEUTENANT F. B. BADT.

There are a good many books extant on the art of electrolytic separation of metals and a good deal has been, and is being written in scientific journals on the same subject, but, strange to say, most of it has been published in the foreign press and very little can be found in American publications.

From this it might appear that but little interest is taken in this subject and that copper refining is being done on a very small scale in this country, while, as a matter of fact, quite the reverse holds true. We cannot explain the dearth of information on this subject any better than by quoting from Mr. Gore's latest book:

> "Much of the information heretofore published on the subject lies scattered about in short articles and fragmentary accounts, in periodicals and books; and statements more or less inaccurate have been made, owing to the writers not having access to the manufactories, the processes in which they have attempted to describe. M. Kiliani and others have observed the privacy with which electrolytic refining works are conducted; he says that 'its effect has been to some extent to make people believe that the operations carried on are based upon discoveries known only to a few and are surrounded by difficulties of a very special and complicated nature.' He writes with 'the object of dissipating this wrong impression, and of convincing all those interested in the subject that whatever is done inside these works can be done by anybody who gives a little attention and study to the subject, the only secrets being slight and immaterial details of practice.' The probable explanation of this is, that each electro-refiner, in consequence of being imperfectly acquainted with the literature of the subject, and of how his fellow refiners were working, has been obliged to ascertain by means of experiments in his own works the practical details of the process; and has thus independently arrived at the same general plan of operating as other refiners, whilst considering his own method a secret."

The foreign books above referred to naturally contain descriptions of plants and processes in vogue in Europe, and very little can be learned concerning the art of electrolytic copper refining in the United States.

It is the object of this paper to give a few practical data in relation to American practice, and to give a short description of the processes in vogue in this country. While the writer has erected some of the largest copper refineries in the United States and examined a few of the other plants, it is for the very reason that Mr. Gore gives, that the data which the writer submits in the following are not claimed to be either complete or to be absolutely correct. The writer has endeavored to tabulate the electrolytic copper refineries in the United States.

This, to his knowledge, is the first attempt ever made, and he hopes that considering the circumstances, his endeavors in this direction will not be misconstrued, but be properly appreciated.

In the following we give a list of electrolytic copper refineries in the United States. It shows us that there are at present 36 dynamos of a total capacity of 1,814 kilowatts used for the electrolytic refining of copper, and that the total output of these refineries (when all are completed) will amount to 3,650 tons per month, or 43,800 tons per year. A capital of over $1,000,000 is invested in these refineries.

The Census report, dated Washington, D. C., July 15, 1891, shows the United States to be the largest producer of copper in the world, its product for the year 1889 being 226,055,962 pounds, or 113,028 (short) tons. The total expenditures involved in this production were $12,062,180, of which there was paid in wages, $6,096,025; in salaries, $120,896; to contractors, $334,443; for materials and supplies, $4,067,970; for taxes, rent, etc., $1,442,846, the total capital invested being $62,623,228, and the total number of employes, exclusive of office force, 8,721. The same Census Report gives some statistics on "copper refining," but unfortunately fails to state how much copper was refined by the electrolytic process.

It is interesting to note that only a part of the furnace material produced in the United States, a small quantity of ore, and nearly all of the mineral from the Lake stamp mills are refined in works, the majority of which are controlled by firms and corporations not directly connected with the mines. In some works copper refining is incidental to the working of other base and precious metals, and in others it is part of a general chemical business One concern failed to report. The returns cover establishments which produced 159,693,252 pounds (79,847 tons) of refined copper, valued at $19,686,561,86. Hence if our list is approximately correct only about 25,000 tons out of 79,847 tons of refined copper were treated electrolytically.

We might construe the following paragraph in the Census Report as referring partly to electrolytic refineries:

> "It is interesting to segregate one group of refiners, which treats exclusively high grade, pure material, like Lake mineral, Arizona bars and Montana blister copper. Works which produced 105,400,664 pounds of refined copper incurred the following total expenses:

EXPENSES IN COPPER WORKS TREATING HIGH-GRADE MATERIAL.

Wages	$326,687
Salaries	42,056
Paid to contractors	4,735
Supplies and material	305,679
Rent, interest, etc	40,462
Total	$719,619
or 0.68 cent per pound.	
Total refined copper, pounds	105,400,664

These figures will demonstrate conclusively that the subject is an important one. It is not intended to tire you with the details of the ordinary process for the electrolytic refining of copper, as this has been described in many books, and the theory and practice are well known. In the above table, however, I mention, in the fourth column, the names of certain processes in vogue in the United States, and it might be well to show the main features of each. The crude material used in the anodes is black copper, or blister copper, containing from 97 per cent. to 98 per cent. of copper. The electrolyte is a solution of sulphate of copper.

In some processes refined copper is a by-product only, especially when it appears in a concentrated form in the mattes of lead smelters and refiners. In the process which we call the "multiple process," anodes and cathodes in each vat are arranged in multiple, in about the same manner as the plates in a storage cell. (See Fig. 1.) In this process there is usually one more cathode than there are anodes. The electrodes are suspended in a vertical position in wooden vats, usually lined with lead. The vats themselves may be arranged either in series, multiple, or in multiple series. For simplicity's sake and for numerous other reasons well known to the electrical engineer, an arrangement of vats in single series is most advantageous.

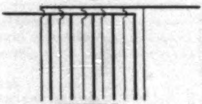
LIST OF ELECTROLYTIC COPPER REFINERIES IN THE UNITED STATES.

Name of Company.	Generators	Number of Vats.	Process (Arrangement of Vats).	Output in Electrolytic Copper per Month, Tons.	Remarks.
1. Anaconda Mining Company, Anaconda, Mont.	5 Edison—60 volts, 1,100 amperes.	320	Partly multiple, partly Stalmann.	350	Plant being extended for ultimate capacity of 900 tons per month.
2. American Nickel Works (Jos. Wharton), Camden, N. J.	1 Excelsior—6 volts, 1,000 amperes.	48 in series.	Multiple.	30*	* By-product.
3. Balbach Smelting and Refining Company, Newark, N. J.	7 Excelsior—15 volts. 2,000 amperes. 1 Excelsior—30 volts, 3,000 amperes.	7 series of 48 1 series of 96	Multiple (18 anodes, 18 cathodes).	650*	* They refine the product of the Orford Copper Company, whose smelters are in New Jersey, and who are general purchasers of copper ores, matte and bullion.
4. Baltimore Copper, Smelting and Rolling Company, Baltimore, Md.	6 Edison*—150 volts, 400 amperes.	....	Hayden.	800	* Generators have sectional fields, which can be plugged for different voltages.
5. Baltimore Refining Company, Baltimore, Md.	2 Edison—80 volts, 700 amperes.	....	Hayden.	300	
6. Boston & Montana Consolidated Copper and Silver Mining Company, Great Falls, Mont	3 Thomson-Houston—multipolar separately excited, 165 volts, 1,000 amperes.	288	Multiple (19 anodes, 19 cathodes)	550	Plant in construction. Dynamo capacity in excess of present requirements.
7. Bridgeport Copper Company, Bridgeport, Conn.	1 Thomson-Houston, 1 Mather, 1 Edison—150 volts, 460 amperes.	3 series of 10	Hayden (100 electrodes in each vat).	400*	* They refine the entire product (black copper) of the Parrott Silver and Copper Company, Butte, Mont.
8. Chicago Copper Refining Company, Blue Island, Ill.	2 Edison—80 volts, 800 amperes.	165	Multiple.	150	
9. Electrolytic Copper Company, Ansonia, Conn.	3 Mather—100 volts, 300 amperes.	75	Smith.	100	
10. Lewisohn Bros.,† Pawtucket, R. I.	1 Excelsior—15 volts, 2,200 amperes.	60 in series.	Multiple (19 anodes, 19 cathodes).	110	
11. Omaha & Grant Smelting Works, Omaha, Neb	1 Excelsior—6 volts, 1,000 amperes.	48 in series.	Multiple.	30	
12. Pennsylvania Salt Manufacturing Company, Philadelphia, Pa.	....	.. ..	Smith.	30*	By-product.
13. St. Louis Smelting and Refining Company, Cheltenham, St. Louis, Mo.	1 Excelsior—16 volts, 2,400 amperes.	48 in series.	Multiple.	60*	* This plant is operated in connection with an electrolytic silver refinery, using the Moebius process.
14. Washburn, Moen & Company, Worcester, Mass.	....	....	....	90	Plant burnt, being erected.

† Agents for the following mining companies: Boston & Montana Consolidated Copper and Silver Mining Company, Montana; Butte & Boston Mining Company, Montana; Arizona Copper Company, Arizona; Huron Copper Mining Company, Lake Superior, Mich.; Tamarack Mining Company, Lake Superior, Mich.; Osceola Mining Company, Lake Superior, Mich.; Kearsarge Mining Company, Lake Superior, Mich.; Santa Fe Copper Company, New Mexico; Peninsula Copper Mining Company, Lake Superior, Mich.

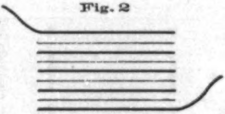
Another process is the "Smith process" (United States patent No. 393,526, Nov. 27, 1888. In this process there are no cathodes of pure copper. The plates of black copper (anodes) are not suspended vertically in the solution, but are placed horizontally. As it would not be practical in many places to cast the plates large enough to give a sufficient amount of surface, several plates are placed side by side so as to form one large plate. These layers are then placed one above the other, at a distance of about 1½ inches, the layers being supported by strips of wood running lengthwise of the vat. The top plate is connected as an anode to the generator, and the other pole of the dynamo with the bottom plate. The spaces between the plates are filled with the solution of sulphate of copper, which forms the only connection between the plates. The copper is dissolved from the under surface of each layer and deposited on the upper surface of the next layer below. The insoluble foreign matters in the copper, including gold and silver, fall upon screens of cotton cloth which are stretched between each two layers. The process is kept up until all the black copper is dissolved and nothing remains but plates of electrolytic copper.

Fig. 1



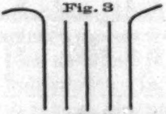
MULTIPLE

Fig. 2



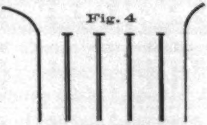
SMITH

Fig. 3



HAYDEN

Fig. 4



STALMANN

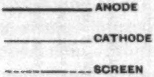


Fig. 2 shows the relative position of plates in one vat. The inventor claims several important advantages over the common vertical multiple arrangement. He claims that the silver or other foreign insoluble matter is removed, that the space required for a given amount of depositing and the amount of solution required are relatively small, that the connection between the plates in one tank is made by the solution only, and that the copper is easily handled and prepared for the vats. The following is his patent claim:

"A vertical series of horizontal electro-depositing cells formed by horizontal plates of the metal to be deposited, separated and supported by insulating supports, and immersed in an electrolytic solution, with screens interposed between the successive plates, substantially as described, whereby insoluble matters are arrested as they are liberated from the lower surface of the upper plate in each cell by the combined action of the said solution and the electric current."

Smith arranges all vats in series.

Another process mentioned in the list is the "Hayden (Fig. 3, United States Patent No. 465,525, Dec. 22, 1891). The Hayden process differs materially from that of the Smith only in that he places his plates in vertical position instead of placing them horizontally, and that he does not use any screens. He uses removable grooved vertical wooden strips to hold the plates in proper position. He also states that it might be advantageous in many cases to cast the plates in small sizes and arrange them one above the other.

Mr. Hayden's patent claims are as follows:

1. In an electrolytic bath having a number of plates unconnected electrolytically, excepting through a solution in the bath, and having narrow partitions extending from opposite sides of the bath adapted to hold the plates in a vertical position and out of contact with each other, stops wholly between the partitions supporting the plates above the bottom of the bath, substantially as specified.

2. In an electrolytic bath having a number of plaets unconnected electrolytically, excepting through a solution in the bath, and having independent grooved side pieces in the bath, the side walls of the grooves forming narrow partitions engaging the lateral edges of the plates to be treated and holding said plates out of contact with each other, stops at the lower ends of said grooves, supporting the plates above the bottom of the bath, the said stops not projecting beyond the faces of the partitions, substantially as specified.

3. In an electrolytic bath having a number of metal plates unconnected electrolytically, excepting through a solution in the bath, a series of grooved strips removable from the tank, driven tightly on to the side edges of the plates and constructed so as to be capable of sustaining the plates within the bath above its bottom, substantially as specified.

Hayden also arranges his vats in series.

Another inventor, Stalmann (United States Patent No. 467,-

350, Jan. 19, 1892, and No. 467,484, Jan. 19, 1892), practically uses the same process as Hayden, only instead of using single plates, he uses, with the exception of the initial anode and the terminal cathode, pairs of plates of crude and refined material, but bringing the two plates together by connecting them by one or more bolts or rivets, so that they become practically a pair of anode and cathode plates with no electrolytic solution between them (Fig. 4).

The claim in the first patent is as follows:

"An electrode consisting of a plate of refined copper material and a plate of crude material with insulating material interposed between said plates and metallic connection between said plates,"

The claims of the second patent are as follows:

1. The method of separating from copper and like metals foreign matter which may be incorporated with them, which consists of arranging in an electrolytic bath an initial anode connected to the positive pole of the source of current supply, and a terminal cathode connected to the negative pole, securing together by conductors, intermediate anodes and cathodes in independent pairs, each pair consisting of an anode of crude material and a cathode of refined material, interposing said independent pairs between the initial anode and terminal cathode of the bath, and passing an electrolyzing current from the initial anode to the terminal cathode through the bath and paired plates, substantially as described.

2. An electrolytic bath comprising a containing vessel, an electrolyte, an initial anode, a terminal cathode, and intermediate pairs consisting each of an anode of crude material and an independent cathode of refined material removably connected thereto and in metallic conductive connection therewith, the several pairs being independent of each other, substantially as described.

3. An electrolytic bath, comprising a containing vessel, an electrolyte, an initial anode, a terminal cathode, and intermediate pairs consisting each of an anode of crude material and an independent cathode of refined material removably connected thereto and in metallic conductive connection therewith, the several pairs being separated on their sides and bottoms from the containing vessel, substantially as described.

3. An electrode, consisting of a plate of refined material and an independent plate of crude material removably connected thereto and in metallic conductive connection therewith, substantially as described.

Stalmann's patent drawings show the vats arranged in either series or multiple. The inventors of the series processes, like the Smith, Hayden, Stalmann and others, claim for their processes very much cheaper first cost of plant, very much less copper carried as idle capital in the vats, much less floor space and higher efficiency in operation, that is, a larger recovery per horse power hour. Whether or not these claims can be demonstrated in practice is yet an open question. One pound of copper will be deposited theoretically in one hour, by a little less than 386 amperes out of a solution of sulphate of copper. If we, therefore, pass 386 amperes through one vat we will deposit one pound every hour, and if we pass the same current through a series of 100 vats, we will deposit 100 pounds per hour. The amount of power expended in each vat, however, is $C^2 \times R$, but as it is always possible to reduce the resistance of a vat by increasing the surface of the plates, we can greatly reduce the amount of power required for the same output in copper per horse power hour. We may also reduce the amperage for a given production by joining more vats in series, or we may combine both methods.

It is therefore always possible to largely reduce the expenditure of electrical energy, and consequently that of the motive power. As a matter of fact, theoretically, any amount of copper may be refined with any given horse power, but as soon as we attempt to largely increase the production of copper with a given power, we find it necessary to largely increase the size and the number of vats and the quantity of copper under treatment. These conditions have been thoroughly examined, and the laws governing them have been explained in the several hand-books extant.

As a resumé it may be stated that if power is cheap we may make the internal resistance of the vats greater, employ small vats and less copper under treatment, requiring less floor space, thus decreasing the first cost of the establishment materially. If, on the other hand, power is expensive, we must make the internal resistance of the vats as small as possible, have a large amount of copper under treatment, requiring a larger floor space and materially increase the first cost of the plant. It can easily be seen that in each special case these conditions must be carefully determined in accordance with principles very similar to those laid down in Sir William Thomson's law.

From these statements, however, we may infer that the claims of the promoters of the so-called series processes are not well founded; because no matter how the plates in the vats may be arranged, we can always arrange the plant for conditions of minimum first cost or minimum operating expenses. It is true that both Smith and Hayden do away with cathodes of refined copper, and thus save quite an item for copper under treatment. On the other hand, there is the disadvantage that particles of the original anodes will stick to the final cathodes of refined copper, and that it will be found difficult to separate them.

As a matter of fact, it is claimed by some assayers and selling agents that electrolytic copper obtained by these two processes is not so pure as copper obtained by a process employing anodes and cathodes arranged in multiple. Mr. Stalmann in his arrangement obviates the latter disadvantage by attaching removable plates of refined material to his anodes.

Another important item in order to obtain a pure deposit is the density of the current. American copper refiners usually adopt 10 amperes per square foot of active cathode surface as a maximum. I am told, however, that some of the refineries operated on the "series" processes go even beyond 15 amperes per square foot, which may account for certain impurities found in the electrolytic copper, and for a greater efficiency of the plant; that is, a larger output of copper for a given number of vats and amount of copper under treatment.

High voltage in an electrolytic plant of course means considerable saving or less drop in conductors, less trouble from poor contacts, etc., and it is probable that for these reasons the so-called series processes show a greater efficiency per given horse power than the processes employing electrodes in multiple. For refineries using this latter process low tension machines and comparatively few vats in series are employed. By adopting as high voltages in the multiple process as in the series process, of course, the same advantages may be obtained.

One advantage of the series process is the fact that each electrode will get its proper amperage per square foot as all the plates are in series, and are of uniform size. In the parallel process some trouble is always experienced in properly subdividing the current in each vat so that each plate will get its share, or, in other words, that the current density will be the same for each square foot of active cathode surface. By proper precautions, however, trouble in this direction may be easily avoided.

Another important item for the obtaining of pure deposits is a good circulation of the electrolyte. This insures a uniform specific resistance in the solution between the parts of the plates. Inasmuch as a solution of sulphate of copper is of greater specific gravity than the acid, which is liberated by the decomposition of the copper upon the surface of the cathode, such acid rises to the surface of the liquid while the solution of sulphate of copper settles toward the bottom. Where plates are arranged vertically, therefore, there is an ascending current before the surface of the cathode, and a descending current before the surface of the anode. If there is not a sufficient circulation of the electrolyte, the free acid attacks the upper portion of the plates more rapidly than the lower portion, and the deposition goes on most rapidly on the lower portion of the cathode.

In American practice, the electrolyte is sometimes kept in circulation by arranging the tanks on the cascade plan, and having each higher vat siphon through a pipe whose orifice is near the bottom of the next lower vat. A better plan now usually adopted is to have a trough run through the whole refinery, and have each vat fed independently by means of a rubber hose. The electrolyte is raised from the collecting tank into the troughs by means of lead pumps or steam injectors. Both, however, are objectionable. The pumps are always getting out of order. and the steam injector means waste of steam, besides having the disadvantage of putting too much moisture into the electrolyte, which again necessitates the adding of free acid continually. A better plan is to have two collecting tanks, which are to be used alternately. Just as soon as one is full it is made air-tight, and a small air compressor driven by an electric motor compresses the air above the solution and drives the solution under a pressure of, say, 25 pounds through a pipe up into the troughs. Smith claims that in his arrangement no artificial circulation of the electrolyte is required. He claims that the free acid liberated over the whole horizontal surface of the cathode in each cell, rises directly through the solution and the screen to the surface of the anode above, and acts uniformly over its whole surface. As the deposition also takes place uniformly over the whole surface of the cathode, uniform electrical and chemical conditions are secured over the whole surface of each plate and uniform liquid resistance between the parts of the plates and the action of the cells is entirely automatic. Smith also claims that less heat is wasted by this arrangement than in installations which require the circulation of the liquid.

We might mention another incidental advantage of the multiple process over the series processes. Some of the patentees claim one-fourth of a cent per pound of refined copper as a royalty. In one instance one plant gives the patentee, under this arrangement, an income of over $100 per day. Taking this in combination with the very doubtful advantages claimed for the series process, we need not wonder that there exists prejudice against erecting electrolytic refineries employing these patented operations.

As to the value of the patents from a legal point of view, I do not care to express an opinion. Some legal authorities state that anybody may arrange the electrodes in series within a vat without infringing, if he omits some of the minor details of the combination patents whose claims have already been quoted.

The power installations of some of the electrolytic refineries which the author had an opportunity to visit, reminded him very much of electric light stations put up about ten years ago. The old motto, "Cleanliness is next to godliness." he found missing, and the plants usually excelled by a total absence of voltmeters or ammeters or other electrical testing instruments showing the conditions under which the plant was operating. Such statements, therefore, as "working current of a thousand amperes," or "density of current of 10 amperes per square foot," which we get from electrolytic refiners, must be taken with several grains of allowance.

I could enumerate a long list of defects that exist to-day in almost all of the electrolytic refineries, and which all may be traced to one cause, and that is the imperfect knowledge of the refiners, of matters electrical. Large amounts of money may be saved in the erection and operation of such plants by considering all the conditions carefully, before making the final plans. One case came to my knowledge where the lead lining was put in the tanks, and plumbers were commissioned to wipe the seams in the ordinary way instead of having the edges of the lead melted together by means of the hydrogen blowpipe. The latter plan should always be followed, as wiped joints will be destroyed by the electrolytic action of the solution in a very short time.

This is not by any means a small matter, as repairing of the lead lining involves large expenses and great loss of time. In one installation, the conductors leading from the dynamos to the tanks are undergound, but run in such a careless manner that they are permanently grounded, often get short circuited, and in a few instances the wood conduit was set on fire. As a matter of fact, all the experiences which we have gone through for the last ten years in constructing electric light and power plants have been of no benefit it seems to most electrolytic refiners. Even in the few installations where a voltmeter is used to indicate the proper working condition of the vats, this voltmeter is carried by two men from vat to vat, and the voltage of each vat is taken in this cumbersome manner.

In a plant just being erected, the writer proposes the erection of a potential board in the office of the superintendent. Both terminals of each vat are to be connected with two contact points of the potential board, and a voltmeter is to be so arranged that by simply turning a switch handle the potential of each vat can be taken almost instantaneously. As a matter of fact, the working condition of each vat may be thus ascertained in a few minutes, while it would take hours for two men to do the same by carrying the voltmeter from vat to vat. One of the occurrences which has to be guarded against is the short-circuiting of an anode and a cathode which, in the multiple plan, means the short-circuiting of a whole vat. Such an occurrence should be detected immediately, as otherwise the contents of the vat may be spoiled. The author proposed, for a plant just being erected, an electric tell-tale in connection with the potential board. This tell-tale might be constructed somewhat after the fashion of a hotel annunciator. Each magnet might be permanently in circuit with its corresponding vat and the tension of the armature spring so adjusted that at the proper potential the armature would be held up, but just as soon as the vat is short-circuited no current or but very little current will flow through the magnet, and the armature will drop, ringing an alarm bell and indicating the number of the vat in which the trouble occurred.

All such appliances are labor-saving devices, and their installation would pay well in a plant where many thousand dollars worth of copper and solution is under treatment and liable to be spoiled. In a number of refineries there are T-rails suspended over the vats in such a manner that a pulley carrying a block and tackle can be used for lifting the plates out of the vats and moving them in the gangways for either washing off the slime or for substituting new anodes for the old ones. Some of the electrolytic refineries in the United States buy copper matte of from 45 to 54 per cent. of copper from the mining companies. In these cases they own their own smelters, resmelt the copper matte, producing black copper of from 97 to 98 per cent. of copper. This black copper is then cast into anodes and subjected to the electrolytic process. Other refineries buy the black copper direct from the smelters owned by the mining companies. For instance, the

Bridgeport Copper Company, of Bridgeport, Conn., buys the entire product of the Parrott Silver and Copper Company, of Butte, Mont., as black copper.

There is another matter which should be mentioned, as it is very often misunderstood by the public and the electric companies. If copper matte contains, according to the assay, 30 ounces of silver or less per ton of matte, no extra charge is made for the silver, but the copper matte is sold at the reigning market price. If it contains, however, over 30 ounces, the silver is to be paid in accordance with the assay. Thirty ounces of silver per ton of matte, of course, means almost double that amount per ton of electrolytic copper. It can easily be seen that electrolytic refineries prefer to buy copper matte containing just a trifle less than 30 ounces of silver per ton.

There are quite a number of points which need the attention of electrolytic copper refiners. There are, for instance, only a few of the refineries which manufacture their own sulphate of copper, but buy it at high rates in the market. Sometimes the cost of freight and haulage doubles its cost. Each copper refinery should have a little installation of its own for the manufacturing of the necessary sulphate of copper, which it can do at a small fraction of what it has to pay for it. The same holds good in relation to the refining of the slime or mud which collects at the bottom of the vats, and which contains the precious metals. Each copper refinery should have its own little plant for the refining of the mud. These little installations can be run at a small expense in connection with copper refineries, and it is almost nonsensical to have other concerns make large sums of money which could be saved by the refiner himself.

A few words as to electric generators may not be out of place. The author prefers separately excited machines for the reason that they cannot be reversed and for other incidental advantages. When water power is used as prime mover a good deal of trouble has been experienced in the regulation of the wheels. As a matter of fact there is no water governor in existence which will regulate so perfectly as the governor of a modern automatic engine under varying loads. By running all the exciters from an independent prime mover (either water or steam) the strength of the fields of the generators will be uniform at all times, whether there are fluctuations in the external circuit or not. The strength of the field of the generators which, with self-exciting machines, is subject to the fluctuations in the external circuit, and is a variable, becomes a constant. The author proposed this arrangement over three years ago for railway and power stations with the very best results.

Before closing, we might mention some processes which have not been introduced in the United States yet, but about which a good deal has been said during the last few years. There are in Europe processes by which electrolytic copper is produced direct from the "matte," containing from 35 to 55 per cent. of copper. From the reports that have appeared from time to time, it seems that these processes have not been a great success. There are other processes which contemplate the extraction of copper direct from the ores by means of electrolysis. The best known are the "Siemens" and "Hoepfner." If we are correctly informed, the Siemens process is now being used in one experimental plant only, which is owned by the Siemens firm. This is at Martinicken fields, near Berlin. Another one which they have in the Caucasus, at their own copper works, is said to have been abandoned because it did not give satisfaction. Dr. Hoepfner's process, it is claimed, is now in operation in three plants; one at Schwarzenberg, which began operations in September, 1890, and two others about which very little has been published. Mr. Hoepfner uses in his process a cuprous chloride solution, out of which a current of one ampere will deposit 2.35 grammes per hour, while out of a solution of sulphate of copper, only one-half of this amount of copper can be deposited per unit of current per hour.

As it was the object of the writer to give practical points on the refining of copper, and as he does not know anything about these two processes from personal observation or from practical experience, he does not wish to comment any further, but prefers to wait until he can see one of these plants in operation in the United States. While these processes do away with the smelting and resmelting of the ores, and while they would make unnecessary the large, expensive plants now used for this process, they seem to be very complicated. Their commercial value certainly should be investigated much more closely by American mining companies before they should be willing to invest large amounts of money.

We finally give below, the approximate cost of a refinery with a capacity of 1,000,000 pounds of electrolytic copper per month:

Building	$30,000
Pavement (asphalt)	2,000
Pipes for steam heating	4,000
Vats	6,000
Lead for lining vats, collecting-tanks and troughs	28,000
Lead burning	1,500
Copper conductors	11,000
Rails for overhead blocks for handling plates	2,000
Sulphate of copper	3,500
Sulphuric acid	1,000
Steam injectors or pumps or air compressors	1,000
Electric generators, switchboard, instruments	30,000
Shafting and belting	3,000
Total	$123,000

To this sum must be added the copper under treatment, which in this case will amount to at least $80,000, and if a steam plant should be required, another $20,000 must be added, bringing the total of this plant up to $223,000. The above items include labor of erection, but do not include freights. These figures, although approximate and true only under certain conditions, will demonstrate that electrolytic copper refineries are expensive plants and should be erected on carefully prepared plans and under the supervision of responsible and competent engineers.

CORRESPONDENCE.

[*From the Engineering and Mining Journal.*]

BLACK COPPER AND BLISTER COPPER.

Editor Engineering and Mining Journal. Sir: In the issue of the Engineering and Mining Journal of the 6th inst., I read with much interest the "Practical Notes on the Electrolytic Refining of Copper," by F. B. Badt.

In these he speaks repeatedly of treating "black copper" by the electrolytic process, while he surely means "blister copper." As I have met a number of metallurgists to whom the difference between these two terms is anything but clear, and it seems desirable not to get technical terms mixed, I think it would be desirable to have the *Engineering and Mining Journal* clear up things. This is the excuse for this letter.

BLACK COPPER (from the German schwarz kupfer) shows in the break a dark appearance; hence the name, and may contain from 75 per cent. to 92 per cent. copper. It contains, besides other impurities, such as iron, lead, etc., always several per cent. sulphur; it is produced in a blast furnace together with matte of 52 to 65 per cent. copper (German, dunnstein or lech). The English copper smelting in Wales does not produce "black copper," unless one would call the "buttons" produced in the "best selected" method by that name. I understand that repeated attempts of refining "black copper" electrolytically, made some years ago, resulted in failure, as the electrolytic copper produced, usually carried from 0.68 to 2 5 per cent. sulphur.

Blister Copper shows, when broken, a true copper color. It carries from 97 to 98.5 per cent. copper and is practically free from sulphur, and it is this metal that is now nearly exclusively used as anodes in electrolytic refining. "Blister copper" can be made by oxidizing matte (or "black copper") in a reverberatory furnace or by the Bessemer process, for the latter process I obtained a U. S. patent in 1865 or 1866. A. Raht.

State School of Mines, Golden, Col., Sept. 8, 1892.

F. B. Badt, Esq., Manager Mining Dept.,
General Electric Co., Chicago.

Dear Sir:—Yours received, and after careful perusal have the honor to reply that I am in accord with what you say regarding the use of "black copper" as anodes in the electrolytic process. I do this not because I desire to agree with you necessarily, but because authorities are rather of the same opinions as those expressed by you, and opposed to the general criticism Mr Raht makes. I have high regard for Mr. Raht, who was Superintendent of the Parrott Copper Works in Montana. If he desires to suggest a distinction of terms as defined by him, well and good, let the criticism be so announced, but if he wishes to "clear up" what he chooses to term confusing, I do not think he is starting on the right track. If he intends to limit the use of "black copper" and "blister copper" in the manner he defines, then he is merely endeavoring to prescribe Montana practice for that of the world.

"Black copper" is the result of a blast furnace fusion, contains over 95 per cent. of copper, and is practically an ingot copper, with an admixture of some oxide of copper; the compound is perfectly homogeneous as a scratch over its surface or fresh fracture will show. It may, and often does, contain other impurities than the oxide, as iron, etc., but it does not as he says "always contain several per cent. sulphur." When toughened it makes the copper of the market.

"Blister copper" is the result of a reverberatory treatment of copper matte and, as may be expected, contains some sulphur. It may carry as high as 92 per cent. of copper, but rarely does. A fracture of the "blister copper" pigs will show a porous surface with pellets or globules of the metal scattered over it. The pig is not so homogeneous as in the "black copper" ingot.

Doubtless Mr. Raht, who used both blast furnaces and reverberatory furnaces at his works, employed the two terms in question to designate the separate products thereof in the manner he mentions, but I am sure the larger number of metallurgists will not agree with him. Mr. Pierce, whose ability as a metallurgist is as unquestioned as Mr. Raht's, ships "black copper" of the character I mentioned above. The Arizona smelters, those of Trinidad, Colorado; and others that might be mentioned in Europe, call the impure ingots I have described, "black copper." I speak now of copper producers, not refiners, as the Orton Works in Baltimore, Md., who may perhaps take greater liberties with the designation of the several grades of intermediate products between matte and pure metal. Freiburg smelters (reverberatory) produce a matte such as Mr. Raht described, and call it "black copper." At Altenau, Clausthal and Andreasberg 600 tons are produced annually of "black copper" averaging 95 per cent. Cu, 2 per cent. lead, 0.085 silver. At Agordo, the blast furnaces fuse mattes till a black copper is produced with 95 per cent. copper.

See Phillip's "Elements of Metallurgy," p. 406.

"Smelting of Copper," Col. Grant-Frances, p. 27.

Greenwood, "Metallurgy of Copper."

All of these refer to "black copper" as the result of blast furnace fusion, which same is refined to ingot copper.

Trusting that this may be clear, and with the remark that the operators of Siemens' electrolytic process call their metal "black copper," I am,

Yours truly,

M. C. IHLSENG.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 8th, 1892, President Sprague in the Chair.

THE REVERSAL OF POLARITY IN PLATING DYNAMOS.

BY PROFESSOR HARRIS J. RYAN.

Shunt machines used for copper plating carbons are frequently found to have their polarity reversed after being stopped. The cause can be ascribed only to voltaic action of the unplated or partially plated carbons, copper anodes, and the plating solution. Any electromotive forces in the bath that would result from the electrolytic action of the dynamo current must be counter to the E. M. F. of the dynamo. Any current that these counter E. M. F.'s might set up through the dynamo when at rest, as is well known, could tend to only maintain the same polarity.

To determine the extent of the voltaic action Messrs. Thayer and Warner, of the Senior class in Electrical Engineering at Cornell, made a copper plating bath of the density that is found to do best for copper voltmeter work. This solution was increased 20 per cent. by the addition of commercial sulphuric acid so as to obtain a solution practically the same as that used by the carbon platers for good tough deposits. In this bath an unplated electric light carbon formed the cathode and good commercial copper the anode. With this cell the observations given in the following table were made.

The E. M. F.'s of the cell due to voltaic action were observed by the deflection of a galvanometer, connected to its terminals immediately after the circuit of the electrolytic current was broken.

The table explains itself, and shows that the difference of potential between the unplated carbon and copper is .17 of a volt. After the deposit of a thin film of copper on the carbon this E. M. F., immediately after stopping the current, was .004 of a volt, and was rising so that at the end of 10 minutes it was .138 volts. When a marked deposit was made by the application of the electrolytic current for one minute the cell gave a counter E. M. F. of .085 of a volt immediately on stopping the current. The return of the voltaic E.M.F. was evident, for two minutes later the E. M. F. of the cell was zero, and sixteen minutes from the time of stopping the current the voltaic E. M. F. of the cell was .130 of a volt. At this point the application of the current for but five seconds produced a momentary return of the counter E. M. F. of the cell. Even after the carbons had been well plated by the application of the electrolytic current for fifteen minutes, when the counter E. M. F. of the cell at the moment of breaking the circuit was .138 volts, there was a strong tendency manifested for the return of the voltaic E. M. F.

Time.	Volts,	Remarks.
11:20	—.170	Carbon unplated.
11:20½	—.004	Current on 30 seconds; lower part of carbon plated; plating on the upper part scarcely perceptible.
11:28	—.012	
11:30	—.138	
11:32	+.085	Current on 1 minute; well-marked deposit on carbon.
11:34	.000	
11:36	—.022	
11:40	—.064	
11:48	—.130	
11:50	+.064	Current on 5 seconds.
11:52	—.124	
12:15	+.138	Current on 13 minntes.
12:20	+.064	
12:36	+.053	
12:55	+.021	
1:45	+.006	

+ indicates a counter E. M. F.
— indicates a voltaic E. M. F.

It is seen, therefore, that with a shunt electroplating machine at work on a bath for plating carbons, there is danger of reversing its magnetism whenever it is stopped before the carbons are well coated. It may be avoided by breaking the main circuit before shutting down, and making it again only after the machine has been brought to full speed. This method would generally require the use of a large and expensive knife switch. Breaking and making the field circuit in the same manner would do just as well.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 8th, 1892, President Sprague in the Chair.

SELECTIVE OR INDIVIDUAL SIGNALS.

BY THOMAS D. LOCKWOOD.

Among the many subordinate branches of invention in auxiliary appliances which surround and form a suitable setting for that grand central figure, the Electric Speaking Telephone, none represents more painstaking thought, has been more thoroughly ransacked, or has constituted a more attractive pursuit than the study of selective signaling.

The problem it presents is both electrical and mechanical, is sufficiently difficult to create the excitement always attending the pleasures of the chase, and sufficiently promising and hopeful of a practically successful issue to be retentively fascinating. Hence its solution has been attempted alike by prominent and capable electricians, by practiced electro-mechanicians and by amateurs; has, from the laboratory and workshop point of view, already many times been achieved in as many different ways, although commercially it yet hangs like the fabled bundle of hay, just a little in advance of the enthusiastic aspirant who strives to clutch it.

THE USE OR FUNCTION OF SELECTIVE SIGNALS IN TELEPHONY.

The business of the average user of the telephone keeps a line employed but for a very small part of the entire business day, and for the remainder of the time the line wire is idle and earning absolutely nothing. If by any means it could be made possible to place several sub-stations in connection with the same circuit, without materially affecting the efficiency of the service rendered, it is obvious that the earning power of the circuit would be largely increased. But a call-bell or other signal is clearly necessary in order that the attention of the station may be attracted when some other station desires to communicate, and as we add stations to a circuit we also necessarily add call-bells; thus introducing two practical objections, viz.:

1. A most incessant and annoying ringing, because a call sent to any station is heard at all stations.
2. The necessity of distinctive signals, whereby when the bell at any station is rung the attendant may know whether the call is for his or for some other station.

At this juncture the selective signal steps in, and its special work or function may be thus stated: That two or more stations being electrically connected with the same circuit, the call-bell at any predetermined or desired one of such stations may be operated, and the call given there, to the exclusion of the call-bells at the other stations, which at the same time remain quiescent, or are silent. We therefore see that the thing to be done is not merely to ring a bell over an electrical circuit; but to select and ring from a distant point a *given* bell on a circuit with which more than one are connected; and at the same time to ensure that the bells at the other stations which are not intended to be rung, shall not be rung.

This statement of function justifies the term "*selective signal*" which I have adopted, and which I conceive to be more appropriate than the old term "individual signal" because more descriptive.

HISTORICAL SKETCH OF SELECTIVE SIGNALS.

The idea of producing a practical selective signal is not (as might from the introductory remarks of this paper have been supposed) one which made its first appearance subsequent to the advent of the telephone.

Appliances possessing the same characteristics, appeared in England many years ago, and several forms have from time to time been patented in that country for use in connection with telegraphy, which closely resemble some of the forms which within the last ten or twelve years have been re-invented and patented here.

One of these[1] is based upon combinations of a plurality of line wires, two or more of which extend from a terminal to each two sub-stations.

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[1] Br. Patent 12039, January 25, 1848, H. and E. Highton.

A second[1] involves two wires together with the use of both directions, and varying strengths of current; and states itself to be "for sounding any particular alarm in a set of telegraph stations."

The third,[2] quoting from the published abstract, is for "transmitting secret intelligence to any one or more stations without the use of extra wires. At each station a metallic disk suitably inlaid with non-conducting portions, is brought to the required position in order to complete the circuit or not, by any electro-magnetic step-by-step movement, thus excluding or including the telegraph instrument at any particular station or set of stations as required."

Three other British patents prior to 1876 are of this class, one of which[3] speaks of "an apparatus comprising a main line, a series of independent branch or station lines at each end, and clocks at both ends working synchronously, which, by means of revolving cams, place the main wire in consecutive connection with corresponding branches at the two ends for definite times."

Another[4] speaks of "apparatus placed in a continuous circuit; the bell at any particular station being caused to ring by the simultaneous advance in all the line wire apparatus, of a cam disposed in a way peculiar to each station, which is instrumental in closing a local circuit by which the bell is sounded."

The final instance[5] of this series is a mechanism in which a disk slotted or perforated at but one point, the location of such point differing for each station, opposes itself to the forward motion of an armature lever, at all times, except when the slot or hole is immediately opposite a pin on the end of the lever. This occurs at no two stations at the same moment, but when it does occur at any station, the pin passes into the slot and the armature is thereby permitted to complete its advance and to close a local circuit and ring a bell.

Several of these suggestions will no doubt be recognized as being found also in some of the more modern selective signal appliances.

In France, too, in connection with railway telegraphy, this class of invention was formerly very popular; and as might be expected, is not neglected by that indefatigable chronicler, the late Count du Moncel, who devotes considerable space to the subject in the "Expose des Applications de L'Electricité."[1]

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[1] Br. Patent 12959, February 7, 1850, E. Highton.

[2] Br. Patent 13427, December 27, 1850, Dering.

[3] Br. Patent, No. 3006, December 1, 1863, Wilde.

[4] Br. Patent, No. 910, March 31. 1865, De Bonneville.

[5] Br. Patent, No. 1801, June 11, 1869, Lyttle.

It is unnecessary to recapitulate much of what he states, and it is perhaps sufficient to say that the plans of no less than thirteen different inventors are described, in which the achievement of selective signaling is at least aimed at; and that some of them evince as thorough an appreciation of the merits of the question, as do many of the appliances patented for the same purpose in America since 1876; while one or two actually are based upon ideas which are found in comparatively recent patents also.

In the plan of Lamothe, for example, which is discussed by M. du Moncel, the gong which is to be struck is by one electromagnet brought within range of a bell hammer, which then by a second electro-magnet is caused to strike the bell.

Another inventor, Bizot, has a plan for selectively ringing bells by means of pendulum hammers placed at the different stations, and adjusted at each station to a different length.

An electro-magnet at each station in the main circuit controls the swing of the pendulums by its alternate magnetizations and demagnetizations brought about by the makes and breaks of the main line current; these being effected by a vibratory circuit breaker at the transmitting station, operated by a similar electro-magnetic pendulum there, which can be adjusted when desired, to the same length as the pendulum at any one of the other stations which it is desired to operate. Only that one to which it is adjusted, will be operated; the others remaining inert. By so arranging *all* of the station pendulums that they can be adjusted, any station is enabled to selectively signal any other.

This apparatus I have described at considerable length, because it is founded on the harmonic principle which later was introduced into telegraphy, and because it contemplates the use in selective signaling of that principle, subsequently more elaborately worked out by German and American inventors and described in their several patents.[2]

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[1] Expose des Applications de L'Electricité, Vol. iii, 1874, pp. 510-528.

[2] German Patent, No. 1944, Maron, December 5, 1877.
Br. Patent, No. 5078, Blake, December 11, 1879.
U. S. Patent, No. 251,097, Currier, Dec. 20, 1881 and others.

Martorey's system, which likewise is described by M. du Moncel, also contains the elements of many subsequent forms. In it, by an electro-magnetic step-by-step motion, an escapement wheel having a number of teeth at least equal to the number of stations on the circuit, to give the signal, is brought to a given position by a given number of pulsations, differing for each station. All the teeth are metallic, and all, except one, are insulated. That one connects with the axis and mechanism, and is part of the alarm circuit. When brought into position, the ringing circuit which it appears is open normally at two points, is closed at one of these points, and at this juncture a single impulse sent over a second wire closes the second break in the local circuit, causing the bell to ring.

In addition to the foregoing examples of the condition of the art, it is not, I conceive, out of order to point out that the principles which govern printing telegraphy are closely related also to selective signaling. Consider for example, the House, Hughes, and Phelps telegraphs, and the form of stock quotation recorders now so universally employed; in all of these a given letter or figure is brought to a suitable point and is then pressed by a final operation against the paper to make the impression.

Now a little careful thought will show that the two problems:

A. To bring a number of typewheels at different stations all to the same predetermined point, and then press the paper against them so that the same letter will be printed at all stations, and

B. To bring a number of wheels at different stations carrying local circuit closing devices (placed at a different point for each wheel) all to the same predetermined point, where any particular one of them will be in position to close its local, and ring its bell when the press movement is made; are not in principle by any means far distant from each other.

They are further related, in that both must have a unison, zero, or starting point, to which they can be brought periodically for a fresh start, to forestall a lack of harmony between the different stations, or to provide means for bringing back an errant wheel which has strayed away from the path of electro-mechanical rectitude.

A good stock of knowledge thus was at the disposal of those interested, even at as early a date as 1876, when the career of the telephone was only beginning. We had the electro-magnetic ratchet and pawl, step-by-step, movement. We had electrically controlled clocktrains.

It was old to trip clockwork electrically; and also to use wheel circuit changers. The unison device in connection with step-by-step devices was old, and its use well known; the relay and its function of closing local alarm circuits was already a classic; wire combinations of various sorts were customarily employed; and the harmonic field had already been well threshed by Helmholtz, Bizot, La Cour, Varley, Gray, Bell, and others.

Why then was it not easy to make at once a thoroughly satisfactory and efficient selective call?

One reason is, that the average inventor or would-be inventor generally refuses, tacitly it is true, to recognize that there is any material of which he may make use. He does not know the state of the art; does not want to know the prior art; does not care anything about the prior art, and often feels affronted when he is necessarily brought face to face with it; and then he gives utterance to the pathetic plaint that "the people of former times were a dishonest race, they have stolen all my new inventions."

It is however clear from what has been stated, that the principles which underlie the problem in its many phases had to a great extent been elucidated, and that much had been done in working out scores of electro-mechanical contrivances, capable of being utilized in apparatus embodying such principles; and for these reasons there has in this branch of electro-mechanics been little scope for the exercise of original broad invention, and consequently little reasonable expectation of substantial pecuniary reward, even for him who might produce a thorough practical success in selective signaling.

These considerations have without doubt had effect in preventing many inventors and mechanicians of the first rank, or of great experience, from vigorously grappling with what was left of the problem. Yet from January 1879 to December 1891, 161 U. S. patents were granted for forms of Telephonic Selective Signaling appliances, all of which I have examined.

A careful analysis shows that quite a large number of general plans have been proposed, and that each of these has been developed in several ways, with greater or less care, and to many degrees of perfection. Several sub-methods can be utilized in many of the general plans, and in a number of cases attempt has been made to provide for several additional functions.

Though at one or two points selective signals embodying a few of these different plans are even yet employed, every telephone man knows that it cannot truly be asserted that success has attended such appliances.

One reason why, has been detailed. Some other reasons may now be given.

1. Some forms of promising selective apparatus have been put into use before being mechanically perfected, and their faultiness has given a figurative black eye to the entire race.

2. Some have been inherently defective, either in principle or construction.

3. Many as soon as crudely devised, have been employed as beasts of burden on which to organize a heavily capitalized stock company, and it has therefore been the great object of the advocates of these, to sell stock, instead of to make a practically successful apparatus.

4. To be commercially successful, a selective signal for a telephone system must be of low cost. In every form which has been put out and which has come under my observation, this requirement has been too prominently insisted upon, and the attempt has been made first of all to make not merely a *low priced*, but a really *cheap* signal. This attempt, I may note, *en passant*, has met with uniform success.

5. Excellency has not been insisted upon.

6. In many instances the apparatus has been required to do too much, or to perform too many functions, and has therefore become complicated; and

7. Usually the places where such signals have been tried, have also been places where intelligent supervision has been at a minimum.

Under the above circumstances, good results need not have been expected, and although they were expected, they did not arrive; and the user, after trying successively many of the different kinds of signals from time to time placed before him, has with the few exceptions already noted, discarded all; and for a long time now has refused to listen further to the voice of the charmer; alleging and with some show of reason, that "the only trustworthy individual signal is an individual line."

DESIRABILITY OF SELECTIVE SIGNALS.

This epigrammatic saying is however in some sort losing its attractiveness, in face of the multitude of conductors which it implies; and to a keen observer of the signs of the telephonic times, it is evident that a very extensive return to party lines in some form will bye and by be inevitable; at all events for residential districts, and from this point of view, it appears that the demand for a simple and trustworthy, but not costly, selective signal is as great as ever, and that, if such a thing is made, and its merits proved, many more party lines could and would be arranged and satisfactorily operated.

To reach practical success, by which is meant to devise a really trustworthy selective signal, economical, simple and easily kept in order, invention is not so much needed—keeping the prior art in mind—as is electro-mechanical skill to mould a homogeneous and perfect apparatus from the immense amount of material we have to choose from; an apparatus in fact which will operate perfectly, not only when the several circuit signals are ranged up alongside each other on the testing room wall, where a screw may be adjusted here, a spring tightened there, and a bell hammer rod bent in another place; but equally well when they are distributed over the circuit, and all at a distance from the operator.

And to aid the adventurer who intends to construct such an apparatus, I will outline the principles upon which this class of work in its several proposed plans, have so far been founded, reference being made in brackets to a single U. S. patent exemplifying each of the several plans.

GENERAL PLANS.

1. *To operate the several signals at each sub-station on a circuit with differing strengths of current.*

Bells depending for their proper operation on a margin of current are however generally untrustworthy. [U. S. Patent—No. 221,512, Nov. 11, 1879, Buell.]

2. *Signals arranged to respond to currents of opposed direction.*

This implies of course polarized bells, and for a line with but two stations, it is probably as good an arrangement as can be provided. [U. S. Patent—No. 218,153, August 5, 1879, Anders.]

3. *Signals involving changes both in strength and direction of currents.*

These are apt to be unreliable for the same reason as are those of No. 1. [Patent No. 242,408, May 31, 1881, Anders and Vail].

4. *Electro-magnetic step-by-step devices, acting to bring the sub-station signals to a ringing point differing for each section, and then to close a local, branch, or shunt circuit, including the local bell, to operate ringing mechanism, or in some way to introduce the bell magnet into the circuit.*

Figure 1, taken from a patent of E. N. Dickerson, Jr., illustrates a ratchet and pawl step-by-step movement operating to close a local circuit in which is placed the bell. Successive pulsations sent from the central station, cause the electro-magnet M in the main line circuit to attract its armature *a*, which after each pulsation is pulled back by a suitable spring (not shown).

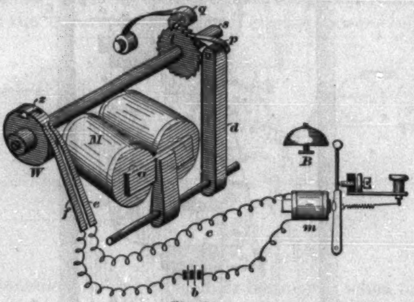


Fig. 1.

The non-conducting wheel W on which presses the two springs *e* and *f*, has at one point on its periphery an inlaid piece of metal *z*. When the wheel is rotated till the plate *z* comes under the two springs, the local circuit of the battery *b* is closed through the bell magnet M, and the bell B, which in practice would be vibratory, rings.

The wheel W can be adjusted on its shaft *s*, so as to be in a different position at each station, and is operated by the main magnet through its armature *a*, lever *d*, pawl *p*, and ratchet wheel T, the retaining pawl *q* holding the shaft *s* between the several forward impulses.

There are of course many other ways of utilizing these principles, one being shown in Fig. 2, which is one of the drawings of a patent taken out by Messrs. Curtis and Crocker.

In it, the main line L comes to the binding screw *b*, passes by the wire *a* to the main electro-magnet A, which acts through its armature *h*, lever H, and pawl I on ratchet wheel K, held at any point by the retaining pawl *l*, and provided with a zero spring *p* and stop *o*. The ratchet wheels K are metal and at a point on their disk surfaces, differing for each station, have a non-conducting piece *n* let in, flush with the said surface.

A contact spring M presses also on the conducting surface.

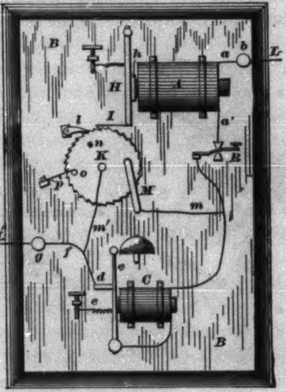


Fig. 2.

After passing the magnet A, and continuing by wire *a'* to a key R, used to call the central station, the circuit divides, one branch passing through the magnet C of the call-bell and thence by way of the bell hammer lever *c*, back contact *d*, and point *f*, to the terminal *g* and thence to the outgoing line L'.

The second branch passes by wire *m* from the point of division to the contact spring M, thence to the ratchet wheel and through the mechanism and by the wire *m'* to the reuniting point *f*, binding screw *g* and outgoing line. Thus as long as the contact spring M presses on the metal face of the ratchet wheel, the bell magnet is short-circuited; but when as the wheel is revolved, the end of the spring presses on the non-conducting piece *n*, the shunt circuit is broken and the bell introduced and rung.

This general plan No. 4 has been much favored by inventors, admits of almost indefinite variation, and is easily associated with auxiliary devices such as unisons, "busy" signals, and lock-outs for apparatus at other stations. If mechanism properly designed and made, be employed, there is no reason why satisfactory results might not be expected. Cheap apparatus has however invariably been tried and the system besides is slow. [No. 212,792, March 4, 1879, E. N. Dickerson, Jr. No. 230,530, July 27, 1880, Curtis and Crocker].

5. *Clockwork Bells.*

a. Those in which a constantly running clock-train brings the bells at the different stations into circuit successively, or removes a mechanical holder from their hammers; when only they can be rung. These are not reliable; it being impossible to keep the clocks together; the clocks also require too frequent winding. [Patent 223,469, January 13, 1880, Bliss].

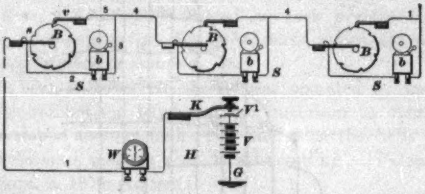


Fig. 3.

b. Those in which at each station is a normally stopped clock-train. All clocks are started by the same initial pulsation, and bring their signals to ringing points, or their bells into circuit at different times.

This idea is one of the most practical that has been advanced, and if the apparatus is well designed, and put together in a good mechanical form, it ought to do good work; unfortunately first-class apparatus is costly enough to remove the gilt from the economical side of the selective signal.

This plan is indicated by the diagram Figure 3, in which however the clockwork is not shown. In the diagram, H is supposed to be the signal transmitting, and S the signal receiving station, at each of which is a metal circuit wheel B, by which in conjunction with the circuit springs *s* and *v*, the bells *b* are when at rest short-circuited. At the transmitting station is a circuit closing key K, a battery V having one pole united to earth or return G, and the other extended to the front contact V′ of the key, and a duplicate W, of the several sub-station instruments.

The first pressure of the key through suitable mechanism starts the clock-works at all stations, and introduces the bells at each station successively, as the wheels B bring their notches under the springs *v*. The wheels are in the present instant supposed to be slow moving, and to have six ringing notches for each complete revolution. They can by suitable mechanism be stopped on each notch to ring a given bell as long as may be desired; and they come to rest after one notch at all stations has passed under its spring.

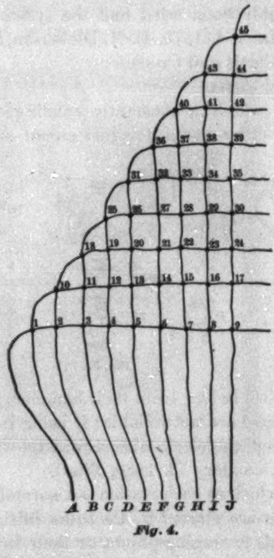


Fig. 4.

Of course the notches are arranged to pass under the springs at different times for the several stations, and this is indicated in the figure, where the bell at the nearest station is introduced into the circuit; the springs *v* of the others being at different positions on the periphery of the wheel. [Patent 232,442, Sept. 21, 1880, Blake].

c. Those in which a normally quiescent clock-train is controlled by an electro-magnetic escapement, and permitted to move by successive beats, as a key at the central station alternately makes and breaks the circuit. [Patent 228,047, May 25, 1880, Eaton].

This could doubtless be made to work all right by putting excellent apparatus into it, but like nearly all step-by-step devices, would generally be slow.

Note.—All of the sub-divisions of class 5, may either have the signaling device directly in the main circuit, or in a local circuit to be closed by the action of the main circuit.

6. *Bells in which all the hammers move forward to strike at the same time, but except the desired one are prevented from striking by an interposed mechanical obstacle, which in the bell of the wanted station is removed.*

This has been faithfully tried. It was operated by an electro-magnetic ratchet and pawl, and was uncertain in operation. It also developed another fault: viz, that even the bells which are not to respond, make a considerable rattling. [Patent No. 219,059, Sept. 2, 1879, Anders.]

7. *Bells in which the ringing is done by circuit arrangement, or conductor combinations. Two series of circuits may radiate from a central to a number of substations, and one of each series caused to enter all stations. No two stations however have the same two circuits. Only when the current traverses both circuits does the bell ring at the station they enter.*

This is very promising. It can not however be applied to a multiple switch-board system. It requires relays and a local circuit bell; but no specially constructed apparatus.

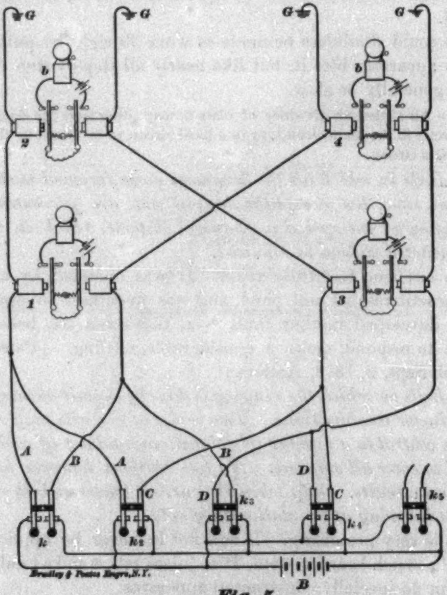
It is also very speedy, the single pressure of a given key will always ring a given bell, and no mistake can be made.

This system is roughly illustrated by the diagram, Fig. 4, in which each outgoing conductor is shown to intersect each other conductor at some one sub-station, and thus afford facilities for producing a selective signal for 45 stations by using but 10 main lines.

For example, if it be desired to signal station 41, it will be necessary to manipulate the lines G and I.

The diagram Fig. 5, more fully illustrates one way in which the idea is worked out, although there are several plans, all of which are more or less advantageous.

A battery B, branches its two poles to each of a series of keys *k*, each of which controls the terminals of two lines; for example, key *k* may control terminals of A and B, k^2 those of A and C, and so on.



Bradley & Poates Engrs, N.Y.

Fig. 5.

At each sub-station two lines both enter and pass respectively through relays, and the concurrent action of the two relays at any station is required to ring the bell, since the bell circuit is led through the armature levers and their front contacts of both.

Taking the conductors in pairs, the number of possible combinations is represented by the expression $\frac{n^2 - n}{2}$ and that number of separate stations can be selectively signaled by the use of *n* wires. [Patent No. 219,188, Sept. 2, 1879, Vail; Patent No. 224,855, Feb'y 24, 1880, Vail.]

8. *Bells working on the principles of harmonic telegraphy.* At each station the spring, pendulum or reed carrying the bell hammer is tuned to respond to a different note or rate of vibration.

The central station transmitting device is adjustable, and can have its rate made to correspond with that of any sub station bell. When set in operation the coinciding bell only will ring.

Operated always by experts, and carefully constructed and adjusted, these might do well. They are, however, too sensitive; tend to become operative as their proper number of vibrations is approached, and in some degree become irregular with temperature changes.

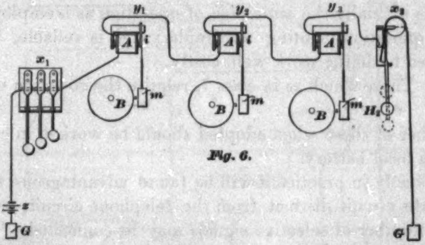


Fig. 6.

Fig. 6 illustrates the idea, and shows a central and three sub-stations.

At the central station is a battery *z*, and three pendulum circuit breakers formed into a single apparatus x'. Each breaks and makes the circuit at a different rate of speed.

The main circuit passes through the magnets A of the different sub-stations successively, each magnet having a reed armature to which a pendulous bell hammer is attached, the normal rate of vibration thereof corresponding to that of some one of the transmitting circuit-breakers. At the end of the circuit is shown a second transmitter x^2, with which, if desired, the sub-stations may be provided.

It has an adjustable pendulum bob, H^2, which can be moved to correspond with that of any bell which it is desired to ring, and which, then being set in vibration, is responded to by such bell. [Patent No. 251,097, Dec. 20, 1881, Currier.]

9. *Sub-station signals in which galvanometers identically constructed and calibrated, make the same deflection with a given current; to ring the bells different currents are sent, which produce different deflections. Each, of course, has its ringing deflection at a different point, which may by a mercury cup close a local circuit.*

These signals ought to work fairly well, if sufficient care and cost be put into them. [Patent No. 262,252, Aug. 8, 1882; Southworth.] It would not be difficult to add severalother classes to this list, but those which have been mentioned are the most important.

The plans of greatest promise are:

A. Those which by clockwork (as in 5*b*) are brought to a ringing point differing for each, the clockwork being then stopped, and the ringing done by a different kind of current. This is virtually the same plan of operation as is employed in the stock quotation printing telegraphs; and is reliable, but when adapted to do the work well, costly.

B. Those which as in class 7, require the co-action of two circuits.

Either of these when adopted should be worked in connection with a local battery.

Probably in practice it will be found advantageous to have a separate circuit distinct from the telephone circuit, on which a great number of selective signals may be connected.

Many sub-methods and appliances have to be thought of in considering this subject; whether we will be content to ring sub-stations from the central station only, or on the other hand give each sub-station the power of signaling others on the same circuit.

If step-by-step apparatus be employed to successively bring the station appliances to their ringing points, some means is required to prevent the bells of intermediate stations from giving a signal as they are passed. Sometimes the time element may here be utilized. But a better way is to have the call-bell in a local circuit open at two points; to let the advance of the apparatus close one of these, and then send a different kind of current to close the other.

We have to consider also whether the introduction of line-in-use signals, unison apparatus, call sending appliances, and telephone lock-out apparatus, are sufficiently important to be added.

The unison or zero device in some form may be considered as an essential auxiliary in all progressive signals, whether of the clockwork or step-by-step orders. Even the most perfect mechanism slips occasionally, and as in the stock printing systems where instruments "throw out" or get out of step with each other, a suitable zero at which all can periodically be brought to rest, and start even, must be certainly required. A "busy" signal which indicates at each station whether the line is in use at any other is useful, but not essential. And lock-out apparatus, usually provided by closing a shunt circuit round the call sending mechanism and telephones, at all stations on a circuit, except the one or two which are actually communicating, is also highly advisable in order that privacy and freedom from interruption may be maintained.

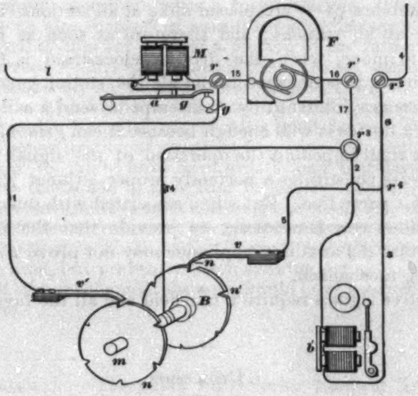


Fig. 7.

But, while some of these devices are necessary, and all useful if reliable, the contriver from this time forward, must keep at all times before his eyes, the undoubted fact that selective signals where heretofore tried, have uniformly been handicapped by being too intimately associated with too many such auxiliary devices, and must be correspondingly cautious about the way in whichthey are added.

Some such mode as is depicted in Fig. 7 is best for this purpose.

In the figure, M is a clock tripping magnet; F the magneto generator for sending outgoing calls, *b* the station bell, and *m* and *n* two metallic wheel commutators mounted on, and electrically connected with the same shaft B. Arranged to press on the periphery of *m* is a contact spring *v′* and similarly placed with respect to *n*, is a spring *v*.

When the station mechanism is at rest, it is desirable to have the generators in the line circuit, but the bells out. Accordingly, the bell is short-circuited through the disk *n*, spring *v*, and wires 14 and 5; while a possible shunt circuit round the generator through disk *m*, spring *v′*, and wires 1 and 14 is open, the spring *v′* being over the notch.

The notches of *m* are placed alike at all stations, but those of *n* differ at all stations; and therefore as soon as the shaft B begins to move, when the driving clock-train is tripped, the shunt circuit is established round all the station generators, and no station can interrupt by an attempt to send a call signal.

Such a device is well enough because it can generally be added without at all impeding the operation of the signal proper, and in any case constitutes a perfectly proper adjunct to the equipment of a party line. But when associated with selective signals the greatest care is necessary to provide that the arrangement is such that the auxiliary appliance may not prove a drag on the signaling mechanism.

Selective signals require a fair field and all the favor they can get.

DISCUSSION.

PROFESSOR CROCKER:—Mr. Chairman, Mr. Lockwood asked me to say a word in discussion of his paper, but I fail to see where there is any room for discussion. It is very complete and puts on record in a clear and satisfactory form, a certain branch of the electrical art which has quite an extensive application. As Mr. Lockwood says, it is not confined to the telephone. The ability to call up one particular office or point on a single circuit containing many such offices or points is certainly very useful. I thought so some thirteen years ago, and patented the method referred to by Mr. Lockwood. It was my maiden effort in electrical invention and I am pleased to see that it still exists. But I was not aware, until I read Mr. Lockwood's paper, that it had not sunk entirely into oblivion. The perfection of the apparatus is the essential point in this matter. It is not a very difficult thing to think of a method. In fact, there are almost too many possible methods.

If the ability of the inventors and makers of this apparatus had been concentrated on one method, I think the results would have been better. One very interesting point is the fact that in 1881 and 1882 it looked as if such devices were entirely unnecessary, simply because each man wanted his separate circuit. Now, in 1892, we are coming back to the position that we had before that time—that is, where the selective signal is useful again. That cycle of change in electrical engineering is quite interesting. We see the same thing in the case of the alternating current which twenty or thirty years ago was the standard form. Then it disappeared for more than ten years, and then reappeared again.

The necessity or desirability of putting a number of offices on one line is apparent to any one who goes into a telephone station and sees the multiplicity of connections required where there are several thousand subscribers from a single station. That would appear to be an almost essential reason for adopting some form of selective signal system for at least a portion of the subscribers.

Mr. Lockwood:—Mr. Chairman, if there are no other gentlemen who wish to say anything on this subject, I would like to add one word which has been called to my mind by what Professor Crocker said. I cordially agree with what he said about the necessity of concentration of attention upon one line, and that the results would no doubt have been greater had that been done. But I should like to add to that, that it largely depended upon which method several inventors concentrated their attention, and I do not think that any amount of concentration of ability devoted to that method in which hammers were brought upward, all at the same time, but prevented from striking by the opportune presentation of a mechanical obstacle, would have produced a good signal system.

The Chairman:—The paper on "Series Electric Traction," having already been read by title, if there is any discussion on that we will take it up now.

Mr. Upton:—Mr. Chairman, I believe I am right in saying that it was the intention to have the discussion on the coming papers go together. There are three papers all on electric railways which should be read and the discussion follow at one time.

Mr. Martin:—That would be a great saving of time, Mr. Chairman, and I move that the three papers on traction be read, and that we have the discussion upon them all at the same time.

[Motion seconded and carried.]

The following papers, by Mr. H. Ward Leonard, of New York, on "A New System of Electric Propulsion," and by Prof. George D. Shepardson and Mr. E. P. Burch, of the University of Minnesota, Minneapolis, Minnesota, on "Electric Railway and Motor Tests," were then read by the author.

The Chairman:—At the time when Mr. Perry's paper was called, as he was not present it was read by title. Being upon the subject of electric traction it is directly in line with the papers which have already been read, and Mr. Perry perhaps would like to summarize in a brief way the particular points in that paper before the discussion takes place.

The paper on "Series Electric Traction," by Mr. Nelson W. Perry, of New York, was then read by the author.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 8th, 1892, President Sprague in the Chair.

SERIES ELECTRIC TRACTION.

BY NELSON W. PERRY, E. M.

In telegraphy, that application of electricity where pre-eminently long distances are concerned, the series system is alone adequate to the demands. In telephony, where more than two instruments are concerned, as in the case of several subscribers on the same circuit, or where as in small towns, several of them are connected on a single wire, they are all connected up in series; and again where larger amounts of energy are to be transmitted to considerable distances, as in the case of the arc light, the series arrangement is the only practical one now at hand. Why then should not this system commend itself for long lines of electric railroads? Mr. T. Carpenter Smith, in a series of valuable articles on "Some Views of Central Station Management," now running in one of the weekly electrical journals, lays down as one of his canons in the establishment of a plant, that it should be so planned as to permit of its ready extension as the demands increase; or, in other words, that in laying our plans we should provide for the future rather than for the present. If this be good advice in the case of electric lighting stations, how much more force must it have, if it be applied to electric railway systems, which, as we all know, are being rapidly extended, in many instances to double and treble the distance originally contemplated, only to be increased again as soon as these further limits are reached.

In railway practice, it is the distance rather than the quantity of current that forms the most important factor of increase, though the latter cannot be neglected in planning for the future.

In street railway construction, assuming the distance to remain the same but the traffic to double, the amount of copper required to maintain the same relative economy must also be doubled. This is a minor matter as compared with the case where both distance and traffic are doubled. In this latter case, to maintain the same economy, the weight of copper must not only be doubled but squared. The limit of extension even on these lines is, however, soon reached, and the only recourse, by existing methods, is to erect at or near the further point another generating station, with all that that implies in the way of additional investment in real estate, machinery, etc., and a duplicate set of employés.

In long lines, the bugbear of the multiple arc system of distribution is the drop in potential at the further end. The potential may be amply sufficient near the generating station, but it becomes less and less so as this proximity is departed from, until, at the further end, the lights which burned brightly near the station, dim down to scarcely more than a dull red, and the cars are operated under heavy loads with difficulty or not at all. There are two remedies for this drop in potential at hand—either to increase the amount of copper in the feeders, which within commercial limits is but a partial remedy, or to increase the pressure, using the same amount of copper which, when the multiple arc method of distribution is employed, introduces as many difficulties as it obviates. There is still a third remedy, and that is to employ the constant current method, in which, under all circumstances, the current remains the same and the pressure varies as the work performed.

In the one case the copper must vary as the demands and distances—the pressure remaining the same—and in the other the pressure varies, the copper and current remaining the same.

In an arc light (series) circuit, the wire will be of the same size whether it be erected to supply one lamp or sixty—whether the distance be one mile or five, and if at any future time it be desirable to extend the line to double or treble the distance, the wire already erected is amply sufficient for the part of the line it covers, and merely requires extension to the desired point to meet all the demands. There are in this country numerous arc light circuits in successful operation, twenty miles and more in length, yet I doubt if there is anywhere in the world a direct current constant potential circuit in operation to-day of anywhere near this length.

If energy can be so much more advantageously distributed to long distances by the constant current method for arc lamps, why should this method not commend itself to the ever-increasing distances to which our electric street railroads are reaching? If the steam locomotive is ever to be supplanted by the electric motor, why should not this method of distribution be the one to be adopted?

The answer is that the constant current method, as heretofore developed, has not been sufficiently elastic to meet the requirements, and although the generation of a constant current involved much less complex machinery, and the cost of distribution to distant points necessitated a much smaller initial outlay, still the difficulties introduced by this method largely outweighed all its advantages.

With the arc lamp or other stationary translating device, arrangements were readily devised for automatically shunting the current around in case of disability of the translating device, or the desire to throw it out of service. Not so with a moving object, such as a street car. No means had been found to shunt the current around the latter, or in case it got out of circuit by means of the trolley getting off the wire, of automatically closing up the gap thus made, and the disability of one car became the disability of the whole line and of every car on the line.

Could this and some other difficulties be surmounted, the constant current possesses many advantages over the constant potential that seem to me would bring it into almost universal use for traction purposes, especially on interurban lines and in other cases where long distances are concerned.

It is my purpose to show how all this has been accomplished, and how the constant current system may be made as elastic in every way, at least on paper, as the constant potential, without sacrificing in any way any of the advantages peculiar to the system, and how it is possible to make each unit in such a system nearly if not quite as independent of every other unit as is the case in the system now in vogue.

The most ambitious and painstaking, and withal successful attempt up to the present time, to adapt the series or constant current method to electrical traction, was that made by Prof. S. H. Short, now of Cleveland, Ohio, and he actually constructed and operated for a considerable length of time, and with some success, two roads, one a single track road at Huntington, W. Va., connecting that city with Guyandotte, about three and one-half miles distant, and the other a belt and double track road on South Broadway, St. Louis, which was, I think, about three miles in circuit. Prof. Short employed two wires, divided up into sections of any desired length, but necessarily not less in number than the number of cars to be operated. These sections were connected together by switches operated by a peg on the trolley mast, the peg on one car throwing the switch in one direction, and that on the succeeding car throwing it in the opposite direction. The current coming upon one line of wires, crossed over through the motor to the other line, by which it was carried to the next car motor, through which it passed back again to the original line of wires. Each car as it passed a switch set it so that the current in the section it was entering would come up on the same wire as it had in the section just left, and the succeeding car would set the switches so that the current would come up to it on the opposite wire, and so they alternated.

Since the continuity of the circuit depended upon the presence of a motor between the two lines, if either of the trolleys got off its wire, the circuit was broken and all cars had to stop for lack of current, just as if we had an arc lamp without an automatic cut-out, and the carbons should become broken or the arc blown out, the entire circuit would be disabled, and any irregularities in the resistance of the arc of one lamp would affect all the other lamps on the circuit.

Since the switch must be thrown to opposite wires by each succeeding car, if one car failed to do this, as would be the case if an odd number of cars were added to or removed from the line, it would again become broken by reason of the failure to throw the switch in the proper direction. Cars had, therefore, to be added to or removed from the route in even numbers. This precluded the inter-communication of two or more systems, for the reason that if a car was run off from one line onto another, it would be a case of the removal of an odd number to the other line, and both lines would become broken. So, too, it was impossible to operate a branched road on which part of the cars took one branch and a part the other, for the succeeding car, after it passed the forks, whichever route it took, would either break the circuit itself, by reason of failure to properly turn the switch, or would leave the switches in such a position that they could not be operated by the next car.

On a single track road where the cars passed each other on sidings, it was necessary to wire the whole route as though it were a double track road, the cars going up one pair of wires and returning on the other which followed the siding. This was the case at Huntington, where, although there was but a single siding, perhaps fifty feet long, where the two cars passed each other, the whole three and a-half miles had to be double wired to enable them to pass, else the two cars would be thrown into multiple with each other and neither could run. I succeeded in devising a means by which if there were but two cars and one siding on the road, the only double wiring required was at the siding, but the plan failed where more cars and sidings were involved.

In Short's system the switches were purely mechanical, exposed to the vibration of the trolley wire and to the elements, and had to fit tightly, else they would burn out. If they fitted sufficiently tightly to obviate this, they were liable, through corrosion or ice or some other cause, to fit too tightly and fail to operate, in which case the line again became broken.

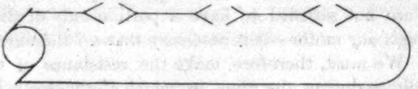


Fig. 1.

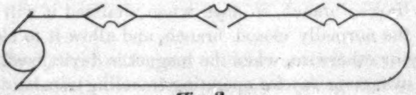


Fig. 2.

I speak more freely of the defects of the Short series system from the fact that it has been abandoned by the Short company, and because its failure has been officially announced by its projectors. My object in dwelling upon it at such length is to point out more clearly some of the defects that must be remedied before the series can become a practical working system.

The system which I am about to explain may properly be described as consisting of a circuit interrupted by a series of breaks in multiples of two. Or one consisting of a circuit interrupted by breaks in multiple series with each other—the breaks being in multiples of two. The conception then would be like Fig. 1.

But in order that a current shall pass around this circuit, it is necessary that *one* of the breaks in each multiple shall be normally closed. It matters not which one is closed, so our plan would now look like Fig. 2.

If a translating device should successively occupy each of these

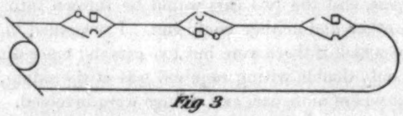


Fig 3

breaks, it would receive current during the time that its terminals connected the ends of the two broken wires, in proportion to the relative conductivity of the two paths open to the current. But we are not satisfied to have a portion only of the current pass through our motor—it is necessary that *all* the current must so pass. We must, therefore, make the resistance of the other branch infinite during the time in which the motor is bridging, through its terminals, the break, or, in other words, the normally closed branch must be broken.

If, therefore, we place an electro-magnet in *series* with the normally broken branch, so that when vitalized it will open the switch in the normally closed branch, and allow it to close again by gravity or otherwise, when the magnet is devitalized, we have provided in a crude way for operating traveling translating devices in series. Our conception now would be represented somewhat by Fig. 3.

By this arrangment, the moving device only receives a momentary impulse as its terminals touch the ends of the break, and would have to rely upon its acquired momentum to carry it to the ends of the next break. It will be apparent, however, that it is immaterial whether the device receives its next impulse by closing the next break in the circuit or whether it skips that one

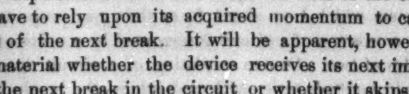


Fig. 4.

and gets in from any other break, either on that circuit or from one connected with an entirely different dynamo. It is this feature that permits the use in my system of sidings on single track roads, and of forks (the skipping of one or more breaks), and the inter-communication between separate circuits (the jumping from a break on one circuit to a break on another),

It is desirable, however, that the moving device be impelled by as nearly a continuous force as possible, instead of by a succession of momentary impulses. Instead, therefore, of having the ends of the normally opened break face each other as represented heretofore, we may extend these wires from some distance parallel with each other, and our plan assumes the form shown in Fig. 4.

And it is desirable that the interval of time during which the device is without current, viz: that occupied in passing from one break to the next, shall be as small as possible. This naturally suggests having the normally open breaks all on the same side of the normally closed breaks, and if these parallel wires be continued so as to overlap those constituting preceding and succeeding breaks, a car or other moving translating device may pass from one break to another without even momentary interruption of current. In Fig. 5, the lower line which I call the "feeder," contains all of the normally closed breaks, represented by mercury cups normally connected electrically by a fork taking into them, which constitutes a double pole switch actuated by an electro-magnet in the manner to be described.

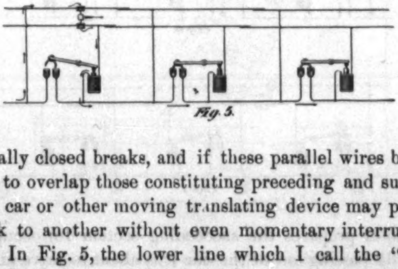


Fig. 5.

The upper pairs of wires, which it will be seen are merely the parallel extensions of the other possible path for the current, are in multiple with the path through the mercury cups. One of each of these pairs of wires, which will hereafter be denominated sections, is connected to the feeder wire on one side of the mercury cups, and the other has a similar connection on the other side. The two wires of each section normally without electrical connection with each other, constitute what has been termed the "series of breaks normally open." If, however, these two wires be connected electrically through the terminals of a motor, the latter will be in multiple with the mercury cups and momentarily receive only the current due to the relative conductivity of the path thus offered. By the arrangement shown in Fig. 5 the electro-magnet which opens the mercury cup switch is in series with the motor circuit, and the latter no sooner receives current than the former is actuated, the switch is opened and the whole of the current which was momentarily divided between the two branches, is diverted to the motor.

If the corresponding wires of the succeeding sections be caused to overlap without making electrical connection with each other, and these laps be staggered as shown, a means is provided whereby the motor prepares the succeeding section for occupancy before wholly passing out of the one it is about to leave and accomplishes this without interruption to the current. Figs. 6 and 7 show how this is done and the arrows show the paths taken by the current as the transition takes place. When the motor is in the position represented in Fig. 5, there is but one path for the current—viz. around the open switch in the feeder and through the electro-magnet by which it is kept open, back again to the feeder.

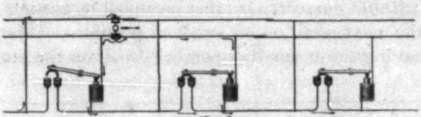


Fig. 6.

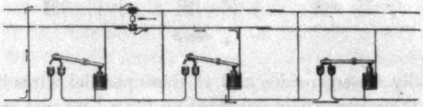


Fig. 7.

As the motor proceeds to the right, one of the terminals first embraces the lower lap, thus offering two paths for the current—one passing through the switching arrangement that controls the section the car is about to leave, and the other through that controlling the section it is about to enter (see Fig. 6). As it progresses still further, this terminal passes beyond this lap at about the same time that the other terminal embraces the other lap. This leaves the first switch without current, and the succeeding switch is automatically opened. There is no break in the circuit, however, even during the short interval occupied by the two switches in acting, for while the further terminal embraces the further lap, both of the switches concerned are short-circuited, as shown in Fig. 7. Before the further terminal has passed beyond the lapping portion of the two succeeding wires, the first switch will have closed and the succeeding switch will have been opened, thus diverting all of the current to the section upon which the motor has now entered.

Fig. 8 shows another arrangement by which a motor is enabled to pass from one section to another without interrupting the circuit. In this arrangement the ends of the sections, while mechanically continuous, are electrically disconnected, but do not lap as before. The break in the wires connected with the switch-actuating device is longer than that between the other two wires. The larger gap, however, is lapped by a subsidiary conductor, which short-circuits the switch actuating the section the motor is about to enter, as shown. As the car proceeds from left to right, before both its trolleys have passed off the section, one of its trolleys has embraced the subsidiary conductor and the electro-magnet controlling that section is cut out of circuit and the switch drops. While the switch is dropping, however, the current continues to flow through the companion wire, through the motor to the subsidiary wire and *around* the switch actuating the section it is entering.

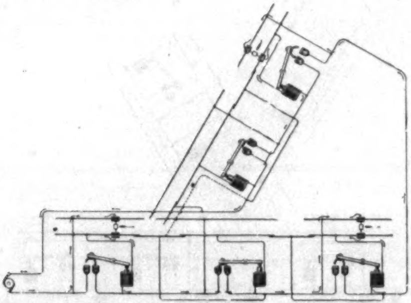


Fig. 8.

Fig. 8 represents a branched or forked road operated by a single dynamo. In the arrangement shown in Fig. 8, where the branch road is operated by the return circuit, a car turning to the left would really skip from the first section of the road to the very last, if we count them in the order in which they receive current, whereas in going straight ahead it would pass from section 1 to section 2, etc., counting them in the same order. In Fig. 9, however, which represents a similar road operated by the loop system, a car taking the left hand branch would pass from section 1 to sections 2 and 3, whereas if it went straight ahead it would pass from section 1 to sections 4 and 5. It will be apparent from what has preceded that in neither of these cases does the skipping of one or many sections involve an opening of the circuit.

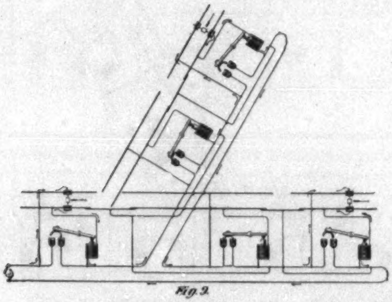


Fig. 9.

But returning to Fig. 8, and following the car still further in its progress from left to right. We left it standing with one trolley on the further wire and the other on the subsidiary or short-circuiting conductor. In this position the current will continue to pass through the motor only so long as the switch actuating that section requires to fall into the mercury cups. Immediately that has occurred, which will be accomplished before the further trolley has passed from its wire, the motor will have been short-circuited and the car will, by its acquired momentum, pass from the system without sparking or other disturbance of the circuit. If the acquired momentum should carry the car to another section of the same system, or to a section connected with an entirely separate system, all that is necessary to enable it to proceed would be some arrangement by which the switch controlling the section upon which it was entering, could be opened by the entrance of the car itself. It will be evident from Fig. 10, which represents two connecting roads operated by separate dynamos, that it matters not as far as the integrity of either circuit is concerned, whether the car goes to the left or to the right; in either case the circuit is closed before it leaves a section, and as it enters a succeeding section it does so before opening the switch which actuates that section, and it is entirely immaterial whether that section belongs to the same system as the one just left, or to one entirely foreign to it. If only the proper switch can be opened the car may proceed on its way on either system of roads. In order to accomplish this automatically, each car may be provided with some such an arrangement as is shown in Figs. 11 and 12: M is a solenoid magnet in series with the motor D; L is a lever pivoted at P and playing between two stops, but when not actuated by the plungers of M, kept in contact with the lower stop by the spring S; B is a local battery; W, W^1 are the trolley wires, and C is the mercury switch connecting adjacent sections of the feeder wire. As long as the motor is receiving current from the feeder, as is the case in Fig. 11, the current must pass through M,

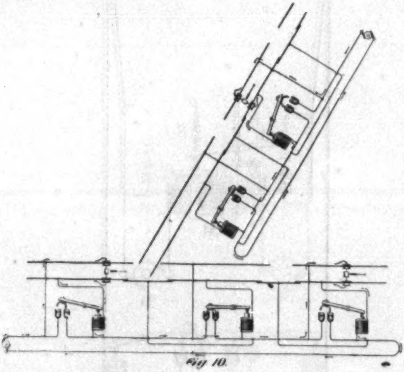


Fig 10.

which, by its pull on the plungers, maintains the lever L against the upper current. As the trolleys pass off from a given section and the motor loses its current as before described, the spring S causes the lever to make the lower contact (Fig. 12), thus throwing the battery B into the circuit W, F, C, F′, W′. The moment the trolleys enter upon a new section this circuit is closed, the local current passes through the feeder switch magnet and opens the switch C, and diverts the current from the feeder to the section just entered. This stronger current actuates the lever L, again bringing it against the upper contact and cutting the battery B again out of circuit. Since the battery B is only momentarily in circuit, the cost of maintenance will be insignificant.

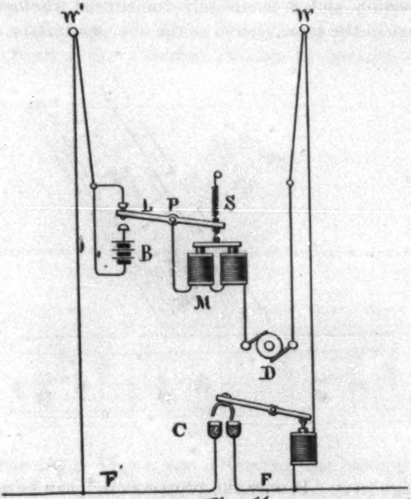


Fig. 11.

On single track roads where turnouts are required, the turnout constitutes merely an additional section to one side of the main line, which is skipped by the car keeping the main track, and taken by the other car in passing. It, like every other section, has its own individual controlling switch, and when both the main track and siding are occupied by cars, their motors are arranged in series with each other, as represented in Fig. 13. By following the arrows, the course of the current can readily be understood.

But both of the plans thus far discussed possess serious faults. In both we have provided a means by which a car may leave one section on a given road and enter another one on the same or another road without breaking the main circuit, *provided the car is always going in the same direction.* In all of the cuts thus far, the car is supposed to be going from left to right, and the switches are arranged accordingly. But if we examine the drawings again, it will be found that in the plan suggested in Figs. 5, 6 and 7, if the car were going in the opposite direction, the circuit would be momentarily broken as it left one section and would fail to divert the current to the section it was entering. In the second plan, outlined in Figs. 8, 9, 10 and 13, the car is momentarily without current whenever it passes from one section to another.

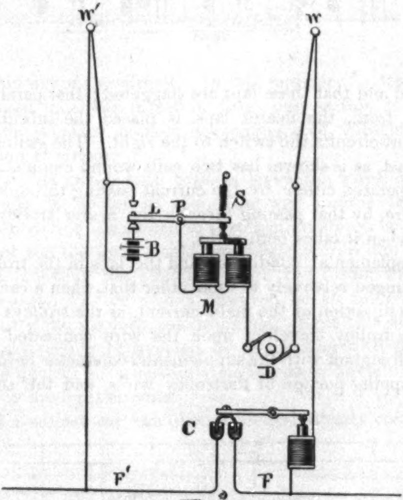


Fig. 12.

By combining the two plans, however, all of the difficulties seem to be removed. Figs. 14 and 15 show how this combination is effected, and the courses of the current when the motor terminals are in different positions on the wire. It will be noticed that, as in the first plan described, the trolley wires of adjacent sections are lapped and that these laps are staggered ; that parallel to, but insulated from, the nearer laps, is placed the subsidiary wire which short-circuits the switch to the right. The switch-controlling magnet, as is shown, has two coils wound cumulatively, and may be operated either by the current passing through the subsidiary wire, by that passing through the nearer trolley wire, or by both when it takes both paths.

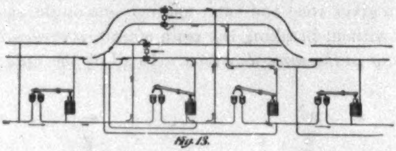


Fig. 13.

The supplemental conductors and the laps in the trolley wires are so arranged relatively to each other that when a car is traveling in the direction of the main current, as the trolleys approach a lap, the trolley traveling upon the wire connected with the switch will contact with the supplemental conductor before reaching the lapping portion of the trolley wires, and the companion trolley will reach its lap after the first trolley has reached its lap and before it has passed beyond the supplemental conductor, while with a car traveling in the opposite direction, the order will be reversed, the trolley upon the wire *not* connected with the switch, first reaching its lap after which its companion trolley reaches first its lap and afterward the supplemental con-

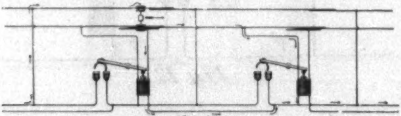


Fig. 14.

ductor. By this arrangement, in whichever direction the car may be going, in passing from one section to another, it first short-circuits the switch controlling the section it is about to enter; next diverts a portion of that current to that switch, and finally all of it, cutting the switch controlling the section it has left entirely out of the circuit. In this manner the transition is accomplished in either direction from one section to another, either of the same or another connecting road, without spark or interruption of circuit.

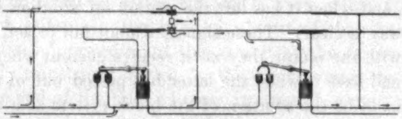


Fig. 15.

Should either one or both trolleys leave the wires, or the circuit through the motor in any other way become broken, the magnet controlling the switch to that section would be left without current and the forks would immediately drop into the mercury cups renewing the circuit through the feeder wire almost instantaneously. There would, however, be an interruption of the current while the fork was dropping. Even this slight interruption may readily be avoided by connecting with the switch any of the devices so successfully employed to obviate the same difficulty in arc light circuits.

Should a second car run upon a section already occupied, the two would be thrown in parallel and both would stop. This arrangement therefore constitutes a most perfect automatic block system, not dependent upon frail human nature, but sure and positive in its action. On interurban electric roads, where it is even now proposed to reach greater speeds than are customary or at-

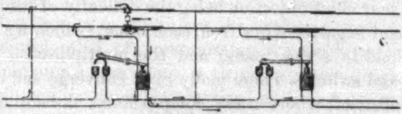


Fig. 16.

tainable with steam locomotives, the importance of this feature cannot be overestimated. Not only will collisions be rendered practically impossible, but by putting semaphores or other signalling devices in the wires connecting the main feeder with the two trolley wires, each car as it enters a section will automatically set a signal at each end of that section showing that the section is occupied, and when it has left the section set another, indicating that the way is clear. These signals cannot fail to act, for being in series with the motor, they *must* receive current when the motor does, and lose it when the latter has passed out of the block. This seems to be the solution of the block system as it eliminates all the uncertainties inseparable from a dependence upon human intelligence and watchfulness.

This system will no doubt be criticized first, on the score that it is a double trolley system which many are opposed to on general principles, and second, on the score of the high potentials necessarily involved in series distribution to long distances. Anticipating these, I will say in regard to the first, that however valid the objections may be to the double trolley on multiple arc roads, they have but little force as applied to series circuits. In the latter, since both the wires on all sections not occupied by cars are of the same potential, the difficulties of insulation of the one from the other disappear entirely. Then again, I am satisfied, with an intimate knowledge of the electric roads in Cincinnati, where with one exception they are double trolley multiple arc roads, that most of the objections raised against them are greatly magnified. They certainly have the decided advantage over the single trolley—of being in their operation independent of the condition of the track. Their chief objection being the difficulty of insulating the positive and negative wires from each other, (which we have seen does not hold in series roads) and the multiplication of wires, crossings and switches where many lines converge and cross. In suburban districts where these complications do not occur (and this system is particularly adapted to such) the double trolley multiple arc system is in many respects superior to the single trolley. Mr. John Kilgour, President of the Cincinnati Street Railway Company, is justly proud of the Avondale double trolley road (a suburban line), and I doubt if there is a more successful road in the country than this.

In regard to the other objection to the series method, viz., the high potentials, the method outlined in this paper permits the use of any number of feeders, each of which may be used to supply a different portion of the road, thus reducing the potential, which in series distribution is the variable quantity, in the same manner exactly that the " drop " is obviated in parallel systems.

It is perfectly clear to the members of the Institute that this question of potential on series circuits is very much misunderstood by the laity. In the parallel system of distribution, the high potential impressed upon the circuit at the dynamo is carried to every translating device on the circuit, even to that at the extreme end of the line, whereas in a series circuit, the high potential spoken of, exists only at the dynamo terminals and is the sum of the electro-motive forces consumed by all the resistance on the line.

As an illustration in point, supposing we have two circuits—one parallel and the other series, the differences of potential between the terminals of the two generators being exactly the same—let us say 2,000 volts. On the multiple arc circuit, no matter how insignificant the translating device may be or where it is located, one cannot handle its terminals without subjecting himself to a shock due to 2,000 volts. If the motor be generating one H. P. or 100, the danger is exactly the same. On the other hand, in the series system, if we are using a current of say 40 amperes, which is a convenient one for street car purposes, if the motor be generating one H. P., the difference of potential between its terminals will be a little less than 19 volts. If it be generating 100 H. P. it would be but 1865 volts, and before it would be as dangerous to handle as a motor on the multiple arc circuit, it would have to be doing work equivalent to 107 H. P.

Not long since, there was a very animated discussion at a meeting of the Institute as to which was the better method of electric street car control—that of commutating the field, or the other one, of employing a rheostat. Both sides of the question were ably handled, and the discussion resulted as discussions usually do when the subject is either politics or religion, viz., in the making of no converts. Both sides to the controversy admitted, however, that in either case there were disadvantages, and the question practically resolved itself into " which was the lesser evil ?" The constant current system simplifies matters at least in this respect, in that neither commutated fields nor external resistances are required in regulation.

As regards economy in transmission of energy to long distances by the constant current method, we have high authority in Silvanus P. Thompson, who, in his "Dynamo Electric Machinery" (edition of 1888), p. 531, says:—"Now, the method of distribution by a constant current is, where power is to be transmitted to long distances, *a much more economical method* than that of distribution with a constant potential, owing to the fact that to the former method thinner, and therefore less expensive conducting wires may be employed."

He might have gone much further than this. As to the cost of installation, the comparison is much in favor of the constant current method on account of the extreme simplicity of the plant required, and the smaller size of both motors and dynamos for the same output. But the economy in plant becomes most obvious where high potentials are used, and these are always necessary where long distances are concerned.

But important as are the economies in installation and in transmission of large energies to long distances, there is another that greatly outweighs them, viz., the economy in operating expenses. In this respect the series method is almost an ideal one. In the parallel or multiple arc method, when a car is going downhill or is being checked, the whole of its $\frac{m v^2}{2}$ is absolutely thrown away in the application of the mechanical brake. Not so with the series system. The mechanical brake need never be used except in the case of an emergency. By merely reversing the position of the brushes, the motor becomes, for the time being, a dynamo in series with the one at the power station, driven by gravity or the acquired momentum of the car, and the energy thus absorbed, while checking the car or bringing it to a standstill, is thrown onto the line for use elsewhere. One of the arguments used in favor of the cable road is that a car descending a grade assists another one which is ascending. If this be an argument where at least 75 per cent. of the engine power is consumed in dragging the cable alone, how much more valid should it be if the same thing could be accomplished where there was no cable to drag. Such, in fact, is the case with the series method. A descending car or one being checked assists another car that is ascending a grade or being started from rest, just as truly as it would if the two were connected by cable, and the energy thus thrown onto the line continues to contribute to the economy of the system to the last turn of the wheels. This is not mere theory. Professor Short demonstrated its actuality on the South Broadway line, in St. Louis, by disconnecting the circuit entirely from the dynamo, and causing one car to partly ascend a grade on one part of the line by electrically braking a descending car on another part of the line. It has also been demonstrated in England. Mr. Edward Manville, M. I. E. E., in a paper on "Series Electrical Traction," read before the British Association, in September of 1888, says:

"It may be well to point out in series running, that owing to the employment of current of constant value, it is impossible for the most inexperienced car driver to damage his motor by either too rapid starting or by reversing while running. Indeed, it is a positive advantage, when descending a hill to check the speed of the car by altering the field connections so that the armature tends to revolve in the *opposite* direction to that in which the car is traveling, as the power that would otherwise be lost in braking the car is actually added to that produced by the generator, and thus large systems with many cars and varying gradients would appreciably reduce the total power required to be generated, and the consequent consumption of coal. At Northfleet, they could actually turn the engine back against the steam, when they were going down hill; that is to say, the power developed by the car going down hill, passing through the circuit, is more than at the engine at the moment."

The importance of utilizing electrical brake power, which is impracticable in the parallel system, is emphasized by some experiments on the Third Ave. Elevated R. R., New York, insituted by Mr. Frank J. Sprague, to determine in what manner the power generated was utilized. The result of these was that 83 per cent. of all the power utilized was consumed in overcoming gravity and the inertia of trains, and only 17 per cent. was actually consumed in traction.

I have endeavored in this paper to show both the advantages and disadvantages in the constant current method of distribution compared with the parallel system. I think that many of the disadvantages which have heretofore been regarded as inherent in the series system are not so, and I have ventured to suggest a method by which many of them that were so grave as to preclude the utilization of the constant current in electrical traction, may be remedied. I am glad to have the opportunity of bringing this subject before such an assemblage of practical men—all the more practical because they are theoretical.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 8th, 1892, President Sprague in the Chair.

A NEW SYSTEM OF ELECTRIC PROPUSION.

BY H. WARD LEONARD.

In the distribution of electricity from a power station for the operation of electric railways, the only commercial method to-day is by the use of a system of constant E. M. F. operating the motors in multiple arc with each other, and at the present time every consideration of economy and automatic regulation seems to indicate that the constant E. M. F. multiple arc system will always be the best for such distributions.

In the use of electric energy by motors operating under conditions of varying speed and torque, the best results as regards economy and regulation are obtained when the electric energy utilized has a voltage varying directly as the speed, and a current varying directly as the torque, for it is evident that under these conditions the electric energy required will be always proportional to the power developed.

If we could operate, from the constant potential system, a shunt wound motor running at a constant speed, and could interpose between this motor and the axle some device equivalent in its effect to an infinite number of different sets of mechanical gears, so that we could make use of any reduction desired, it would enable us while using a constant power to increase the torque as we decreased the speed and *vice versa*, which is just what is desired in railway practice where the least torque is required when at full speed on the level, and the greatest torque is required at the slow speed in starting and in operating on a grade. Numerous and very ingenious devices have been invented for accomplishing this variable mechanical reduction, but on account of the complication, noise and unreliability they have never proved successful.

The writer has recently devised an electrical method of securing all the results which could be obtained from such a set of gears described, with a freedom from the noise, wear, complication and rigidity which such a set of gears would necessarily involve.

Following is a general description of the arrangement proposed, as indicated by Fig. 1:

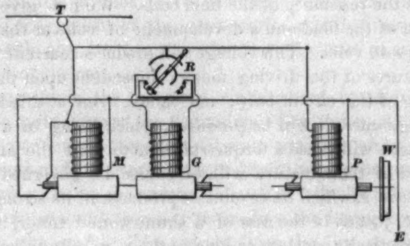


FIG. 1.

T,	trolley.
M,	motor portion of power converter.
G,	generator portion of power converter.
P,	the propelling motor for the car.
R,	the regulation and reversing rheostat in field of G.
E,	the connection to ground.
W,	the car wheel.

Each axle is driven by a gearless motor, either directly or by means of a connecting-rod. The fields of these motors are excited directly from the constant E. M. F. of the line and independently of the armature circuit. Beneath the car and between the axles there is suspended a motor-generator, each armature winding being in a seperate field The motor portion of the motor-generator—which will, for convenience, be called the power converter—is shunt wound and connected just as a shunt motor is for use upon ordinary constant potential circuits. The field of the generator portion of the power converter has its field connected across the line and has inserted in it a regulating and reversing field rheostat. This field circuit is independent of the armature circuit. The generating armature of the power converter is in metallic connection with the armatures of the gearless propelling motors. It will be noticed that this circuit including the armature is a distinct and separate metallic circuit having no connection with the line in any way.

Suppose now that our shunt motor is running at full speed and that our controlling rheostat in the generator field circuit is at its central position, so that the generator field circuit is broken. Although the generator armature is being driven at full speed it is revolving in a field having no magnetism except the residual magnetism, and hence produces practically no volts. Let us now move our controlling switch so as to place the generator field across the line, but with a resistance in series with the field, of ten times the resistance of the field coils. We now give a slight excitation of the field and a development of volts at the brushes of perhaps 40 volts. This voltage will produce a current through the armatures of the driving motors dependent upon the ohmic resistance of this circuit only; and hence, even at this low voltage, a large current will be produced, which being in a field of full strength will cause a torque sufficient to start the armature. The speed of the armature will, of course, be governed by the counter E. M. F. which its revolution produces in its strong field; and hence, just as in the case of a shunt wound motor, its speed will be practically constant so long as the E. M. F. supplied is constant.

If we now gradually increase the magnetic field of the generator by cutting out resistance by moving the controlling switch, we will gradually raise the E. M. F. of the armature circuit, and with it the speed of the driving motors. Since these armatures are revolving in a constant field, the torque they produce will be exactly proportional to the current in them, and the current will automatically flow exactly as is required to produce the necessary torque to maintain a speed such that the counter E. M. F. will approximately equal the E. M. F. supplied by the power converter. Thus it will be seen that the speed of the car will be dependent upon, and proportional to the E. M. F. supplied by the power converter, and the torque or tractive effort will be dependent upon, and proportional to, the current supplied by the power converter.

Let us suppose that 60 amperes flowing through the armatures in fully excited fields will produce a torque sufficient to move the load when upon a grade. It is evident from what we have seen that 40 volts from the power converter will produce this current. Hence, by an expenditure of 5,400 watts in the secondary circuit, or a total power, including field excitation, etc., of about eight H. P. we can start a fully loaded car upon a grade.

Under the existing systems, we would need the same 60 amperes in the same fully excited field, but would necessarily use the full voltage of 500 volts, and, therefore consume energy represented by 30,000 watts, as against possibly 6,000 in this system. The current from the line in starting the car under ordinary conditions by this system would be about 12 amperes at 500 volts, instead of from 60 to 100 amperes at 500 volts.

In practice, the controlling switch lever can be instantly thrown from its central position to its extreme position for full speed. The field magnetism of the generator is rapidly increased, and consequently also its E. M. F., which in turn causes a gradual acceleration of the car.

The current in the armature circuit, and consequently the torque, is quite large in the beginning; but the E. M. F at this time is quite low, so that the total watts are low. As the inertia is overcome and the counter E. M. F. begins to approximate to the impressed E. M. F., the current falls off and finally becomes constant at an amount necessary to produce the torque required to maintain the speed. The current from the line, and hence the power, gradually increases from zero to the amount required at full speed, but at no time, either at the start or during the acceleration, is the energy from the line greater than that required when we are operating at full speed. It will be noticed that the effect is the same as though we first operated through a set of gear wheels, giving an extremely great reduction of speed and then rapidly changed the ratio of gearing, until finally we operated at full speed, with no reduction.

With our hypothetical gears we could, when running at speed, rapidly increase the ratio of gearing, so that the movement of the car would tend to drive the shunt motor faster and faster This would convert it into a generator, forcing current back into the system, which production of electrical energy would act as a brake and gradually bring the car to rest.

Just so, if, while running at full speed, we suddenly place our switch lever at its central position, the field of the generator will gradually reduce the strength, and the counter E. M. F. of the propelling motors will soon exceed that of the generator. The momentum of the car will now be driving our gearless motors as generators, which will supply current to the former generator, operating it as a motor, causing it to drive the shunt motor coupled to it as a generator, which, in supplying energy to the line, will act as a brake, and smoothly but rapidly bring the car to rest by converting the energy stored up and represented in the movement of the car into electrical energy, which will tend to relieve the work at the central station. Similarly, a car descending a grade and tending to accelerate in speed, can be made to move at any desired speed without the aid of any mechanical brakes, and the energy represented by its falling weight will be converted into electric energy and the car will become a moving feeder supplying energy to assist the generators at the central station in the operation of other cars.

It will be evident from what has preceded, that with this power converter system we can propel a car upon any practicable grade with a consumption of power no greater than is required to operate the car at full speed upon a level, by merely reducing the speed to the required extent.

In street railways of from five to ten cars, this is of great importance, for it means that we can equip a road with about six H. P. per car, as regards the engines and dynamos, and that our conductors can be reduced to about one-third of the amount at present necessary, for we will never require more than 20 amperes at the distant point, where to-day we have to provide for 60 amperes, with the same loss and the same initial E. M. F.

Under the rheostat system, the plant is severely taxed when an unusual crowd must be moved from a certain point. and it is then, when it is of the greatest importance that no break-down should occur, that it usually does occur. With this power converter system we could, upon a five-car road, start up and move with perfect safety 10 or even 20 cars from the most distant point on the road, though of course at a reduced speed, but the crowd would be handled with perfect success and without subjecting any portion of the plant to any unusual strain.

In the large cities it is no unusual sight to see an electric car moving at the slowest possible speed for perhaps several blocks. Perhaps 12 amperes are required to obtain the necessary torque. This, at 500 volts, is 6,000 watts. The power required for this slow motion by the proposed system would not exceed one-fifth of this amount

Following is a tabulated statement (Table 1) showing the results we may expect to obtain by this system in operating with fully loaded car under three different conditions:—First, at 12 miles per hour on level; second, at three miles per hour on five per cent. grade; and third, at one mile per hour on level.

LEONARD SYSTEM OF ELECTRICAL PROPULSION.

TABLE 1.

DUTY OF CAR. SHOWING VARIOUS LOSSES EXPRESSED IN WATTS.

Various Losses Involved.	8 Tons at 12 Miles per Hour on Level.			8 Tons at 3 Miles (or 5 Tons at 5 Miles) per Hour on 5 per cent. grade.			8 Tons at 1¼ Miles per Hour on Level.		
	Full Speed, 1-6 Full Torque; Armature Current, 10 Amperes.			¼ Full Speed, Full Torque; Armature Current, 60 Amperes.			1-10 Full Speed, 1-6 Full Torque; Armature Current, 10 Amperes.		
	Power Converter.		Driving Motors.	Power Converter.		Driving Motors.	Power Converter.		Driving Motors
	Motor part.	Gen. part.		Motor part.	Gen. part.		Motor part.	Gen. part.	
Field.	250	275	250	250	60	250	250	25	250
C^2R in armature ...	160	60	60	250	2000	2000	20	60	60
Friction.....	60	60	120	60	60	30	60	30	10
Foucault currents, hysteresis, etc.	200	400	400	200	50	50	200	10	10
Total.	670	795	830	760	2170	2330	530	125	330
Total watts wasted . .	.....	2295	......	......	5260	....	......	985	....
Watts of work done	.....	4000	....	......	6000	....	.. ..	400	... .
Total watts absorbed .	......	6295	....	......	11260	....	... ..	1385	....
Amperes at 500 v. . .	......	12.6	....	......	22.5	....	......	2.8	... .

In arriving at the losses, as indicated, the motor part of the power converter has been assumed as having the following features:

E. M. F. 500 V., current capacity for ten hours' continuous duty, 15 amperes; resistance of shunt field winding, 1,000 ohms, armature resistance, 1.1 ohms.

The generator portion of the power converter and the driving motor are assumed as having the following features:

E. M. F. 500 V.; current capacity for ten hours' continuous duty, 40 amperes; resistance of field, 900 ohms; armature resistance, 0.55 ohms.

The rolling friction with gearless motors on good level track is assumed as 20 lbs. per ton. Car is assumed to be eight tons in weight full loaded and five tons for moderate load.

We find that with 12 tons moving at 12 miles per hour on a level we will require 12.6 amperes which is practically the same as by present series motor systems. With eight tons moving at three miles per hour upon a five per cent. grade 22.5 amperes will be required, which is about one-third of the power required by present systems. With eight tons at one mile per hour on level 2.8 amperes will be required, which is about one-fifth that of present systems. With five tons moving at five miles per hour on five per cent. grade 18.5 amperes will be required, which is about 40 per cent. of the power required by present systems.

Let us examine some of the advantages that this method seems to offer over the existing methods, starting at the car and considering the entire equipment back to the boiler.

In order to place before you the opinions of some of the best authorities on the questions involved, I shall quote freely from "The Electric Railway," (Crosby and Bell); Parshall's "Methods of Electrically Controlling Street Car Motors," "Comparative Test of High and Low Speed Engines in Electric Railway Work," by Charles W. Wason (*Electrical Engineer*, April 27, 1892); "The Practical Operation of the Gearless Motor," by S. H. Short (*Electrical World*, April 16, 1892); "Load Diagrams of Electric Tramways and the Cost of Electric Traction," by A. Reckenzaun (*Electrical Reveiw*, London, March 25, 1892).

The cost of car equipment will be increased by the cost of the motor-generator, but as a partial offset to this we have saved the rheostats, two expensive controlling switches and a complex system of wiring.

Our motors, having constant and fully-excited fields, will operate absolutely without spark under all conditions. The control of the car will be entirely accomplished by a small switch and rheostat, handling never more than one-half of an ampere, and occupying a space of one foot square and one inch deep over all.

As regards efficiency, we will have the advantages of the present system under all conditions. For long runs upon the level we will, by a suitable switch, connect the driving motors directly to the line and secure an efficiency of 90 per cent. for our motor.

As regards depreciation, we will have the advantage of no rheostats or controlling switches to burn out: and with no sparking and no connection with the field circuit we will have the minimum liability of burning out armatures.

Our fields will have no tendency to burn out, since they are not subject to the excessive currents which the present series fields are. The current in our fields will be independent of the load.

As to field windings and rheostats in existing methods, Parshall says:

"With 25 H. P. motors, an external resistance of 10 to 12 ohms is required."

"Lessening the duty of the rheostat is a very important point, since as yet it has been found exceedingly difficult to construct a cheap rheostat that could be placed under the car in the small space available and dissipate so large an amount of energy as is required when the car is to be run for a considerable time at a speed so low as two or three miles an hour. Any method of control that has lessened the energy to be dissipated in the rheostat has in general been considered with favor, since there has been a corresponding diminution of trouble in each case that the energy to be dissipated has been lessened."

"The range of speed without the use of a rheostat is determined by the limit to which it is safe to heat the magnets."

Crosby and Bell say:

"In using this method (commutated fields), the principal difficulty has been met with in disposing of the excessive heat necessarily generated in the compact mass of field windings."

"The practical problem has been to secure a convenient rheostat."

"The principal sources of loss in our present street railway motors are the regulating devices and the gearing."

"With the motors and the gearing generally employed, the average commercial efficiency of the combination is probably not often in excess of 65 per cent., giving a total commercial efficiency for the system, from engine to car wheel, of 39 per cent.; this of course is but an estimate. But taking all the factors into consideration, it is probable that the average of the roads now in operation would fall quite nearly to the point indicated. In very few cases would it fall below 30 per cent.; in still fewer would rise about 40 per cent."

Regarding the power required to start a car on existing methods and to operate it upon level and grades, Crosby and Bell say:

"With the ordinary car equipment of two 15 H. P. motors, and the usual speeds, from eight to twelve miles per hour, experience has shown that five to six electrical H. P. is necessary on nearly level tracks."

"The amount of current ordinarily taken in starting a car, is momentarily more than 50 amperes, which, at the ordinary voltage, corresponds to about 25,000 watts."

Reckenzaun says:

"If we calculate from the accepted coefficients of resistance to traction on common tram-rails, we find that an ordinary tramcar will require but three to four H. P. for its propulsion when once in motion."

He says of Thomson-Houston car:

"The maximum current at any time was 75 amperes."

Of Sprague motors:

"Here, again, we observe a maximum current of 95 amperes."

"Westinghouse motors—maximum current, 75 amperes."

Short finds that 80 to 100 amperes are required to start a car, and says:

"On this road the traffic is very heavy, although grades are light."

Leaving the car, let us now consider the line. It will be evident from what we have seen that we can reduce the amount of copper to one-half the present requirements, as we never will require the enormous currents at present called for in starting and upon heavy grades. Or, to put it in another way, with the existing conductors, we could run twice as many cars as at present, with the same loss in the conductors.

Now let us look at the generators and the prime movers, whether steam engines or water wheels.

Under existing systems, for roads of from five to ten cars, it is necessary to install about 20 indicated H. P. (rated at one-fourth cut-off) per car, and about 16 kilowatts per car in generators. Also about 20 H. P. per car in boiler capacity. This large equipment is necessitated by the occasionally very large demands for power and the inefficiency consequent upon this.

Under the proposed system, it is not necessary to provide power in excess of 15 H. P. for any car under any conditions, and since in practice most of the cars will be operating at less power than this, we need only install engine, dynamo and boiler capacity of 8 H. P. per car, instead of 20. Or, to express it in another way, we can operate with existing boilers, engines and dynamos at least double the number of cars they can at present supply.

Crosby and Bell recommend, for a five-car road:

"An equipment consisting of two 40,000 watt dynamos, one 80 H. P. high-speed, simple engine, belted directly to them, and two boilers of about 50 nominal H. P. each."

Now let us look at the economy of the operation of the station. With the extremely fluctuating loads of existing systems, the economy of the entire generating plant is very low. The stations of three roads which have been tested, give for the combined efficiency of engine and dynamo 40 per cent., 54.6 per cent. and 62.8 per cent. respectively. If the load can be kept approximately constant, the combined efficiency of engine and generator should be about 75 per cent., and in the proposed system the load will be sufficiently uniform for us to expect an efficiency equal to this and because of the nearly constant load we can produce a horse power on about 25 lbs. of water, while in present practice for small roads about 50 lbs. of water per horse power is a fair figure; and the best published result thus far obtained, even when the average horse power rose to 750 H. P., is 28 lbs. per H. P., as found by Wason, at Cleveland, in operating a total of 71 motor cars.

With the present systems the average indicated horse power per car is about 12 horse power, which, on account of fluctuating load, requires at least 36 lbs. of water per horse power or about 420 lbs. of water per car per hour.

With the proposed system we will operate with an average of about eight indicated horse power per car, which, on account of the steady load, will be produced with about 25 lbs. of water per horse power, or 200 lbs. of water per car per hour. That is, we require about 50 lbs. of coal per car by present systems, and about 25 lbs. of coal per car per hour by proposed system, or a saving of 50 per cent. in the coal and water required in favor of the proposed system.

On this subject of fluctuating loads and their effect, Crosby and Bell say:

"A record of ten minutes on a recording ammeter may give some faint idea of the condition of things. It will be seen that at one point the output jumped from zero to 150 H. P. and back inside of a single minute, and during the latter five minutes shown in the diagram there were no less than 25 sudden *variations* of 50 to 100 H. P., each taking place within a few seconds. The road from which this record was obtained is four miles in length, and was operating seven cars at the time of the test."

Reckenzaun says:

"These abrupt changes have the effect of reducing the efficiency of the whole system to a comparatively low figure."

Church says (*Electrical Engineer*, April 27, 1892) that the best compound engines will show an economy of only 28 lbs., and the usual compound engine "an average duty not better than 35 to 40 lbs. The same is true of every form of non-compounded engine, whether high-speed or low speed, both of which show a tremendous falling back of fuel duty under variable load."

Let us now examine the comparative first cost of a railway of moderate size, say, from five to ten cars, equipped by present systems and by the proposed system.

The detailed figures per car are given in Table 2.

TABLE 2.

SHOWING PROBABLE COMPARATIVE FIRST COST PER CAR BY PRESENT AND PROPOSED SYSTEM.

	Present System.	Proposed System.
Steam plant, generators and conductors per car (steam plant 1,000, generators 700, conductors 500	2200	1100
Motors (2 15 H. P. equipments)	1800	1400
Power converter	0	900
Controlling switches, cables, rheostats, etc.	200	30
Total first cost per car	4200.	3430.
Saving in favor of proposed system per car		$770.

Table 3 gives a summary showing the features of the proposed system as compared with the corresponding features of the present system.

TABLE 3.

	Present System.	Proposed System.
First cost of steam plant, generators, conductors and car equipment per car	$4200.	$3430
Amperes at 500 volts required to start full load on level	75	2.8
Amperes at 500 volts for full load at full speed on level	12.5	12.5
Amperes at 500 volts to start full load on 5 per cent. grade	125	10
Amperes at 500 volts for working speed on 5 per cent. grade	60	22.5
Amperes fed back to system in coming down 5 per cent. grade	0	10
Pounds of coal per car per hour	50	25

The features of the proposed system which seem at first sight to be very objectionable are :—The increased cost of the car equipment, and the fact that we are adding an additional machine, having two fields, two armatures and three bearings; but, as we have seen, there is only an apparent increase in the first cost, for the saving in the generators and distributing plant far exceeds the additional cost of the car equipment; and the use of the motor generator for elevators, traveling cranes, etc., has demonstrated that, as regards the attention it requires and the depreciation it suffers, it has a marked advantage over the rheostat or commutated field used in the present methods of operation.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 8th, 1892, President Sprague in the Chair.

ELECTRIC RAILWAY MOTOR TESTS.

BY PROF. GEO. D. SHEPARDSON AND EDWARD P. BURCH.

The experiments here discussed were undertaken for the purpose of obtaining definite information about the performance of electric railway motors under various conditions. There is a diversity of opinion about the relative merits of regulation by rheostat and by commutated fields, the use of one or two motors on a car, value of wetting rails, causes of "bucking" and other points. It is commonly understood that electric roads using rheostat regulation require larger capacity at the generating station than those using the commutating fields. It is also known that one large road has substituted rheostat regulation for commutated fields on all its motors. One large road has reduced the pressure on the lines to a figure considerably below 500 volts, with marked reduction in the repair bill.

Much has been said and written, but the data published are meagre and in some cases misleading. It was therefore thought desirable to obtain data from careful tests under certain conditions in the hope of subtantiating some of the claims made, and of bringing out some new facts. The tests were conducted for the most part by Edward P. Burch, a senior in electrical engineering in the University of Minnesota, in connection with his graduating thesis, and by courtesy of the Minneapolis Street Railway Company, especially the master mechanic, William Cooper. Most of the tests were made upon a No. 6 Sprague double reduction motor of the type described by H. F. Parshall before the Institute May 21, 1890, and April 19, 1892. Some experiments were also made upon a single-reduction motor. Each machine is used at the repair shop for testing armatures, being anchored to a heavy crib sunk eight feet into the ground. Each car axle is replaced by a short shaft carrying a heavy cast-iron wheel with solid web, upon which two oaken blocks are bolted for use as a brake. This was made into a Prony brake by removing the angle irons that held it to the floor and bolting on a plank to which an iron plate was fastened, so as to give a knife-edge bearing, at a distance of 63.1 inches from the centre of the shaft. This plate rested upon a knife-edge supported upon a platform scale by which the turning moment could be measured.

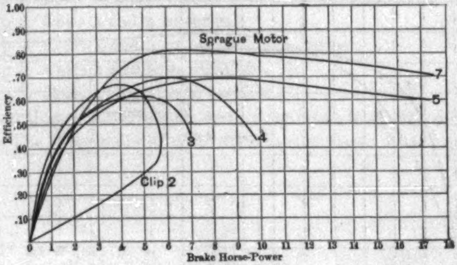


Fig. 1.

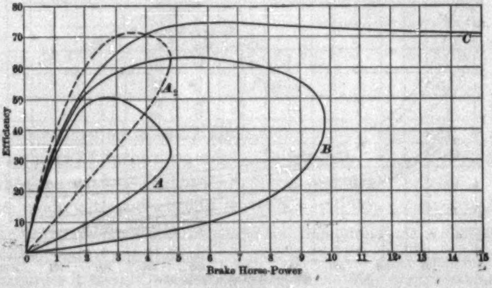


Fig. 2.

The mechanical horse power delivered at the car wheel is

$$\text{H. P.} = \frac{2\pi R P N}{33{,}000} = \frac{P N}{1{,}000},$$

in which *P* is the pressure on the scale, *N* the revolutions of the car axle per minute. The speed was measured by a common speed indicator with soft rubber tip to prevent slipping. For slow speeds a stop watch was used, the time for five revolutions of the car wheel being noted. Current and potential were measured by new Weston instruments, about 12 feet distant from the motor.

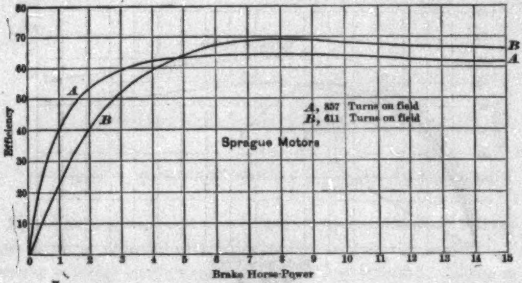


Fig. 3.

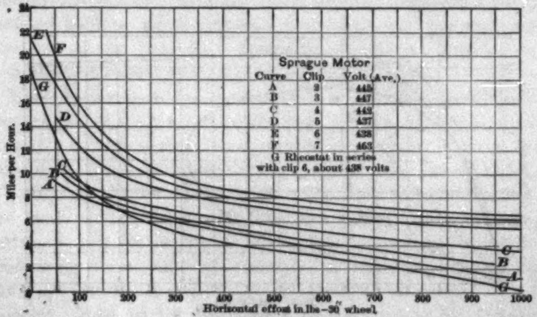


Fig. 4.

The Sprague motor armature has 56 sections of eight turns each of No. 12 B. & S. G. wires. The speed is reduced 11.765. The field coil as used at first was in three sections—coil A, 700 turns; coil B, 820 turns; and coil C, 620 turns—all being No. 12 B. & S. G. wire. By means of the barrel switch these coils are thrown into the following combinations:

Clip 2. Coils A, B and C in series.
Clip 3. Coils A and B in series.
Clip 4. Coils A and B in parallel, C in series.
Clip 5. Coils A and B in parallel.
Clip 7. Coils A, B and C in parallel.

Several sets of efficiency tests were made with each combination. The three pairs of coils were then replaced by a single pair of coils connected in series and having several terminals, so that the number of turns used might be altered. This was used in connection with a Thomson-Houston rheostat. Efficiency tests were made with and without the rheostat. Only a few of the tests and curves obtained can be given in this paper. In comparing these curves among themselves and with the result of other experiments, it should be noticed that the voltage was considerably below that for which the motors were designed, and that the power delivered was measured at the car axle. Different armatures were used on different days, and the curves in different places are not strictly comparable; but the curves given in any

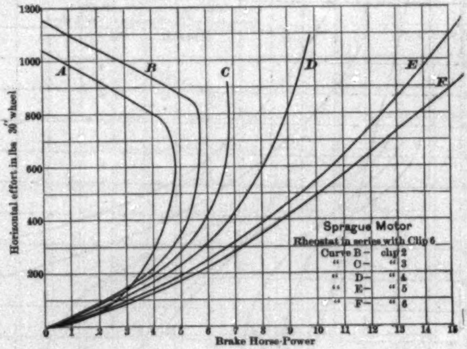


Fig. 5.

one place were taken with the same armature and with approximately the same voltage. Each of these armatures had been recently repaired, and although they had been well baked, there may have been some leakage through the green shellac. Foucault currents in the cores were probably greater than in new armatures. With the same armature and fields, the efficiency is a few per cent. higher cold than when hot. It has been the aim to run as nearly as possible at the temperature of ordinary running, so that the fields were comfortably warm but not hot. It is confidently believed that the efficiency of these machines would be

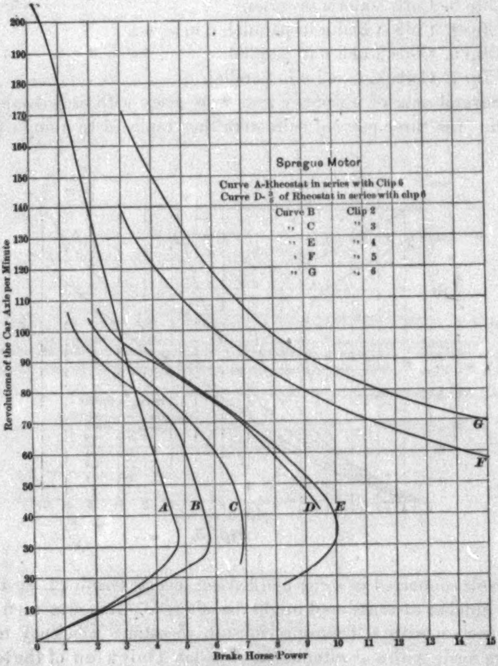


Fig. 6.

considerably above 90 per cent. if taken at armature shaft and with full voltage.

Fig. 1 shows efficiency curves for the motor with different combinations of the field coils.

In Fig. 3 curves A and B show efficiency of the machine with

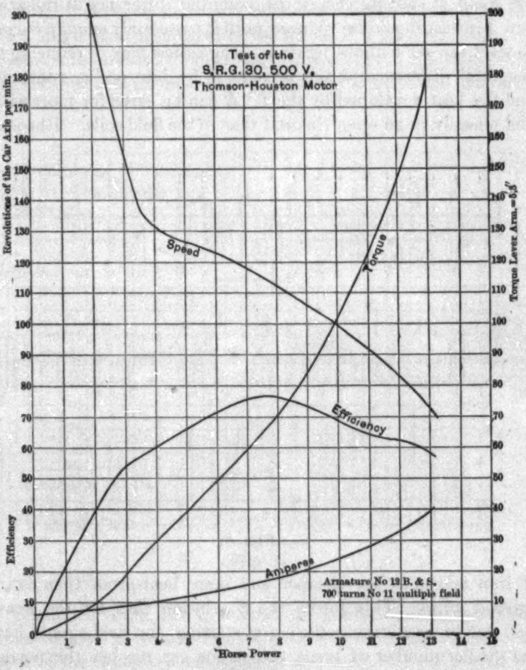


Fig. 7.

857 and 611 turns of No. 7 wire in the field coils. It will be noted in Figs. 1 and 3 that the motor has highest efficiency with the larger number of turns on field coils until the iron is saturated when the better efficiency is given with smaller number of turns and consequent lower resistance in field. This shows the advan-

tage of working on the fifth and seventh clips with barrel regulator, or of cutting out part of the turns on field when the load is heavy. On starting the car, the current is very heavy and the field is saturated with a small number of turns; hence the torque with a given current is nearly the same whatever the windings of the field. The speed and counter-electromotive force are small at starting; hence the potential difference at the armature brushes should be reduced until it passes only enough current to start the car without jerking. The easiest way of reducing the potential difference at the brushes is by interposing resistance, and the heat developed by the $C^2\,R$ can be cared for more easily and cheaply in an open rheostat than in the field coils. Rheostats of iron and mica are cheaper and more heat-proof than cotton covered wires. This forces the conclusion that for the heavy load due to accelerating the car at starting, the field should have the smaller number of turns, and as the car reaches the normal speed, and the horse power and current becomes less, the efficiency of the motor would be raised by increasing the number of turns in the field. This conclusion is exactly the reverse of the common practice of manufacturers who sacrifice some efficiency for speed at small horse powers. It is also in line with the experience of the road mentioned, that replaced the commutated fields with a single series coil of No. 7 wire, and later with No. 5 wire.

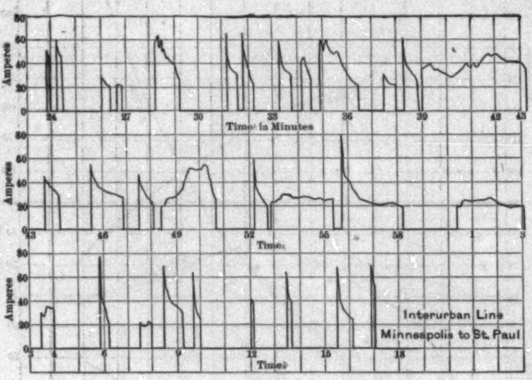


Fig. 8.

This heat loss could be saved, and the average efficiency raised, by using the two motors in series at the start, and afterwards cutting out one, or putting the two motors in parallel if necessary, as the speed increases and horse power decreases. An average of 12 tests on the road shows that with two motors on a car it requires twenty-seven per cent. less current if only one is working.

In Fig 2, curve A shows the efficiency of the Sprague motor with a single pair of coils, when the whole of a Thomson-Houston rheostat is in circuit; B shows about three-fifths of the rheostat in use; and in C it is all out. The dotted line A^2 gives the efficiency as calculated from the difference of potential about the motor alone. Curve A^2 is not strictly comparable with the others on account of the necessary variations in the voltage at the motor, but it does show the increased efficiency to be obtained by utilizing the superfluous voltage in a second motor in series. In this connection it is interesting to note that the resistance of the rheostat as calculated from current and potential difference, decreases with increase of current. This is as expected, since the larger part of the resistance is due to surface contact between the plates of iron in the rheostat, these being pressed together more closely as the

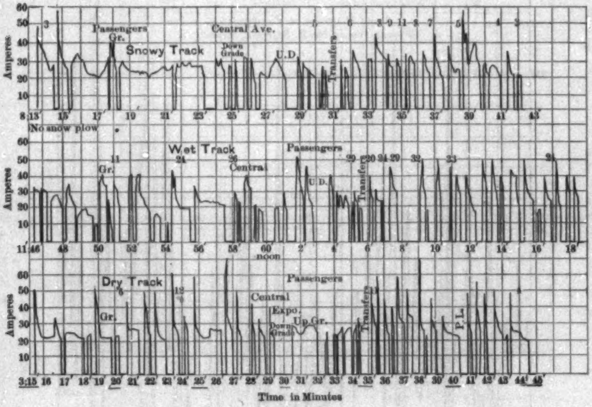


Fig. 9.

iron heats. A similar effect, but much more marked, has been noticed in earlier experiments by one of us, with alternate plates of carbon and iron.

Figs. 4, 5 and 6 show curves of torque, speed and horse power, referring to the No. 6 Sprague motor. In Fig. 5, A shows horse power and horizontal effort on a 30 inch wheel; B, C, D, E and F clips 2, 3, 4, 5 and 7, respectively. In Fig. 6, A shows speed and horse power, with all of the rheostat in; E with three-fifths of rheostat in; B, C, D, F and G clips 2, 3, 4, 5 and 7, respectively. These may be compared with results presented by Professor S. H. Short before the Chicago Electric Club, March 28, 1892. Fig.

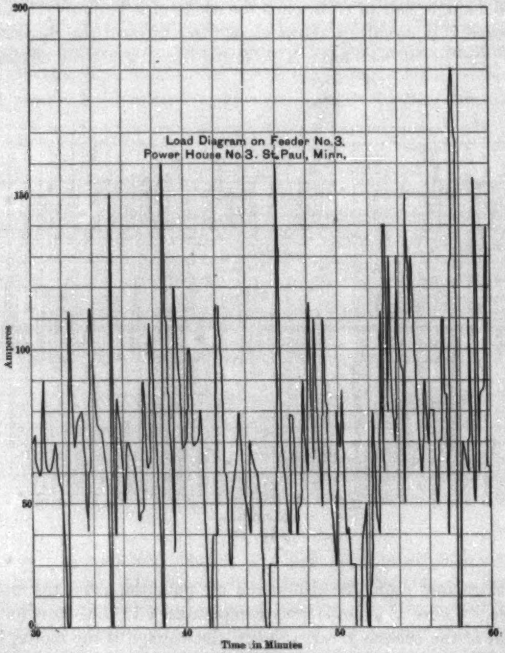
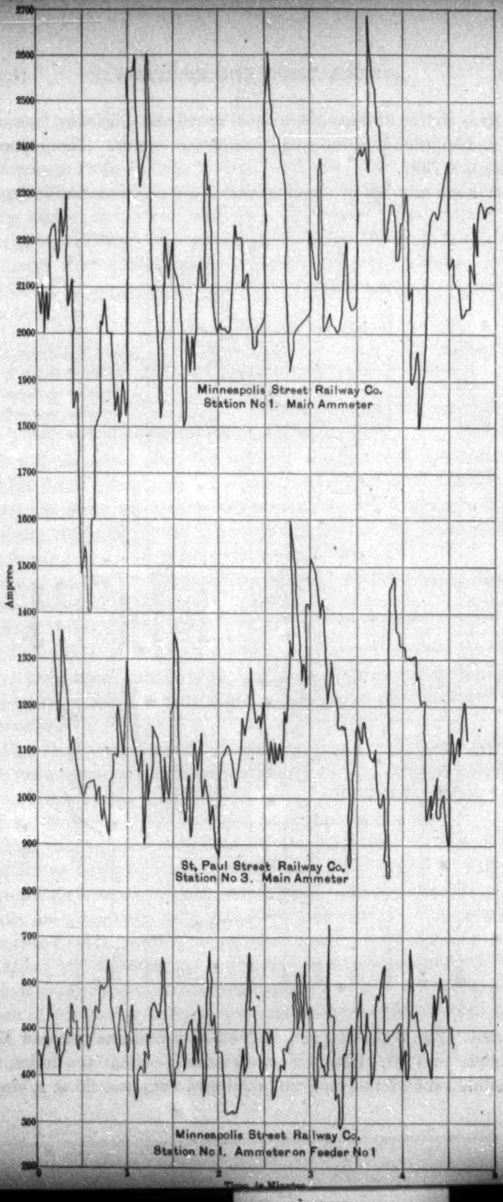


Fig. 10.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Minneapolis Street Railway Co.
Station No 1. Main Ammeter
St, Paul Street Railway Co.
Station No 3. Main Ammeter
Minneapolis Street Railway Co.
Station No 1. Ammeter on Feeder No 1

7 gives curves of currents, torque, speed and efficiency from a 15 H. P. Thomson-Houston single reduction motor. The reduction ratio is 4.7857.

A large number of running tests were made on cars in regular service. Fig. 8 shows a run on the interurban line between Minneapolis and St. Paul. The distance is about ten miles, the schedule time 55 minutes, cars equipped with two 25 H. P. single

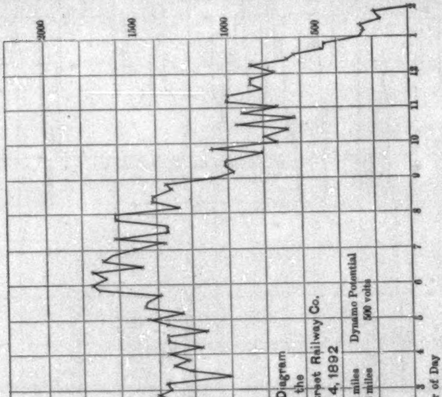

Fig. 12.

reduction motors. Special trains sometimes make this trip in 30 minutes or less. Fig. 9 shows three runs on the University Line of Minneapolis on successive days, with the same car and same driver. The lower curve was taken when the rails were dry and dusty, middle curve with wet rails and upper curve when the ground was covered with snow eight inches deep on the level. The snow plow had been over a part of the track about an hour before. The curves show variation of time, number of passengers being noted.

	Feb. 5.	Feb. 6.	Feb. 7.
Condition of Track	Dry.	Wet.	Snowy.
Number of Stops	38	50	36
Average voltage	445	450	440
Average amperes	15.15	14.9	17.5
Average amperes while working	23.4	20.9	28.1

It will be noted that the current was less with wet rails, although there were more passengers. The load at the station is about the same for a rainy day as for dry, the difference being probably made up by the increased leakage on the lines, and by the larger number of passengers on some cars.

Fig. 9 shows variation of current on a single car. Fig. 10 shows variation of current on a feeder to line with five cars, the average current being about 60 amperes. Fig 10 is plotted from readings taken at intervals of two seconds. The ammeter stops some little time at the high and low points, showing that these are actual values of the current and not due to the inertia of the ammeter.

Fig. 11 gives the results of readings one second apart taken on the main ammeters in Minneapolis and St. Paul and on one feeder in Minneapolis. There were about 190 cars on Minneapolis lines, 100 on St. Paul lines, and 32 on the one feeder.

Fig. 12 gives the daily load of the Minneapolis power house, the curves being plotted from readings of the ammeters taken at ten-minute intervals throughout the day. In this case the power units are unusually large, there being two triple expansion condensing Corliss engines, the constant loss from friction in engines, shafting and dynamos averaging 218 H. P. Indicator cards were taken simultaneously by means of electro-magnetic attachments on each of the twelve indicators, the same current also giving signals to the observers at the ammeters.

A series of 13 tests were made for ascertaining the effect of wetting or greasing the rails on a curve. Two cars were coupled by a spring dynamometer and the pull noted while rounding the curve at ordinary speed, which was kept nearly uniform by means of a brake on the motor car. The average pull on dynamometer is as follows:

	Dry.	Wet.	Greased.
Outer track	500-800 lbs.	500 lbs.	300 lbs.
Inner track	650 "		400 "

The inner rail was wet or greased in each case; but on the inner track the greasing was not so thorough on account of the lateness of the hour. It is noticed that wetting the rail reduces the power required about one-third, and greasing reduces it one-half. Readings of the ammeters on the car correspond in general with the dynamometer readings.

The "bucking" of street car motors is a subject of considerable interest to street railroad men, although it seems never to have been discussed in public. From the silence of the books and papers, and from the ideas current among the railroad employés, the conclusion is reached that little is known about its conditions, causes or remedy. It is therefore thought wise to present the results of observations and tests, with a theory for the cause and remedy. It sometimes happens that an electric car suddenly stops as if by collision. In some cases the car refuses to go further, but usually it goes ahead immediately as if nothing had happened. This is known among railroad men as "bucking." To explain the action it is noted that in common practice the motor fields are connected between the armature and trolley, the armature being grounded at the negative brush. If a second ground occurs on any part of the machine except between the fields and trolley, a heavy current passes to ground, giving an intensely strong field, while the armature is short-circuited by reason of the two ground connections. It therefore acts as a dynamo on short-circuit, and becomes a powerful electric brake that stops the car very suddenly. This agrees with the facts, since a second ground, either from loose connecting wires or broken insulation always causes bucking. Evidently, if the ground occurs entirely outside of the machine, no bucking follows, since the series motor cannot excite its own fields and act as a dynamo unless the armature connections are reversed. It is common practice to connect the two motors together between the fields and armature to equalize the work of the two machines. When a ground occurs in such a case, both armatures act as brakes, and the bucking is more violent than when the "equalizer" connection is not used. A loose reversing switch has been known to reverse the motors accidentally and spasmodically. There are many cases of bucking where no evidence of a second ground can be found. Doubtless some of these may be accounted for by a temporary ground which burns itself out—a common method of removing grounds and crosses. This explanation will not hold in many cases. Extended inquiry was therefore made among railroad men in order to gather as many facts as possible. Some of these reports were incredible at first, but seemed to be honest testimony, and were afterward corroborated by others.

It may be well to state that the term "bucking" is used with some looseness, being sometimes applied to cases where a motor acts badly for a variety of reasons, but as used in this article it is to be taken in the sense above noted. There is a tradition among the car drivers that a motor is very liable to buck if the current is full on while running downhill, but no case of this sort could be substantiated. Cases are reported of motors bucking when the circuit is opened for an instant, as may be caused by the trolley jumping on or off, or by the car passing over a dead rail or over a very dirty track. It is not plain that this is a true case of bucking, but rather of a sudden cessation and re-application of the propelling force.

Sometimes a car will buck at regular intervals. Examination of the motor shows a small bright spot on one brush which rolls to and fro at the contact between brush and commutator. This is intensely bright like a ball of melted metal and grows larger until nearly three-fourths of an inch in diameter, when it suddenly explodes with considerable noise and a blinding flash, and at the same instant the motor "bucks." The car goes on again at once, and the cycle is repeated again in the time required to run from one-half to four city blocks at ordinary speed. In such a case the bucking is stopped by shifting the brushes.

One case is reported where a motor bucked hard, although the brushes were removed so that the armature was entirely disconnected. Here the field was charged by being in parallel with the field of the other motor, and a cross in the armature coils short-circuited it. The foreman of the repair shop where our efficiency tests were made states, that on six or eight occasions he has seen a motor buck because the carbons were not properly placed in the holders. In testing armatures rapidly, the carbons are sometimes placed carelessly, so that the presser foot rests partly on the carbon and partly on the holder. This presses the carbons against the commutator so as to give good contact until the carbon wears off a little, when the presser foot no longer feeds it in. Vigorous flashing occurs, culminating in a buck, the shock of which may jar the presser foot into its proper position, when the motor runs on as if nothing had happened, leaving behind no trace to indicate the cause of the trouble. Cases have been reported where a wire on the track touched the motor as it passed, thereby grounding it and causing it to buck. Similar grounding may be caused by water or mud being thrown upon certain parts of the motor. The motors with Gramme ring armatures are known to buck more frequently than those with drum armatures. These buck worse when run on the "loop," *i. e.* with part of the field coils cut out, some of them bucking regularly every time the loop is cut out. This is one reason why the use of the loop has been abandoned on this road except in a few cases. The machines with commutated fields would buck when the field coils become old, although they showed no signs of being grounded. Bucking became much less frequent when the commutated fields were replaced by a single pair of series coils. It was noticed that bucking was very commonly preceded by flashing at the brushes, though flashing was not always followed by bucking. The explanations offered are various. One is that the iron in the fields varies in magnetic quality so that one motor works harder, and by some means acts upon the other as a dynamo. Another is that one motor becomes reversed while the other does not. Another is that bucking is caused by poor brushes or brush holders. One explanation offered was that a partial break in the circuit, such as may be caused by shaking a loose connection, induces a high electromotive force that causes a flash to span the commutator. In searching for a reasonable explanation for all these cases in which no second ground connection existed, it was noticed in each case that bucking was more liable to occur when the field was comparatively weak; also where the armature reaction was greater. In each case the neutral line is shifted and the coils under the brushes are in an active field. The large current in short-circuited coils further distorts the field, and the sparking at the brush becomes excessive, causing a vigorous flashing. As soon as the counter-electromotive force in the sections leaving the brushes rise above 20 volts, the arc is carried around from one brush to the other, thus grounding the positive brush and short-circuiting the armature, which then acts as a dynamo and causes the motor to buck. Such bucking is usually less vigorous than when caused by a dead ground on account of the resistance of the arc. This explanation covers all the cases noted.

Such being the causes of bucking, the remedies are plain. Armature reaction must be reduced. This militates against the conclusion drawn by Mr. Parshall, in his recent Institute paper, that the strength of field is of little importance, and the windings of the armature should be a maximum. The liability to bucking would be reduced even when there is a large armature reaction, if the armature is divided into a large number of sections.

An obvious remedy for bucking caused by direct grounding would be to connect the fields between armature and ground instead of between armature and trolley. With such an arrangement a ground on the fields would simply cut out a part of the coils and cause the machine to word harder. A ground at the brush between armature and field would cause a sudden forward impulse on account of the field requiring some time to lose its magnetism, and would blow the fuse. A ground on the armature would likewise cause a sudden forward impulse and blow the fuse. With such an arrangement there would be no true bucking except in case of the flashing, as noted. An incidental advantage of this arrangement is that the difference of potential between the field and the frame of the machine is reduced to a few volts caused by a drop through the field; hence there is less danger of grounding the field. The increased difference of potential between armature and core is only a few volts and would not affect the insulation very much.

APPENDIX.

The following tables of data were not accessible to those who took part in the discussion of the paper on "Electric Railway Motor Tests," either at Chicago or New York. They give the data for curves in Fig. 1, and partial data for Figs. 4, 5 and 6, and are now printed in order to make the paper as complete as possible. [EDITOR.]

SPRAGUE MOTOR No. 6.

CLIP 2.

C.	E. M. F.	E. H. P.	Revolutions per minute.	Torque in pounds at axle.	H. P.	Efficiency
4.7	452	.85	105	12.5	1.31	.46
7.2	444	4.28	87	30	2.61	.61
11.4	456	6.97	70	66	4.62	.66
15 7	444	9.31	53	100	5.30	569
22.6	434	13.15	37	155	5.74	.436
28.1	440	16.56	28	200	5.6	338
30.3	444	17.88	19	220	4.18	.234
34.	440	20.08	8	250	2.	.10

CLIP 3.

8.2	458	5.03	92	31	2.85	.57
11.9	446	7.14	79	56	4.42	.618
16.5	444	9.81	62	95	5 89	.60
18.7	444	11.13	57	110	6.27	.563
23.6	444	12.26	45	150	6.75	.557
29.2	444	17.37	35	195	6.83	.49

CLIP 4.

7.0	440	4.13	111	20	2.22	.537
7.6	444	4.53	101	25	2.53	.557
10.6	448	6.38	90	45	4.05	.635
15.5	448	9.27	76	85	6.46	.696
23.3	444	13.86	54	155	8.37	.603
30.8	432	17.84	44	210	9.24	.512

CLIP 5.

10.9	444	6.48	122	32	3.9	.602
12.2	434	7.1	116	40	4.64	.654
15.6	444	9.28	97	65	6.3	.678
20.7	448	12.42	86	100	8.6	.693
31.3	432	18.15	69	165	11.38	.628
44.1	420	24.86	58	260	15.08	.607

CLIP 7.

11.0	450	6.63	134	40	5.36	.808
14.2	442	8.43	106	65	6.89	.805
19.0	444	11.31	85	105	8.93	.789
24.1	442	14.26	80	138	11.04	.774
35.3	444	20.96	70	220	15.4	.733
42.3	432	24.57	63	275	17.38	.708

EDW. P. BURCH.

Discussion.

The President:—I find, gentlemen, that the time at our disposal is very brief. Three papers on important subjects have been presented to you. Among the references which have been made from time to time in these papers, are some to persons who are not present, and they should take part in the discussion where their figures or statements, selective or otherwise, are brought up for criticism or comment. Furthermore, some of these papers have not been accessible for general perusal or criticism until this morning, and the majority of those present are ignorant of their contents except as they have been generally stated here. That is, there has been no time to digest what has been stated, to enable one to make as clear a criticism as he would care to. I particularly feel that myself, because there are two or three matters which I am anxious to criticise in a more extended manner than it is possible for me to do here to-day. It has been proposed, on account of the fact that there is one more important paper to be read, and which has been prepared with a great deal of care, that we should limit the discussion here on railway matters to a few general statements occupying fifteen or twenty minutes, and that the paper on "Oil vs. Air as an Insulating Medium" be read, and that the complete discussion of the railway papers be postponed to the next meeting of the Institute.

The propositions here are radical, one going back to a system that I for one strongly feel is a system of the past, and the other suggesting for consideration a system that I unhesitatingly condemn. It is apparent that under the present conditions, with the time at our disposal, the discussion would not be fair to those who have spent much time in the preparation of these papers.

Most plans must be judged by their practical results. What interests the street car man or the buyer of securities is, first: What will be the investment per car mile operated by an electric system? And, second: What will be the cost per ton mile or per passenger mile carried? Now, all things must stand or fall upon those decisions. A scheme may be beautiful theoretically; it may accomplish everything as desired from a physical standpoint, but it will be eventually accepted or rejected in the financial world according to whether it produces a return upon invested capital greater than some other method of operation.

Before taking it upon myself to make any criticism upon the statements made in these papers, and I do not know that I shall enter into the discussion on account of the limited time, I will ask those who have criticisms and comments to make, to be as brief and direct to the point as possible. The papers are open for discussion.

Dr. Emery:—Mr. President, it would add very much to the interest of the paper on street railway motors, and give us a better opportunity of studying the effects throughout the range of the experiments, if the current used during all the tests were stated. It is so stated only in relation to one test. The names of the makers of the motors are given, which is not the customary way when the makers do not take part in the tests, and a conclusion is drawn, and impressions given which it is not certain would be warranted if the facts were presented in a little different way. To my mind, the curves which are of the greatest value in the study of the effect of a commuted field are shown in Figs. 4 and 5. Unfortunately, the current is not given for either of these figures. Many of the figures show that when the coils are in series the efficiency falls off rapidly at the higher speeds, where it is expected to use such coils in parallel, but the weight of evidence from the diagrams is that the series arrangement is very valuable at speeds less than six miles per hour, or perhaps would prove so at even higher speeds if the relative currents could be studied.

I repeat the request that a complete record of the current be furnished in connection with the other observations.

Mr. Bradley:—Mr. President, I would like to call attention to a patent granted to me March 7th, 1889, showing a similar arrangement to Mr. Leonard's, but not so complete a demonstration of the loads, amperes, etc., required. The arrangement of the generator G and of its field and motor is different from my plan. And I would like to call attention to the fact that with this arrangement or with mine, it is possible to use an alternating current on the motor. In my case, the fields of the generator G are charged from its own armature and the motor in series.

Professor Thomas:—One of the facts in the presentation of Professor Shepardson's paper that struck me most impressively, was his statement concerning the variation of load with a large number of cars running. I am not familiar with the facts upon other roads, and would like to ask what is the experience of others upon roads using a large number of cars. It has been the general belief that increasing the number of cars operated upon a line, tended to a smooth load diagram; but this very excessive want of smoothness in the diagram is astonishing. I would like to inquire whether others have observed the same thing or not.

Mr. Leonard:—I have looked after that point as far as I was able to, but Professor Shepardson's paper is the first one that has given positive data as to the matter that was reliable. I have learned from those who are operating several large stations. and I have been present in such stations as Cleveland, Boston, and other places, and it is a fact that an excessive rise of current will occasionally occur, and it has to be provided for at times. It will only occur perhaps once in a day, or once in several hours, but there will be times when many cars will start simultaneously and there will be a tremendous rise of current due to that fact, and the central station must be in a condition to stand that draft.

[President Sprague turned the chair over to Manager Webb.]

Mr. Sprague:—Mr. Chairman, in the paper which has been presented by Mr. Leonard, advocating a three-motor system for the operation of street cars, he is of course arguing from the position of the inventor, if invention it may be called. His quotations have been selective, and I do not think fairly represent the facts as they exist on railroads to-day. The experimental field through which railroads have gone in the past five or six years, has been somewhat unnecessarily ignored, and certainly the methods of operation which have been tried by them for railroad propulsion in the earlier railroad days, and which are now being tried and adopted by the larger railroad companies, have been absolutely lost sight of.

I will only point out very briefly a few of the salient points which occur to me. Simplicity of machinery is, of course, a desideratum. The tendency must be towards simple mechanical appliances, towards as simple electrical apparatus as possible. Railroad men are discussing to-day the advantages of the gearless, the single-geared and the double-geared machine, and it is not by any means a settled fact that the abolition of gears will characterize the street car motor of the future. It certainly is not settled in my mind, and I do not think it is settled in the minds of those who are building motors. Prophesies are dangerous. If I may go back some four years ago—there was then published a paper by a gentleman now well-known in electrical fields, in which it was stated that a multiple arc system of distribution for twenty cars, on a ten-mile road, was an impossibility. It was therein mathematically demonstrated that not less than 2500 horse power would be required to operate such a system. It so happened, at the time that paper was published, that exactly that number of cars, on that length of line, and with grades running from one to ten per cent. were operated with 175 horse power. At a later date, not less than forty cars were in operation, carrying fully as many passengers as the average car carries to-day on other railroads, and operating them under difficulties which were greater than those which are met with ordinarily, and with only a 375 horse power steam equipment.

I am a defender, I must confess, of the commutated field arrangement of operating motors even if I did originate it. In connection with this I may state that in my earlier practice I also used motors sometimes in series. Street car practice only requires a reasonable variation in speed. Ordinarily we make the running speed seven or eight miles an hour. To do this it is sometimes necessary in the larger cities to operate a car for a short distance at perhaps three or four miles an hour, precisely as the cable cars in the lower part of Broadway will be operated at a maximum of four miles an hour. Then again it is necessary to run at a mean speed of seven or eight miles an hour, and at other times, with a light load and clear block, on a straight track, fifteen miles an hour. With a view of saving power I some five years ago tried the plan of operating two motors in series when running at slow speed. They operated ordinarily perfectly, and so far as reduction of speed was concerned, this plan brought the machines well within the required limits of speed in street car practice. Where that system failed in its application was when the machines were running on a heavy grade or a slippery track, the wheels not being connected, one motor would get ahead of the other and develop all the necessary counter-electro-motive force, and the other would stand still.

Another object we had at that time in doing this was to reduce the current demand from the central station when on grades, but when we started and had a large number of cars in operation, it was found better to use more current and get up the grade more quickly than to keep several cars on long grades running with less individual current and at slower speeds. You must bear in mind that the demands on street car practice are continually increasing. We have increased the necessary power of the motors and we have increased the running time of the machines. What people are anxious to do is not to get up a five or six or eight per cent. grade at one-fifth or one-eighth the speed on a level, but they want to get up that grade as rapidly as possible, and in this way the returns on investments in railroads will increase in a far greater ratio than the saving of the cost of power, or the saving of depreciation, when running at slower speeds.

Power in a motor is not determined by the necessary effort of getting the car into operation. Any motor which will propel the maximum load at its highest efficiency up a grade at a reasonable speed can be so connected and so arranged without extra apparatus as to start that car on any grade and under any load.

Now this system propose by Mr. Leonard, is somewhat similar to certain suggestions which have been made in regard to storage battery work. One would suppose from the claim made that the laws of motor government were somewhat new. It may be said that they are part of the fundamental laws of electro-dynamics and cannot be changed. Five methods have been proposed of operating a car by storage batteries. The first is where there are four groups of cells; the motors wound with fixed coils and left in a fixed condition in relation to each other; and the electro-motive forces at the terminals of the motor varied in the ratio of one, two, three or four, by variously grouping the sets. Here of course, the field varies with the armature but practically is fully excited all the while. The second method is a grouping of two separate batteries in parallel circuit to form a closed loop. Here the operation of the motor is very much like the operation of the arm of a Wheatstone bridge. The terminals are moved up and down, and there is created a variable difference of potential at the terminals of the machine. Another method had been proposed to excite the field separately. That is where we approach this method of Mr. Leonard. Here the field of the motor is excited by connection at the points of maximum potential and the potential at the terminals of the armature is varied. A fourth method is where the batteries are arranged in various groups, and the field excitation instead of being by the current flowing through the armature is from a coil with the initial potential of one of the groups. The switch arrangement is such that the current flowing through the field is constant. Still a fifth method something similar in idea consists in exciting the field by an independent section of the batteries. These are some of the precedents in storage battery work in that line.

I noticed in one part of Mr. Leonard's paper that it was stated that the total energy required for getting under way or running up a grade is no more than that used on a level. It seems to me that he has forgotten that a motor when starting has to exert energy in two ways. It has to accelerate the load and it must provide traction, and this must necessarily require more energy than when running at a fixed speed.

In the equipment which has been suggested by Mr. Leonard, it is said that we should have for the regular motor equipment two 15 horse power motors, with a power converter having two parts of different capacities. In the method shown, there is a simple movement of one switch to detach the motor from one system and throw it on the line, when wishing to take current from the main line and propel the cars directly instead of through the medium of this power converter. In this case, whatever the capacity required, the capacity of the converter in both parts, in the motor and in the generator, must be sufficient to supply the energy furnished by the line at the time of transfer. In other words, if you are going to pull your car up to a point where you are exerting thirty horse power at the car axles, no matter whether you are on a grade or a level, there must be, when you want to swing over on the line, a demand on the dynamo part equal to that, and a greater demand on the motor part, so that instead of having two parts, one of which is only one-third the capacity of the other, there must be a very decided increase of equipment. Increased weight on a car, is one of the greatest objections to-day to the single-geared machine. It is a greater objection to the gearless machine, but the moment that you add five or six thousand pounds, as it might perhaps be, to a car, in additional apparatus, there will be some very practical objections raised to a system of that kind.

I think it is well to act on the suggestion that has been made, and postpone this discussion until the next general meeting in New York. Still, if any members desire to make brief remarks, I trust we have the time to hear them. I did not intend to take so much of the limited time.

Mr. Leonard:—Mr. Chairman, in answer to some of the points that Mr. Sprague has raised in his discussion, (if discussion it can be called) of my paper, I would say that the principal corrections I wish to make, are some errors on his part in assuming certain points in connection with my article. One of the last of these points, which if fresh in your mind is the assumption on his part that the motor part of the power converter is only one-third in capacity of the propelling motor. He also spoke of the necessity of having power in the converter equal to the maximum power required at the axles in case we are going to swing over from the motor generator to the line, but it is only contemplated that such a switching from the motor generator to the line will be done at a time when we are operating at full speed on a level, which is not the time when the largest amount of power is required on the system.

As to the full speed upon a grade, and the necessity of going at full speed upon a grade, I will call attention to the fact that while the street car motors of to-day go at as full speed as possible upon a grade it is but a fraction of their full speed.

The motor generator, instead of weighing five or six thousand pounds will in reality not exceed three thousand pounds.

Mr. Sprague also incorrectly assumed that I said that no more power would be required in starting upon a grade than running at full speed upon a level. The current in the armature in the motor will necessarily be exceedingly large to accelerate the load. But during the time that we are taking that very large current through the armature of our motor to accelerate the load starting on a grade, it is being produced by the power converter at perhaps thirty or forty or fifty or sixty volts, and therefore a small amount of energy is taken from the line.

Mr. Sprague has said something about other points in the system as regards anticipation by storage batteries and other converters. That is rather foreign to the discussion of the paper and is a matter which, if it ever should be discussed, should be discussed in the courts.

Mr. Sprague:—A hurried discussion of a paper of this kind is apt to be misunderstood, motives are liable to be misconstrued and somewhat incorrect conclusions arrived at. Apparently, there is a misunderstanding as to the conclusions of this discussion, and it had better be postponed until there is more time to be devoted to it.

Mr. Martin:—I move, Mr. President, that we adjourn this discussion at this time, and give Dr. Williams an opportunity to read his paper. [Carried.]

The following paper by Dr. J. B. Williams, of New York City, on "Oil vs. Air as an Insulating Medium," was then read by the author:

A paper read at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 8th, 1892, President Sprague in the Chair.

OIL VERSUS AIR AS AN INSULATING MEDIUM.

BY JAMES BOWSTEAD WILLIAMS, M. D.

All who have attempted to enlarge in the smallest degree the bounds of our knowledge, recognize the importance of the faithful collection of facts which, though seeming separately insignificant, may yet when brought together resolve themselves into distinct and important scientific truth. By combining the results of investigations carried on from different standpoints, we obtain a view of the subject more complete than any single mind, however masterly, could give us. Hence every item of information has its importance, as it may prove to some other investigator the exact clue needed to solve some long-pondered problem. Acting upon this principle, the writer now offers an account of some experiments made by himself to determine the comparative insulation resistance of oils and air; and it has appeared to him that the results obtained from these experiments are specially important from the fact that *all* of the apparatus used was absolutely insulated, and that the only leakage that could take place was through or across the various substances experimented upon.

The relative powers of oils and air to resist disruptive discharges and also to insulate alternating currents, will not be considered at this time, as the writer proposes to reserve those subjects for a future paper.

The instruments used in the experiments from which the results about to be set forth in this paper were obtained, were an electrometer, a discharge key, a 6 inch spark coil and a Grenet cell to operate it, and apparatus for holding the oils and other substances while they were being tested.

The electrometer was a special form of quadrant electrometer, designed and made by the writer in San Francisco, in 1886, for the purpose of testing insulating material generally, and particularly for determining the comparative I. R. of very short pieces of insulated electric conductors. Its quadrants and needle system are similar to those of Lord Kelvin's electrometer, but the mounting of the quadrants and the needle suspension are different. No glass is used for insulating the various parts, this insulation being effected entirely by means of the best quality of hard rubber which is well paraffined. The suspension is of the bifilar type, is about six inches in length and is provided with adjustments. The replenisher and gauge are contained in a separate vessel, and when they are electrically connected with the electrometer proper the combination can be used in the same manner and for the same purposes as the "White pattern" quadrant electrometer. The insulation of this instrument is so perfect that its quadrants, when charged to potentials of about 6,000 volts, will retain their charge for several hours without any perceptible leakage.

Before the electrometer is used for comparative testing, it is so adjusted that the quadrants, when fully charged, will be charged to any desired potential of from one to six thousand volts.

The discharge key resembles those in common use, the only difference being in the trigger, which is so constructed as to insulate as highly as any of the pillars of the key.

This instrument was used when time measurements were taken.

All of these instruments were illustrated, and the methods of using the same in making comparative tests of insulated wires explained, in the *Electrical Engineer* of March 30 and April 6, 1892, in a serial by the writer, entitled "Insulated Electric Conductors."

With this apparatus, using the same potential as that used in these experiments, the "dielectric after-working" of Boltzman can be demonstrated, if a few inches of a highly insulated wire are tested. The variation of the capacity of a condenser having a small bare wire for its inner coating can also be shown.

The apparatus in which the oils were first tested consisted of a glass cup to contain the oils and two circular brass disks, parallel to each other, which were immersed in the oil and between which layers of oil could be formed.

These disks were each 2½ inches in diameter, one of the disks, the lower, being fixed and the other, the upper, movable. The movable disk was soldered to the end of an adjustable screw passing perpendicularly through the center of the hard rubber cover of the cup, which was 3 inches in diameter. The fixed disk was supported at the middle of a U shaped device formed of hard rubber tubes, both of the arms of which were cemented to the cover of the cup. One of these arms passed through the cover and extended about 6 inches above it, and contained the wire which was soldered to the fixed disk. The disks were absolutely insulated from each other and from the earth, and were separated one-thirty-sixth of an inch by each revolution of the adjustable screw. The apparatus in which the second tests of the oils were made. will be described later.

The method of testing the oils was briefly as follows. The needle of the electrometer, the induction plate, and one pair of quadrants were electrically joined to the frame of the instrument and to earth, and the second pair of quadrants joined to the upper disk.

The lower disk was connected with the binding screw of the upper contact of the discharge key, the lever of the key being depressed and held against the lower contact of the key by the trigger. The lever of the key was joined to earth. Oil was then poured into the cup, sufficient to cover the upper disk when this disk was separated from the lower disk by one-half inch, this being about the effective range of the screw.

The quadrants, and of course the upper disk were then charged by sparks from the coil, to a potential of about 2,300 volts, the illuminating apparatus and scale having previously been adjusted so that when the quadrants were fully charged the image of the cross-wire would settle at the upper end of the scale, at 300, say.

As the quadrants and disks were absolutely insulated at the potential used, there could be no leakage of the charge as long as the lower disk remained insulated from the earth; but as soon as this disk was connected with the earth, either through the medium of the discharge key or by having its electrode touched with a wire fastened to the gas pipe, leakage would take place through the layer of oil, whatever its thickness might be between the disks; and the rate at which this leakage took place was indicated by the rate at which the image of the cross-wire passed down the scale, *i. e.*, by the rate at which the electrometer was discharged.

Before entering into a description of the experiments I wish to refer to a phenomenon which occurs when all good insulating material is tested by means of charges of high potential, if the material forms the dielectric of a condenser.

Take any highly insulated wire and form say four inches of it into a condenser by closely wrapping the dielectric with tinfoil; and then carefully paraffin the ends to prevent surface leakage. The condenser thus formed, is absolutely insulated from earth, and connected with the electrometer and discharge key in the same way as the condenser formed by the two disks and the layer of oil between them, *i. e.*, by connecting the wire with the quadrants, and the tinfoil with the upper contact of the key. The quadrants are then charged to potentials of 2,000 volts or more. This causes the needle of the electrometer, and with it the image of the cross wire to be violently deflected to the right or left, depending upon which pair of quadrants is used. The needle then vibrates, but each succeeding vibration becomes quicker and shorter until the image finally settles at 300, say.

Now, if the trigger of the key be made to release the lever, connection is made between the tinfoil and the earth. A sudden "drop" of the image then occurs, which is due to the rapid absorption of a portion of the charge by the dielectric. The needle again vibrates, but the image soon settles at 250, say, if the wire be *very* highly insulated, and will remain at 250, thus showing that there is practically no leakage. But if the wire be a little less highly insulated, the drop still occurs, but the image simply "hesitates," as it were, at 250, say, and then passes slowly down the scale in proportion to the amount of leakage which is taking place through the dielectric. If the ground circuit be now broken by depressing the lever of the key, the quadrants again charged to the same potential as before, and the ground circuit established, the drop is then very slight, oftentimes nil, and the falling of the image along the scale is now due to the leakage *through* the dielectric and not to absorption *by* it.

No absorption of the charge by the dielectric can be detected as long as the outer coating of the condenser is insulated. If the image *should* move, there is leakage in some portion of the testing apparatus, which *must* be remedied if the tests are to have any value. This drop is observed only when the best kinds of insulating material are tested by the method just described. It cannot be detected when the poorer qualities of insulating material are subjected to the same tests on account of the rapid leakage through them.

The potential of the charge was determined in the following manner:—The length of spark produced by suddenly discharging the quadrants in air was approximately determined by bringing a pointed wire, held in the hand, close up to the electrode of the charged quadrants. Two bright tin disks, each 1⅛ inches in diameter, were then rigidly cemented upon the ends of two stout hard-rubber tubes which were well paraffined. These tubes were fastened together at the other ends in such manner that by inserting a wedge between them, the disks could be separated or made to approach each other and still remain substantially parallel at the striking distance of the spark. One of the disks was joined to the quadrants and the other to the discharge key. The quadrants were then fully charged and the key closed. If a spark passed between the disks, they were separated a little further apart and the test repeated. If a spark did not pass, the disks were brought a little closer together and the test again repeated. After several trials, the disks were separated at a distance corresponding to the exact length of the spark.

Slips of thin writing paper were then inserted between the disks until they exactly filled the space between them and the thickness of the layer formed by the paper measured with a Brown and Sharp's micrometer gauge.

A suitable spark electrometer provided with parallel plates would have given the spark length by a direct reading, but the writer did not have the time either to make or procure one.

The length of spark was found to be .03 of an inch. From tests made with a Thomson static voltmeter, this length would correspond to a potential of about 2300 volts. (A spark .026″ long = 2,000 v.)

But whatever the potential of the charge may have been—whether it was two, three or five thousand volts—the same potential was used in *all* of the experiments described in this paper.

The oils tested were rosin, paraffin, linseed, cotton-seed and castor. The rosin oil was what is known as London oil. The paraffin oil had a specific gravity of 24° B. All of the samples were procured from wholesale dealers, and were declared to be of the best quality.

The first set of experiments include tests of paraffin and rosin oils, air, and a specimen of a well-known insulated wire. This last test was made to show what took place when the dielectric of insulated wires was tested by means of charges of high potential, and also to make a rough comparison between specific resistance of this dielectric and that of the oils about to be tested.

The temperature during the tests was 82° F., and the humidity, as indicated by a Lambrecht's polymeter, 80.

It was first ascertained, by testing, that the quadrants used and the disks were absolutely insulated. Each separate experiment is designated by a number.

1. Paraffin oil was then poured into the cup and the disks separated one-ninth of an inch. The discharge key was not used in these tests in order to save time, a wire fastened to the gas pipe being used to ground the lower disk. The connections were properly made and the quadrants fully charged. As soon as the image of the cross-wire had become stationary, the electrode of the lower disk was touched with the ground wire. The image immediately flew down the scale, apparently as fast as if the *upper* disk had been touched with the ground wire. That this sudden flight of the image was not due to the "drop"—*i. e.*, to absorption of the charge or the sudden rush of the electricity *into* the oil and not *through* it—was proven by repeatedly charging the quadrants, and then discharging them through the oil. In fact, the amount of leakage was so great that, while the lower disk was kept grounded, the quadrants *could* not be charged.

If leakage through a very thin layer of any good dielectric be not *too* excessive, the quadrants can always be recharged, wholly or in part, even while the leakage is going on; but, in this case, the leakage was too great even when sparks were rapidly poured into the quadrants.

2. The disks were next separated one-fourth of an inch, but no better results were obtained than at first.

It is evident that there must have been less leakage through a layer one-fourth of an inch thick than through one one-ninth of an inch thick. But, in both cases, the movements of the image were too rapid to be timed.

3. The cup was then emptied of the oil and a few drops introduced between the disks, and still no different results were obtained.

4. The oil was then heated to about 280° F., to expel any moisture that might be present. But this made no difference in the tests; there was still great leakage, even after the oil had cooled.

5. Rosin oil was then subjected to the same tests as the paraffin oil, but no better results were obtained from it than from the paraffin oil.

6. All oil was removed from the disks and a layer of air one-eighteenth of an inch thick tested. The drop was about five divisions, but after that the image remained stationary, thus showing that the insulation resistance of the air, at the potential used, was absolute.

7. A piece of the insulated wire before referred to, about five inches long, was connected with the quadrants and the quadrants charged. The edge of a pocket knife blade was then held transversely upon the dielectric, when the image passed rapidly down the scale. This experiment was repeated and the same result obtained. The ends were then paraffined and the specimen again tested. Result almost the same as before paraffining, thus showing that there had been some surface leakage at first.

Many of the insulated wires now in use yield the same results when tested exactly as this present sample was tested. But there are also many which give excellent results when thus tested, and there are a few in which there is very little leakage even when several feet are formed into a condenser and tested at this same potential.

It is important to remark that tests of different sections of the same wire, having a vulcanized dielectric, may not always give the same results. For example, a three inch piece of a certain wire may have a very high I. R., while the next three inches may have an I. R. much lower than that of the first. Where this is the case, it will almost always be found that the dielectric of the second sample contains some flaw, or else has particles of wood, metal, etc., embedded in the mass.

By using this method of testing, the superiority of one insulated wire over another, whenever samples a few inches long can be obtained, can easily be demonstrated. It will readily be seen that this fact may become of great importance in legal or other cases in which the comparative insulation resistance of wires is involved. Decisive results can be obtained in many cases from lengths of two or three inches only.

The second set of experiments included tests of all five oils, paraffin, wax, ordinary air, and steam. The temperature was 80° F., and the humidity 75.

In the second set, the layers of oil were formed between an upper disk nine-sixteenths of an inch in diameter, and a lower disk 1½ inches in diameter. This lower disk was the flat bottom of a tin box. Its sides were covered with a thick layer of paraffin wax, so that no leakage could take place except through the layers of oil. The disks were mounted the same as the larger disks, and were absolutely insulated from each other and from earth.

1. Paraffin oil was first tested in a layer ¼ of an inch thick, but the falling of the image was too rapid to be accurately noted. A layer ½ inch thick was then tested and gave a fall of 207 divisions in one minute.

2. Rosin oil tested in a layer ¼ of an inch thick, gave a little better results than the paraffin oil; but when a layer ½ inch thick was tested, the fall was 177 divisions in one minute.

In these two tests of the two oils all effects due to the drop were eliminated by repeated chargings, so that the fall due to leakage alone was noted.

3. Linseed, cotton-seed and castor oils were each tested in ½ inch layers, but as the fall of potential was so much greater than that of the other two oils no further tests were made of them.

Of the three, castor oil had the lowest specific resistance; and there was but little difference between the linseed and the cotton-seed oils, what little difference there was, being in favor of the linseed oil.

4. A layer of paraffin one-sixteenth of an inch thick and free from air bubbles, was next formed between two tin disks, each 1⅝ inches in diameter, and tested like the layers of oils. When the ground circuit was closed the drop was just 5 divisions. The quadrants were again fully charged, and the falling of the image noted. This fall was one division in two minutes, or 30 divisions in one hour.

5. A brass tube 10 inches long and five-sixteenths of an inch in diameter was next used for a receptacle for oil. In the center of this tube was a straight copper wire one-sixteenth of an inch in diameter, the wire being kept concentric with the tube by means of two hard rubber plugs with holes through their centers. The tube contained rosin oil. The wire was connected with the quadrants, and the tube absolutely insulated from the earth and neighboring objects.

The quadrants were then charged and the tube grounded. The image passed down the scale too rapidly to have its rate of falling timed. The hard rubber plugs were carefully paraffined so that all of the leakage took place inside the tube.

A torrent of sparks was poured into the electrometer, but it could not be charged on account of the rapid leakage through the oil.

6. Three plates of glass each 3 by 4 inches and one-eighth of an inch thick, were carefully cleaned and then dried over a lamp. Before they had cooled, one was covered with a film of rosin oil, one with a film of paraffin oil, each film having an area of about two square inches, and the third had a film of paraffin wax about one inch square formed on it. A No. 20 copper wire was fastened to the quadrant electrode, and its free end placed in contact with the center of the film on the glass. The glass was insulated and the quadrants charged. The image remained stationary as long as the glass was insulated, but as soon as any conducting body, the hand for instance, came in contact with *any* portion of the glass plate, leakage over the film of material on the glass occurred. In each case there was a slight drop, but the quadrants were immediately recharged. The rate of falling of the image with the glass plates covered with the oils was practically the same, the fall being about 5 divisions to the second; but with the paraffin wax the image remained stationary.

Repeated chargings failed to alter the rates of falling of the image.

7. The last tests were made with air and steam, and in both tests the layers were formed with the device used to ascertain the potential of the fully charged quadrants. In the first of these tests, the disks were separated a trifle more than .03 of an inch.

The drop in this test was just two divisions, but after that the image did not move in 20 minutes. The writer has made this same kind of test in former years, and has allowed the instruments to stand for hours, and yet there would be no leakage through the layer of air.

The last test was made with a layer of steam of the same thickness as the layer of air in the test just described.

The quadrants were fully charged and the ground circuit closed. Steam was then generated in a test tube and the mouth of the tube held directly beneath the disks. This enveloped the disks in steam and also formed a layer of steam between them. As soon as the layer of steam was formed, leakage took place through it, and the leakage increased as moisture was deposited upon the disks (thereby reducing the distance between them) until a bridge of water was formed, when, of course, the quadrants were immediately discharged. From many tests, the writer found that, as nearly as he could determine, the rate of fall, when the layer was composed entirely of steam, was about 20 divisions in one second.

He would state, however, that as he was entirely alone during these experiments upon steam, and as his attention was necessarily divided between several parts of the apparatus, each of which required continual watching, he cannot vouch for the accuracy of these figures; but he has every reason to believe that they are approximately correct.

Several times during the tests, oil was accidentally smeared upon the surface of the paraffined hard rubber, and in every case the hard rubber would have to be recleaned and reparaffined before its surface insulation was restored. From a careful consideration of the foregoing tests, the results thereof appear to the writer to prove, when continuous high tension currents *alone* are concerned:

1. That air, even when it has a humidity of 80 per cent., is infinitely superior to oil for insulating conductors carrying such currents.

2. That pure paraffin is far superior to oil, both for insulating conductors and for preventing the escape of the current across the surface of glass

3. That air having a humidity of 80 per cent. is superior to paraffin as an insulating medium.

4. That oil will not prevent leakage of the current across the surfaces of glass.

5. That whenever oil covers the surface of hard rubber, even when the latter is paraffined, it destroys the high surface insulation of the paraffined hard rubber, that is, the low surface insulation of the oil has been substituted for the higher surface insulation of the hard rubber, when used bare and clean, or when covered with paraffin.

The writer would state that the results of the present tests, which were made in New York City, correspond substantially with results obtained by himself and his brother, Mr. R. W. Welty, while they were making the investigation of insulating material and its applications, which is now being described in the serial before referred to.

A few words relative to the use of the oils in practice, may not be amiss here. To use oil for insulating conductors, there must be an enclosing vessel of iron, or its equivalent, for the oil and conductors. If long tubes are employed for this purpose, two things are indispensable—plugs in the ends of the tubes and means for keeping the wire or cable concentric with the tube, and the individual wires of a cable separated from each other. For the latter purpose, fibrous material is probably the most convenient means. Jute, when pure and dry, is an excellent fibre, both for the oils when used alone and when thickened with rosins, etc., for by reason of the peculiar nature of the fibre, a larger amount of insulating material can be placed within a given space than if a closely-lying vegetable fibre is used ; and there will also be fewer points of contact of fibre between the wires and the inside of the tubes. Experiments made by the writer show that there is but little difference in the specific resistance of different kinds of vegetable fibre when the fibre is clean and absolutely dry. The resistance of such fibre is high as long as it remains in this condition. But it is almost impossible to keep it so. It may be a matter of surprise to many to learn that cotton which has been dried, and which therefore possesses good insulating properties, loses these properties in less than ten minutes if exposed to a damp atmosphere.

Whatever means are used for the purpose last above stated, they should have a high specific resistance and allow the use of as much oil as possible.

The plugs must be formed of insulating material which is not injuriously affected by the oils. The best hard rubber, or some special vulcanized rubber or gutta percha compound, appears to be the best for this purpose. Paraffined wood is very apt to be of little use in a short time, for no matter what kind of soft porous material is paraffined by allowing it to absorb the melted wax, the wax rarely hardens in a solid form, but usually contains numerous small spaces and these spaces absorb moisture by capillary attraction. It is for this very reason that a dielectric which is formed of fibre saturated with paraffin soon loses its high insulation when exposed to the air. The microscope will demonstrate the presence of these minute air spaces.

All oil should be carefully heated, sufficient to expel all moisture and air, before it is used. If paraffin oil is used, it is better to subject the oil to a comparatively low temperature and exhaust the air by means of a vacuum than to heat the oil to a high temperature. After the moisture has been removed, the oil must be kept excluded from the air or it will again absorb moisture.

If oil is used as a covering for insulators made of inferior glass or porcelain, to be of value it must contain no moisture or dirt; and if used in damp and dusty localities, means must be provided for the self-renewal of the oil, so that fresh surfaces of oil may constantly be exposed. But if insulators had surfaces formed of glass which contained a large proportion of pure silica, better insulation would be effected, if these surfaces were not covered with oil. The writer once found a molded glass vessel, having about the same dimensions and area as the "pony" insulators, which had a surface insulation vastly superior to that of the "pony" insulators. If insulators were made wholly or in part of *such* material a covering of oil would be worse than useless, for experiments have shown that when glass or other material does prevent surface leakage, covering it with oil at once ruins the high surface insulation.

Experiments made upon the expansion of paraffin and rosin oils show that when paraffin oil is heated to 280° F., a column of it 3½ inches long and thee-fourths of an inch in diameter, expands one-half of an inch; and that if a column of rosin oil of the same dimensions is heated to the same temperature, it expands three-eighths of an inch. These facts should be taken into consideration when using these oils in pipes, for if the pipes are full and the oil gets heated, it will expand and probably either expel the plugs or exude at the ends. If it exudes, dust and moisture will collect upon the film of oil and cause surface leakage. This leakage at the ends of pipes containing telephone or telegraph wires, might not be detrimental to the working of the lines, but if the pipes contained wires carrying high tension currents, serious trouble might arise.

The efficiency of oil to fill holes in dielectrics depends greatly upon circumstances. If the holes or gaps should be formed in the dielectrics of wires conveying weak currents, the oil, if not too sluggish, would have plenty of time to move into and fill such spaces; but if in the dielectrics of wires conveying strong currents; the oil must move very quickly or a disastrous burn-out is the result. And not only this, but the oil, becoming hot expands, and the pipes, if of lead, are apt to be strained or broken by the increased pressure from within, outwards. If the burn-out occurs near the ends of the pipes, the plugs may be expelled. If the filling of spaces were the *only* thing required of the oils, the lighter ones would accomplish this purpose better than the denser ones.

But the heavier oils—*e.g.*, heavy and light paraffin oils—are much better for insulating purposes than the lighter.

In conclusion, the writer would state that he feels warranted in believing, unless the contrary can be demonstrated, that dry air, if employed to form a portion of the dielectrics of insulated conductors and if the conditions upon which its highest efficiency depends are fulfilled, is an insulating material superior to oil and even paraffin. In making this statement, he takes it for granted that his method of testing—a method which he does not remember ever to have seen published before his use thereof over five years ago—yields results, the accuracy of which can be relied upon. This belief has been established after several years of experimental work upon insulating materials and the manufacture of different kinds of insulated conductors. Certainly air is cheaper than any other insulating material, and it is not destroyed or injured by intense currents. Burn-outs do not affect it, and if it can be kept dry, which is the great difficulty attending its use, the conductors which it insulates will have a high insulation resistance, and a low electrostatic capacity—two things which are of the greatest importance in electrical engineering.

THE PRESIDENT:—It is unfortunate that this paper was not presented at a time when there could have been a full discussion upon the questions that arise. I am sure that some of the conclusions which have been arrived at by Dr. Williams are entirely contrary to the generally accepted belief, and there are a number of members of the Institute who have made experiments on high potentials and oil insulators. Since, however, Dr. Williams intends to repeat his experiments in New York, and inasmuch as the discussion can take place at that time, when it will be more instructive than the very brief discussion which could be had now, I will not call for remarks on the paper, but ask that members of the Institute, appreciating the importance of it, will be prepared at the next meeting. [Adjourned.]

The following paper by Professor Carhart, was read on June 6th. Owing to the absence of the author in Europe, the revised copy was not received in time to be printed in regular sequence.

A paper presented at the General Meeting of the American Institute of Electrical Engineers, Chicago, Ill., June 6, 1892, Vice-President Lockwood in the Chair.

RELATION BETWEEN THE ELECTROMOTIVE FORCE OF A CLARK CELL AND THE DENSITY OF THE ZINC SULPHATE SOLUTION.

BY PROFESSOR H. S. CARHART.

It is now well known that there is an inverse relation between the E. M. F. of a Clark cell and the density of the solution of zinc sulphate; that is, an increase in the density of the solution is accompanied by a decrease in the E. M. F. of the cell. This relation for the Daniell cell I investigated ten years ago.[1] A similar investigation for the Clark cell was recently undertaken at my suggestion by two students in my laboratory, Mr. T. E. Barnum and Mr. E. A. Cheney. They have also undertaken to determine whether the temperature coefficient changes with a change in the density of the solution, provided there are no crystals of zinc sulphate present.

The importance of this investigation is connected with the fact that the temperature coefficient of the Clark cell containing zinc sulphate crystals is almost exactly double the value exhibited by cells containing no crystals. Lord Rayleigh's value is 0.00077 at 15° C.;[2] Dr. Fleming found 0.00082 for cells with crystals. My own cells, containing no crystals, have a coefficient of 0.000386 per degree C. at 15°.[3] Dr. Kahle in Berlin has obtained a value almost identical with that of Lord Rayleigh for cells containing zinc sulphate crystals. Half of this so-called temperature coefficient is due to the dissolving of the crystals with rise of temperature, or conversely to recrystallization when the temperature falls. A large change in the E. M. F. of a standard cell with change of temperature is clearly a disadvantage, especially because of the difficulty of ascertaining the temperature with exactness. Another incidental but important disadvantage, due to the presence of the crystals, is the slowness with which the cells reach their equilibrium after change of temperature. Time for diffusion must be allowed before the cell assumes a value corresponding to its new temperature. Hence such cells exhibit a time lag, sometimes amounting to several days, and differing for different cells. Such considerations as these led me to make a cell without zinc sulphate crystals. It is then important to know what variations in E. M. F. will follow any departures from the density of the zinc sulphate solution adopted as the "normal" solution; in other words, what decrements in the E. M. F. of the cell correspond to given increments in the density of the zinc sulphate solution.

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[1] *American Journal of Science*, Vol. xxviii., Nov., 1884.

[2] *Phil. Trans. Royal Soc.*, Part ii., 1885.

[3] *American Journal of Science*, Vol. xxxviii., Nov., 1889.

A series of twenty-four cells, two for each density of solution, were made, and their E. M. F. at one temperature was determined by comparison with a Clark cell containing crystals and kept very near 15° C. The E. M. F. of this comparison cell was assumed to be 1.435 volts at 15°.

Table I shows the results of such a comparison.

TABLE I.

Per cent. of $Zn\,S\,O_4$ in solution.	Density of solution at 20° C.	Resistance to balance.	Electromotive force.	Comparison Cell.	
				Resistance.	Temperature.
5	1.036	9728	1.4850	9400	15°
10	1.062	9683	1.4777	9404	14.7
15	1.096	9645	1.4724	9400	15
20	1.131	9621	1.4686	9400	15
25	1.164	9595	1.4647	9400	15
30	1.206	9566	1.4605	9400	15
35	1.249	9543	1.4567	9400	15
40	1.293	9525	1.4540	9400	15
45	1.343	9506	1.4488	9415	15
50	1.383	9459	1.4444	9400	15
Saturated at 15° C.	1.417	9453	1.4398	9423	14.5
Saturated at 20° C.	1.429	9420	1.4380	9400	15

The comparisons were all made by the potentiometer method, and the "resistance to balance" in each case was the resistance between the points of derivation of the derived circuit, the main circuit having a constant resistance of 10,000 ohms.

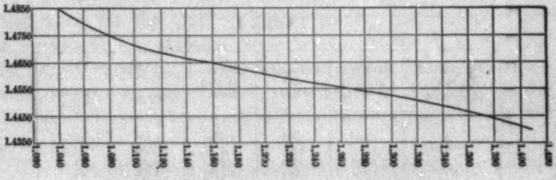
Plotting the electromotive forces of the table as ordinates and the corresponding densities as abscissæ, the curve of the diagram was obtained. The minimum rate of change of E. M. F. occurs with a density of about 1.265.

The further question of temperature coefficient was investigated by determining it for each cell. For this purpose the cells were placed in large test-tubes which were in turn immersed in a water-bath kept at the desired temperature.

Table II. gives the data for the five per cent. cell.

TABLE II.

Comparison Cell.		Five per cent. Cell.		
Temperature.	Resistance.	Temperature.	Resistance.	E. M. F.
14.5°	9414	15.1°	9740	1.4850
15.5	9413	16.6	9737	1.4840
......	......	17.3	9730	1.4837
14.7	9411	18.4	9726	1.4831
......	......	20.0	9720	1.4822
14.7	9412	21.2	9716	1.4815
......	......	22.0	9713	1.4810
14.7	9412	23.0	9711	1.4806
......	......	24.0	9706	1.4800
14.7	9412	26.0	9697	1.4787
......	......	29.0	9687	1.4771



RELATION BETWEEN E. M. F. AND DENSITY.

If these temperatures and electromotive forces are plotted as coördinates the resulting curve, exhibiting the relation between them, does not differ sensibly from a straight line. We may therefore write

$$E_t = E_{15}\,[1 - a\,(t - 15)]$$

as the formula connecting E. M. F. and temperature. The value for *a* which satisfies the observations of the table is 0.00039.

Table III. contains the data for the coefficient of the 10 per cent. cell.

TABLE III.

Comparison Cell.		Ten per cent. Cell.		
Temperature	Resistance.	Temperature.	Resistance.	E. M. F.
14.7°	9404	15.0°	9683	1.4777
14.5	9405	16.0	9680	1.4772
15.5	9406	17.8	9677	1.4760
15.3	9404	19.4	9672	1.4756
14.9	9405	20.6	9665	1.4747
15.0	9406	21.6	9663	1,4742
14.8	9406	23.2	9657	1.4734
14.9	9408	24.2	9655	1.4727
14.9	9408	26.1	9647	1.4715
15.0	9407	27.5	9643	1.4710
15.0	9407	29.6	9637	1.4700

These observations give a slightly smaller value of *a* than those on the 5 per cent. cell.

Each cell was tested in a similar manner, and the variation from a temperature coefficient of 0.00039 was small and probably within the errors of observation of the experimenters. It appears probable then, that the temperature coefficient of Clark cells not containing crystals is independent of the density of the zinc sulphate solution, and is only half that belonging to cells with zinc sulphate crystals in excess.

DISCUSSION.

THE CHAIRMAN [Vice-President Lockwood]:—Each one of us who has performed electrical measurements must have been considerably chagrined to note the discrepancies resulting where no discrepancy should exist, in operating Standard Cells and apparatus worked by them; and any information which we and the world at large can get which will tend to make the Standard Cell anything less of a *Standard Sell* than it often is, must unquestionably be of great value.

MR. WILLYOUNG:—Mr. Chairman, I think Professor Carhart has not stated the case with entire fairness to himself. A great many very thorough tests have been made on those cells, and I think the maximum inaccuracy obtained for some time past has been only about four parts in ten thousand, while a number of cells that have recently been tested at Johns Hopkins', show an actual variation of only two parts in ten thousand; so that the cell is really, I think, much more accurate than Professor Carhart has claimed it to be.

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